# Lean Six Sigma Black Belt

## Volume 2

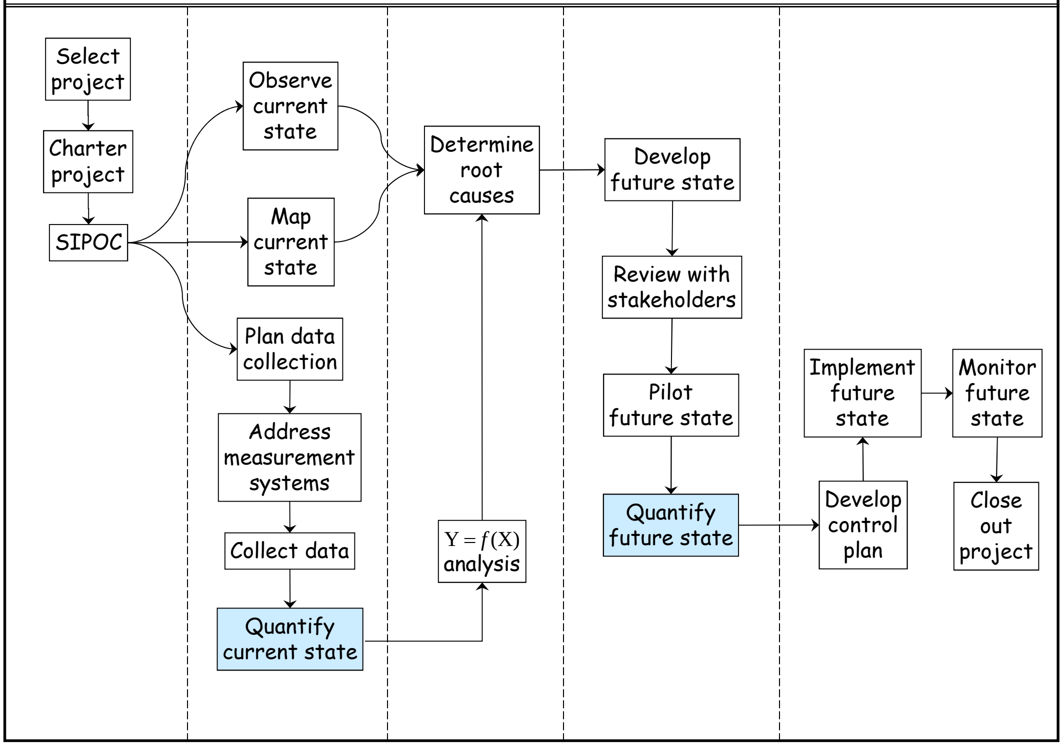
Written by
Russell A. Boyles, PhD
Six Sigma Master Black Belt
*russell.boyles@gmail.com*
971-226-8676

Presented by
ETI GROUP
Oregon: 503-484-5979
Washington: 360-681-2188
*www.etigroupusa.com*

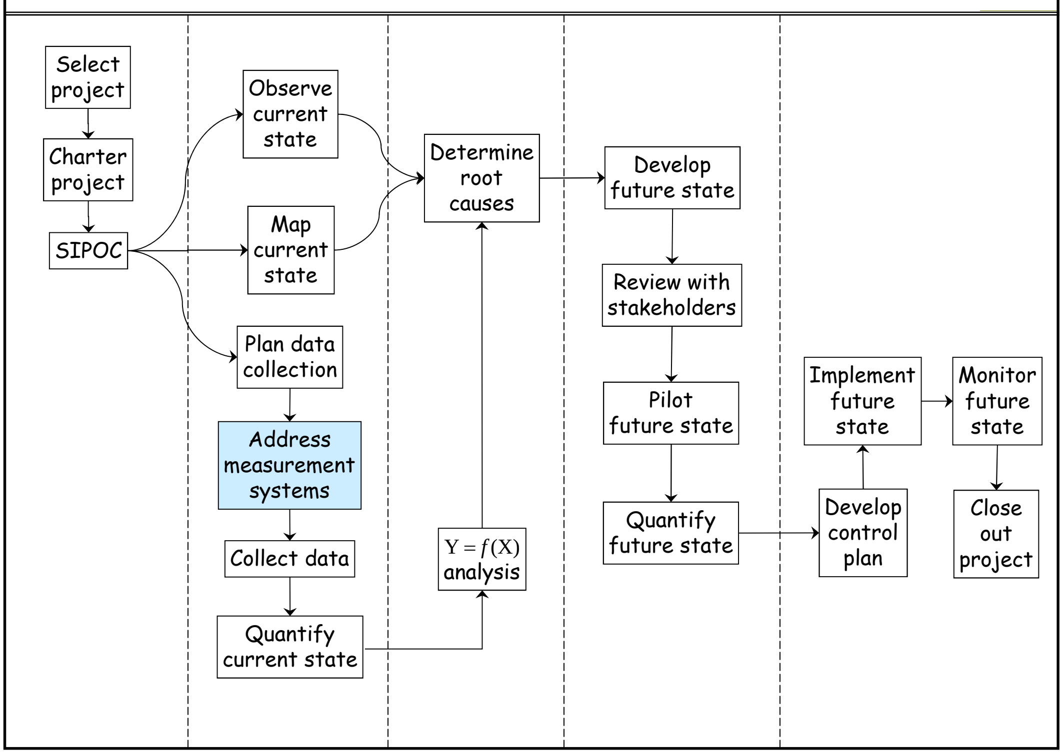
# Lean Six Sigma Black Belt, Vol. 2

## Course outline with slide numbers

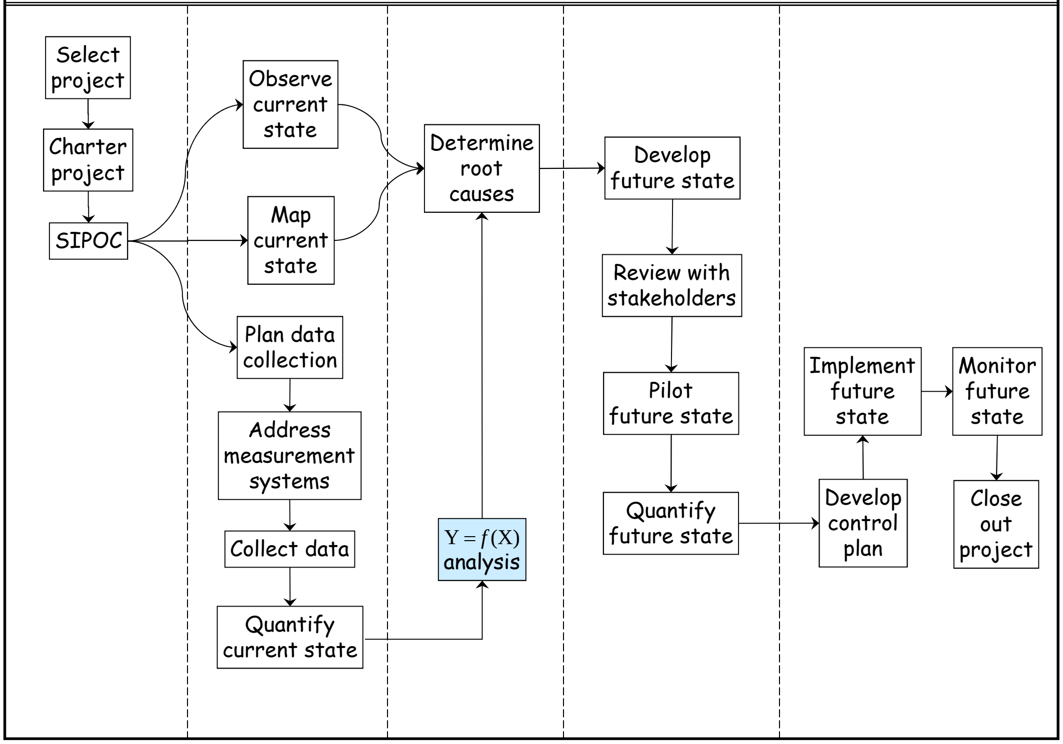
## LSS Vol 2, Sections 2-5, 29



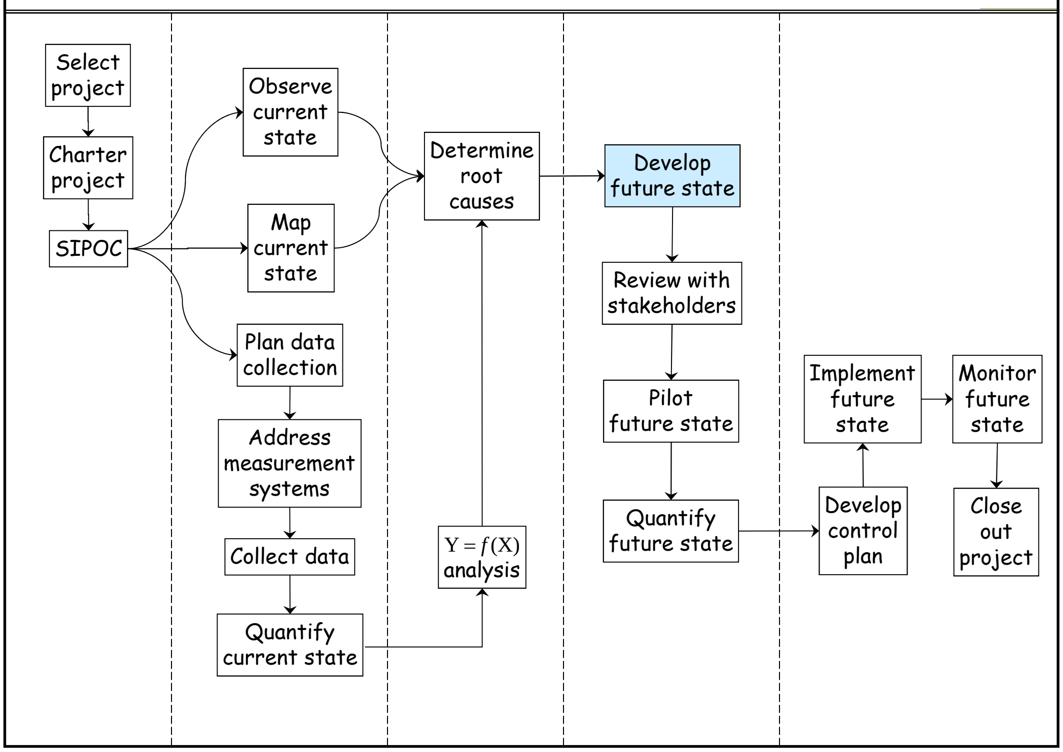

## LSS Vol 2, Section 6



# LSS Vol 2, Sections 7-12, 24, 26, 27




# LSS Vol 2, Sections 13-23, 25, 28



## Recommended default preferences for JMP

*File* ↓ *Preferences* ↓ *Platforms*

| Platforms | Options |
|---|---|
| Attribute Chart | ✓ Attribute Gauge Chart<br>✓ Show Agreement Points<br>✓ Connect Agreement Points<br>✓ Show Agreement Grand Mean |
| Bivariate | ✓ Show Points<br>✓ Fit Line |
| Contingency | ✓ Mosaic Plot<br>✓ Contingency Table<br>✓ Tests<br>✓ Count<br>✓ Row % |
| Distribution | ✓ Summary Statistics<br>✓ Horizontal Layout<br>✓ Histogram<br>✓ Outlier Box Plot<br>✓ Frequencies<br>✓ Separate Bars |

Uncheck any options not shown here

## JMP defaults (cont'd)

*File* ↓ *Preferences* ↓ *Platforms*

| Platforms | Options |
|---|---|
| Distribution Summary Statistics | ✓ Mean<br>✓ Std Dev<br>✓ N<br>✓ Minimum<br>✓ Maximum |
| DOE | ✓ Suppress Cotter Designs<br>✓ Optimality Criterion → Make D-Optimal Design |
| Fit Distribution | ✓ Diagnostic Plot<br>✓ Density Curve |
| Fit Least Squares | ✓ Profiler<br>✓ Summary of Fit<br>✓ Analysis of Variance<br>✓ Effect Tests<br>✓ Plot Regression<br>✓ Plot Residual by Predicted |

Uncheck any options not shown here

## JMP defaults (cont'd)

*File* ↓ *Preferences* ↓ *Platforms*

| Platforms | Options |
|---|---|
| Fit Nominal Logistic | ✓ Logistic Plot<br>✓ Likelihood Ratio Tests<br>✓ Profiler |
| Life Distribution | ✓ Show Points<br>✓ Show Statistics<br>✓ Show Confidence Area<br>✓ Interval Type → Pointwise |
| Model Dialog | ✓ Center Polynomials<br>✓ Emphasis → Minimal Report |
| Oneway | ✓ Means/Anova<br>✓ All Graphs<br>✓ Points<br>✓ Mean Diamonds<br>✓ Connect Means<br>✓ X Axis Proportional<br>✓ Points Jittered |

Uncheck any options not shown here

## JMP defaults (cont'd)

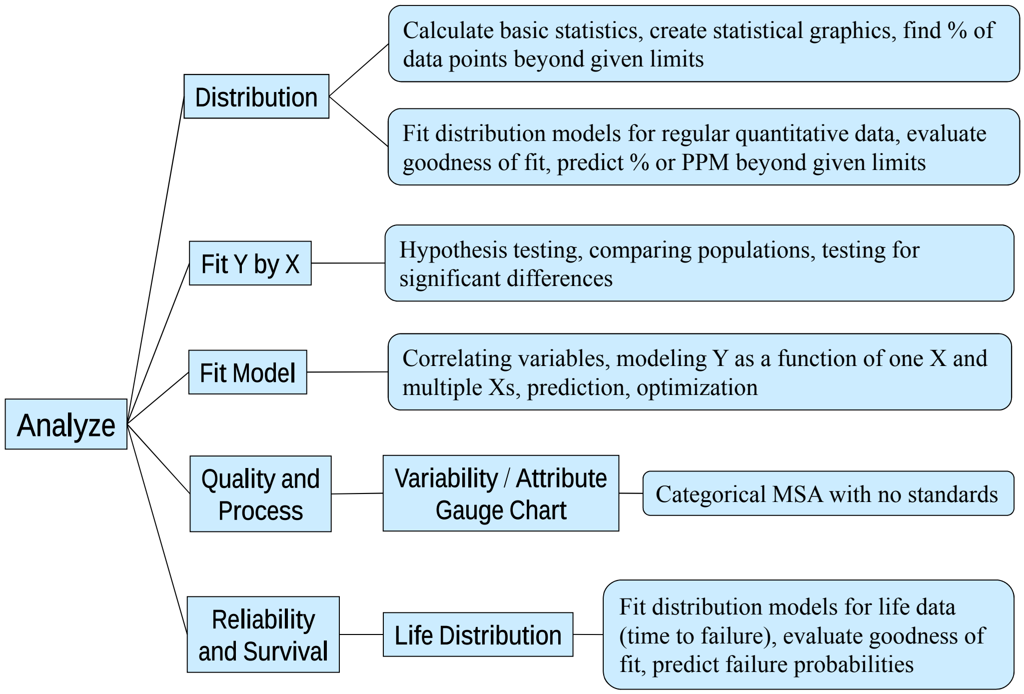
*File* ↓ *Preferences* ↓ *Platforms*

| Platforms | Options |
|---|---|
| Overlay Plot | ✓ Overlay Y's<br>✓ Separate Axes<br>✓ Show Points<br>✓ Connect Points<br>✓ Line Width → Thin<br>✓ Automatic Recalc |
| Variability Chart | ✓ Variability Chart<br>✓ Show Points<br>✓ Show Range Bars<br>✓ Connect Cell Means<br>✓ Show Separators<br>✓ Show Group Means<br>✓ Points Jittered |

Uncheck any options not shown here

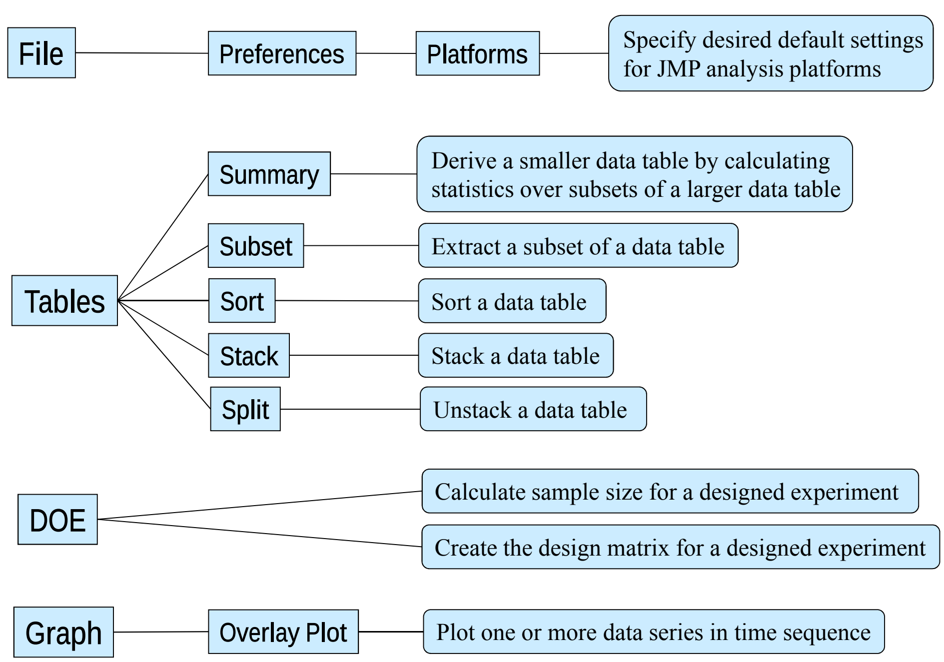
# 1 JMP menu map

1




# JMP menu map (cont'd)

2



## 2 Basic Statistics and Statistical Graphics

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- Frequency histogram
- Cumulative distribution function
- Percentiles
- Box and whisker plot
- JMP distribution analysis
- Data validation
- Distribution analysis options
- Plotting data in time sequence
- Saving analyses and data tables

Notes

4

Y variables are characteristics of parts or transactions that determine customer satisfaction, or lack thereof. They provide the data from which project metrics are computed. In sections 2 and 3 we focus on *quantitative* Y variables. Examples include:

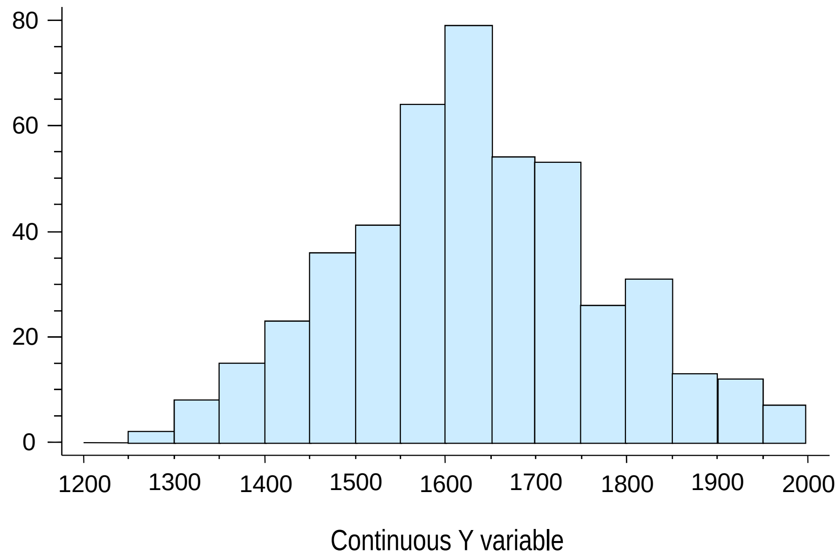
- Properties:  physical, chemical, electrical, optical, . . .
- Distance, time, dimensions, cost, quantity
- Event counts (when there is not a discrete number of opportunities for the event to occur)

JMP uses the term *continuous* for quantitative variables, and often uses the term *nominal* for categorical variables.

## Frequency histogram



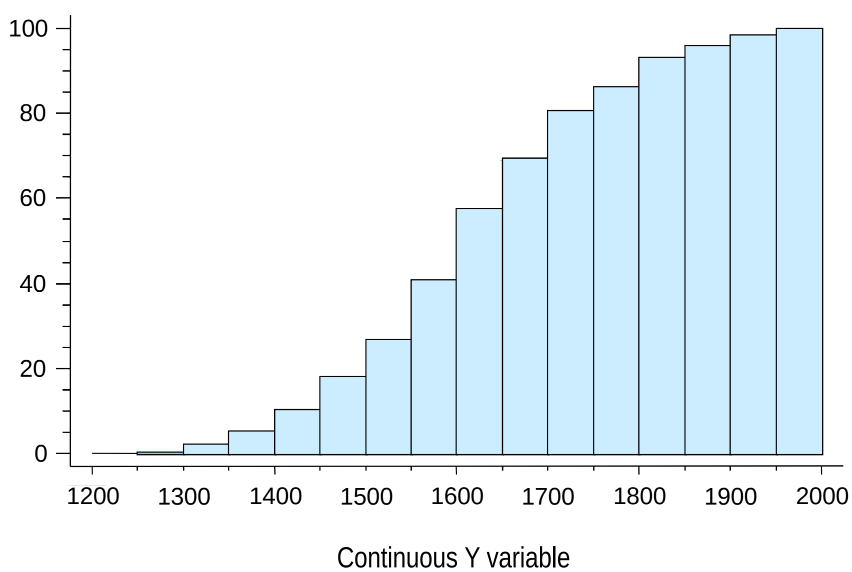
*Number of data points in each bin*




## Cumulative percentage histogram



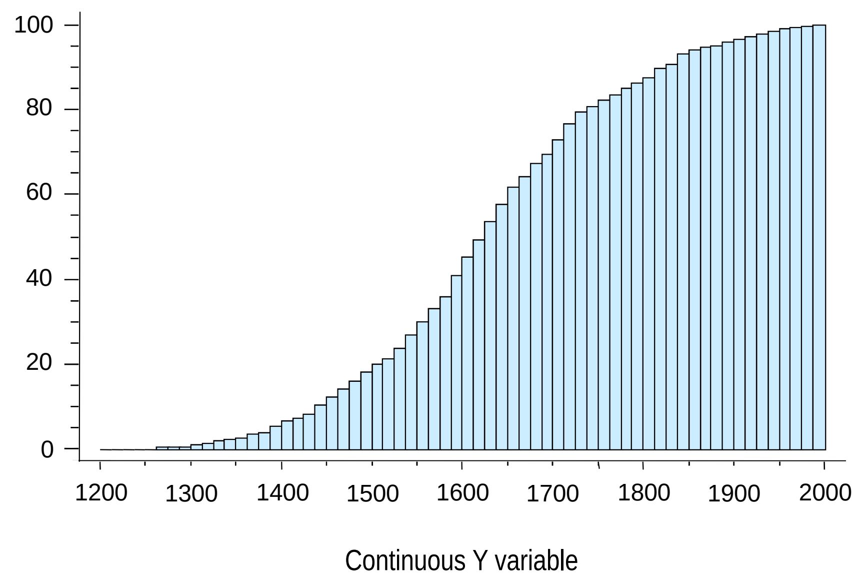
*Percentage of data points $\leq$ upper limit of each bin*



## Cumulative percentage histogram (cont'd)



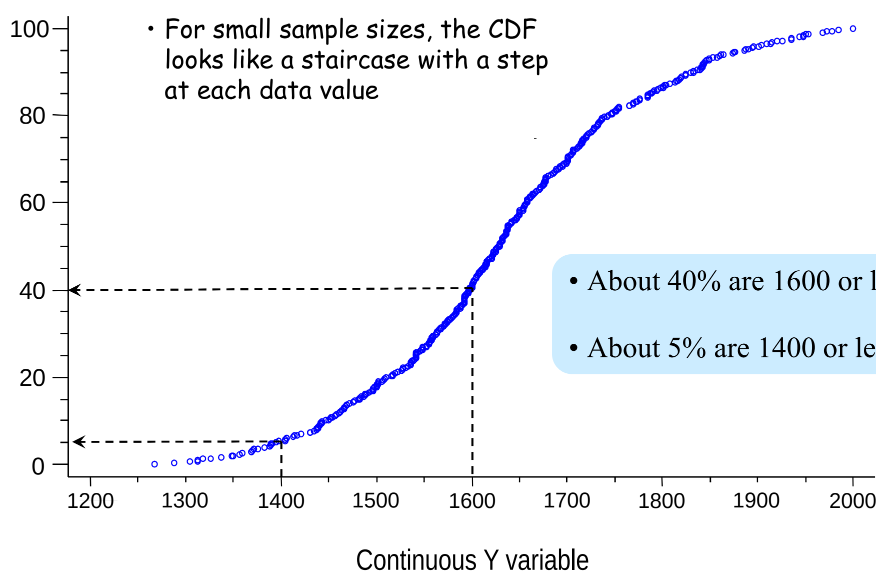
*Made the bins smaller*




## Cumulative distribution function (CDF)



- Bins are so small they isolate individual data values
- For small sample sizes, the CDF looks like a staircase with a step at each data value




- About 40% are 1600 or less
- About 5% are 1400 or less

## Percentiles



A *percentile* is a value that divides a population or data set into two groups, based on a stated percentage

10% are less than the **10th percentile**, 90% are greater

25% are less than the **25th percentile**, 75% are greater

50% are less than the **50th percentile**, 50% are greater

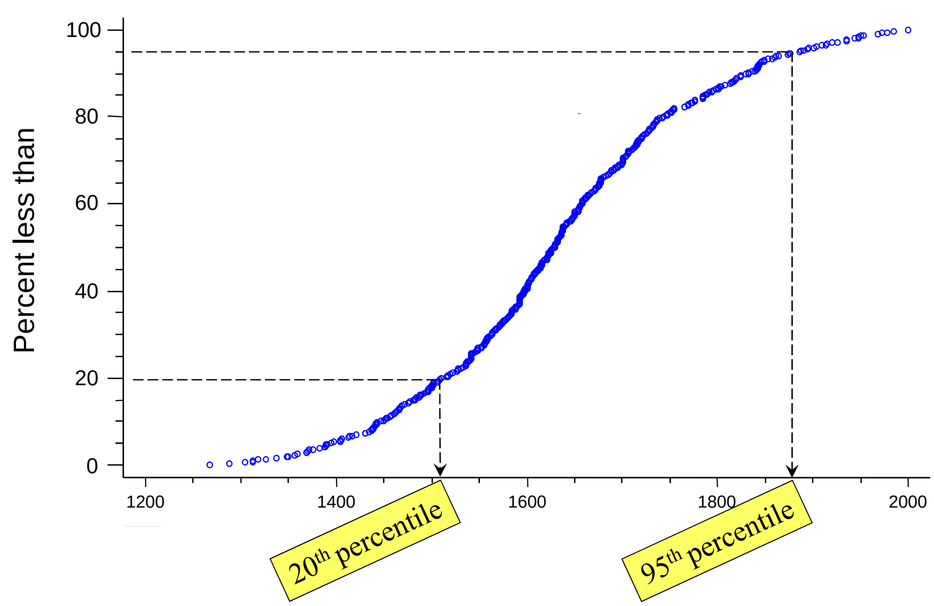
75% are less than the **75th percentile**, 25% are greater

90% are less than the **90th percentile**, 10% are greater

## Percentiles (cont'd)



*Illustration of 20th and 95th percentiles*



## Common percentile-based data summary

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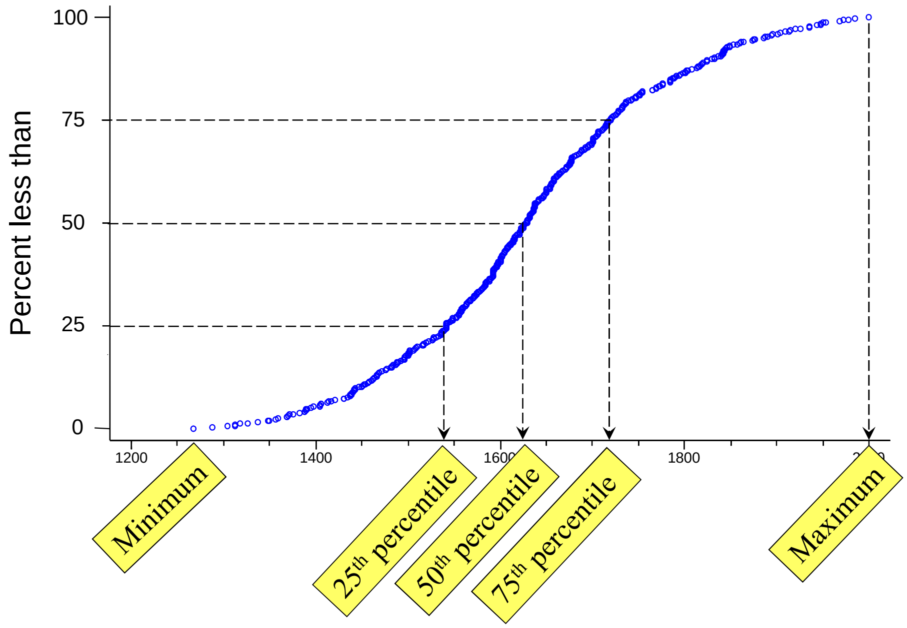
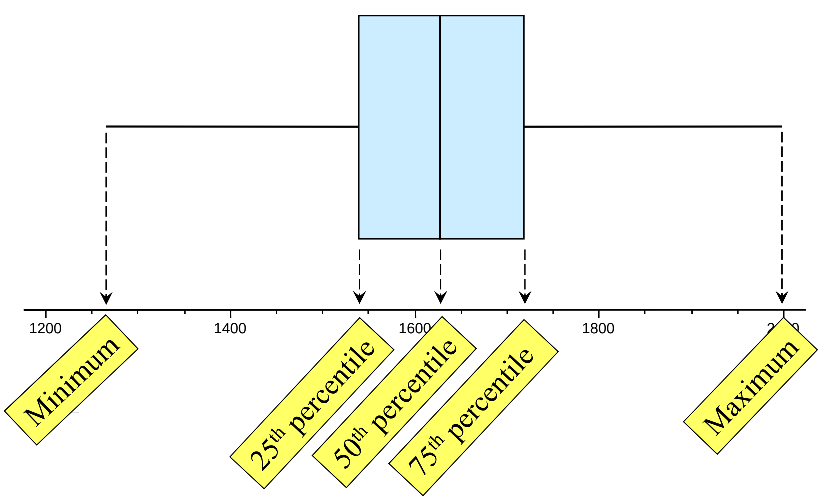



## Box-and-whisker plot

12



## Rule for plotting points separately

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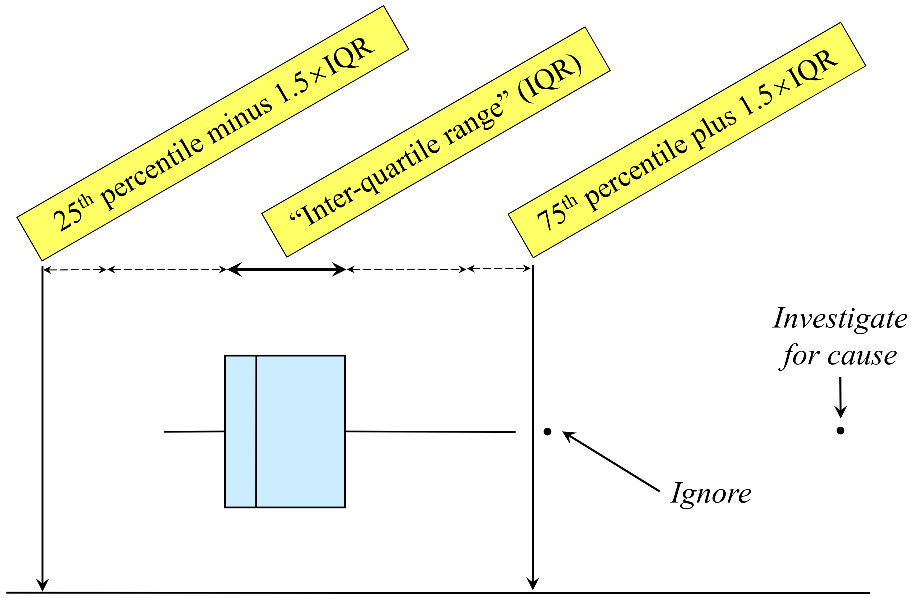


Points plotted separately may or may not represent assignable causes

## Notes

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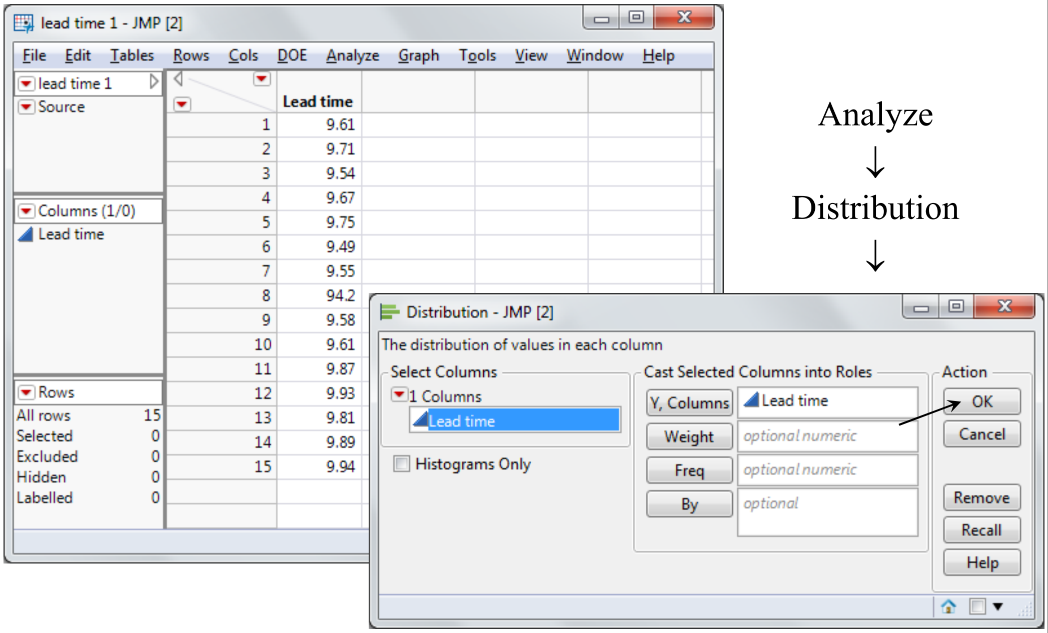
## JMP distribution analysis

File → Open → *LSSV2 data sets* → *lead time 1* → Open

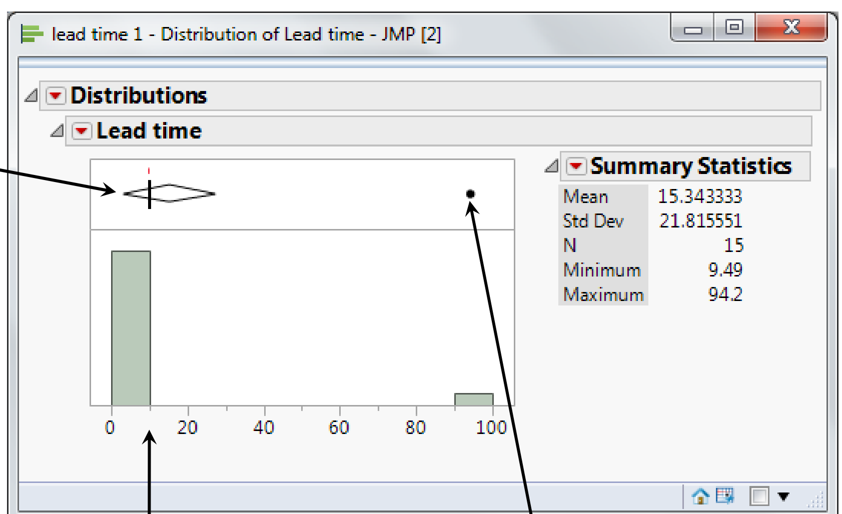



Analyze
↓
Distribution
↓



## Data validation

?
(later)

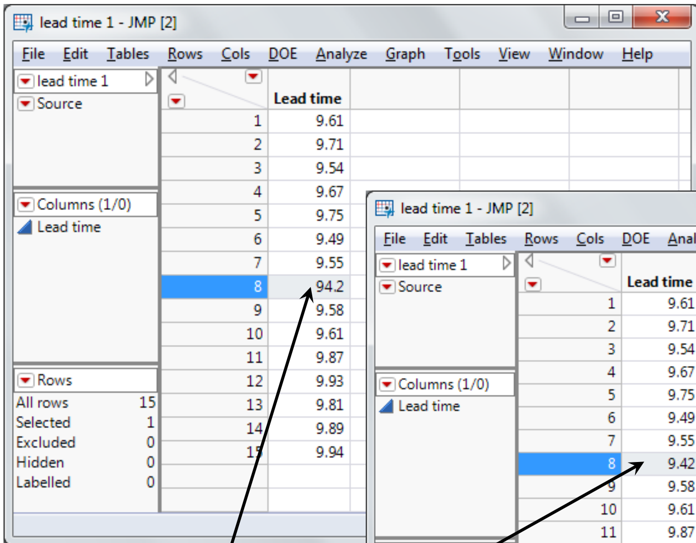


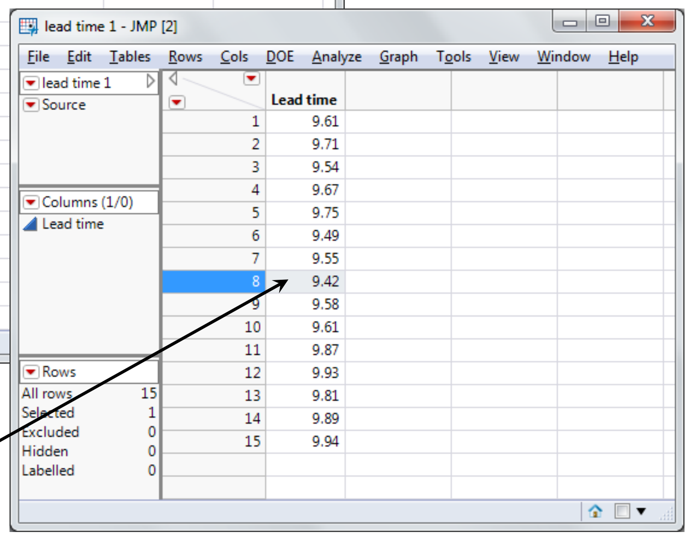

Frequency histogram

- Outlier
- Not always visible in the histogram
- Click on it
- Look in the data table

## Data validation (cont'd)



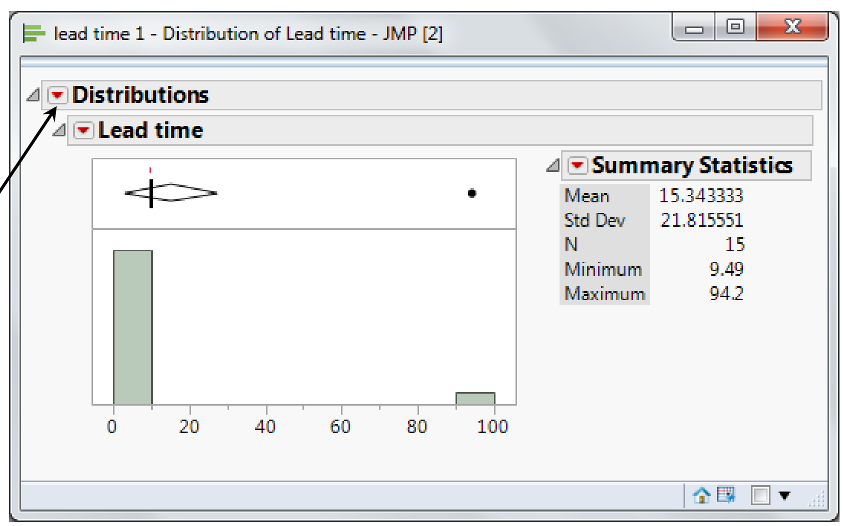



- ✓ Data entry error
- ✓ Enter the correct value
- ✓ Go to next slide

## Redoing an analysis






- Click on this red triangle
- Gives context specific options
- Select *Script*
- Select *Redo Analysis*
- See next slide

## Redoing an analysis (cont'd)

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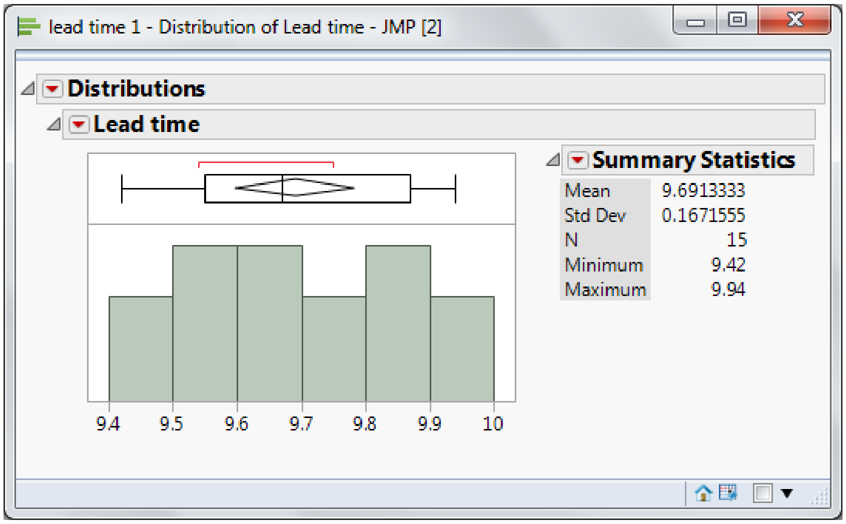


Note the change in the histogram and the summary statistics

## Cleaning up the box plot (optional)

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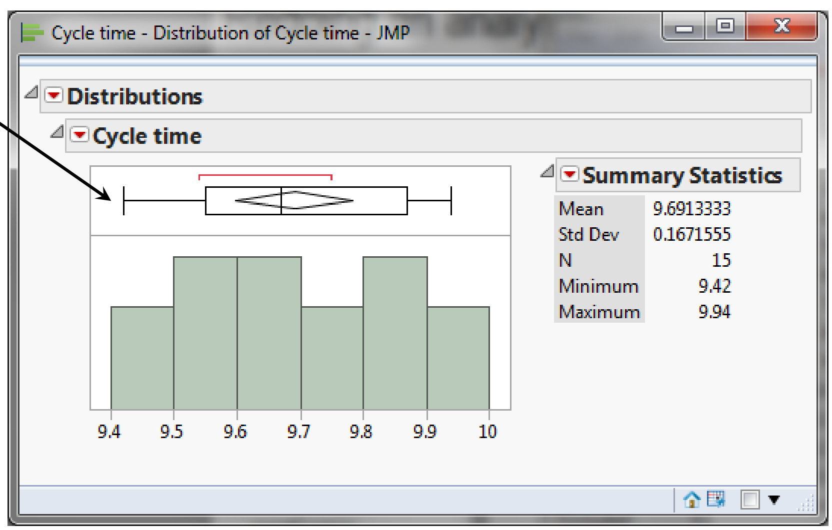
- Right click in here while holding down the *Alt* key
- Select *Customize* → *OK* → *Box Plot*
- Uncheck *Confidence Diamond* and *Shortest Half* → *OK*




- What remains is the box and whisker plot
- JMP calls it *Outlier Box Plot* because its main purpose in this context is to show outliers

# Distribution analysis options

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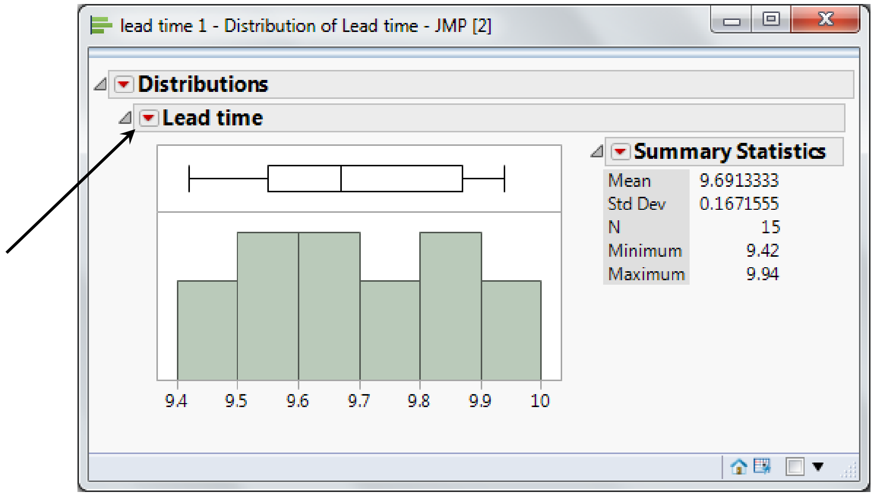
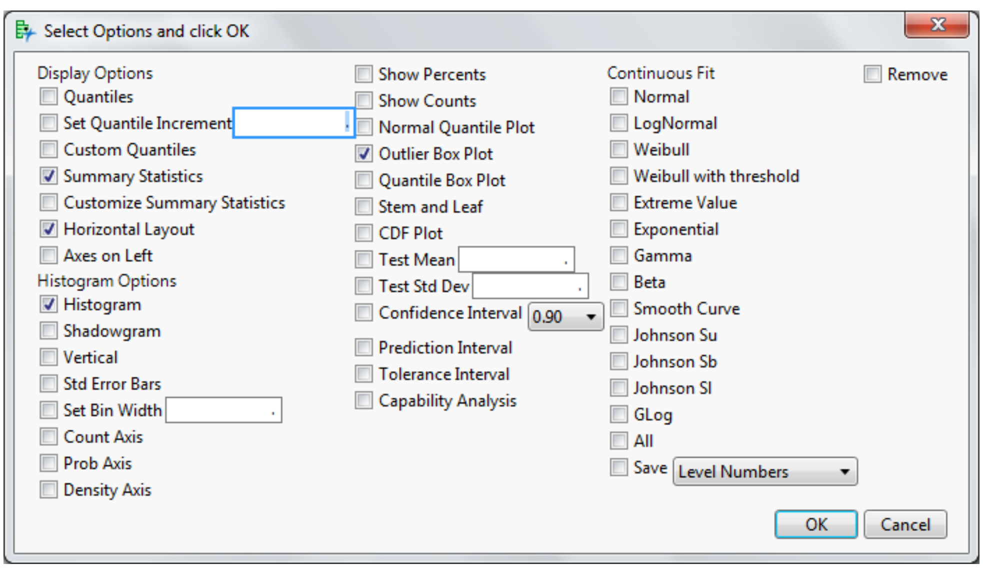



- Click on the red triangle next to *Lead time* while holding down the *Alt* key
- This will show the analysis options for the *Distribution* platform
- See next slide

# Analysis options (cont'd)

22




Just for practice:

Uncheck *Summary Statistics* and *Outlier Box Plot* → Check *CDF Plot* → OK

## Cumulative distribution function (CDF)

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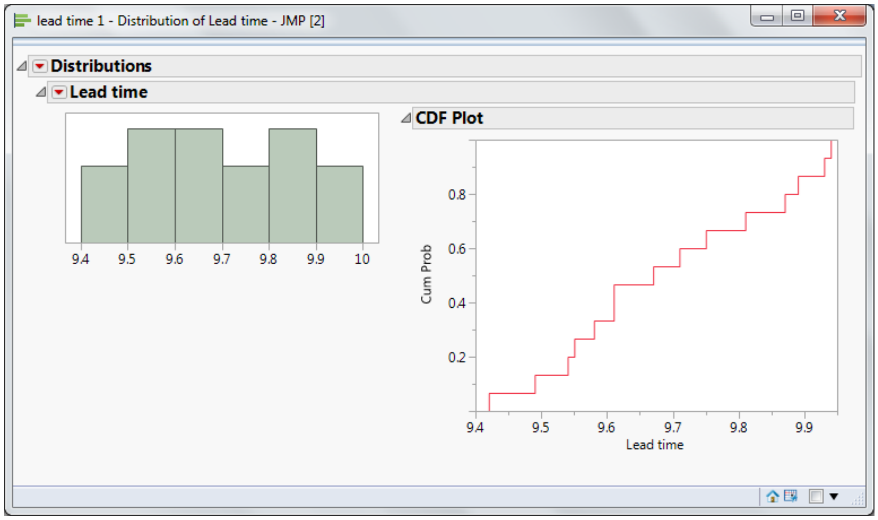


- Plots the proportion of data points ≤ each value in the data set
- The step size at each data value is usually 1/N, where N is the sample size
- If the same value occurs twice in the data set, the step size there is 2/N

## Modifying JMP plots

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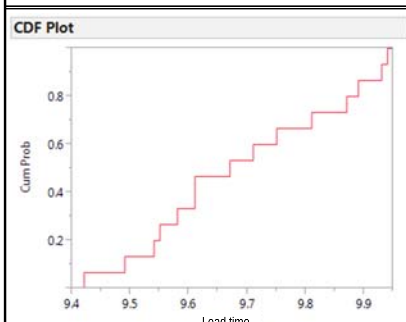
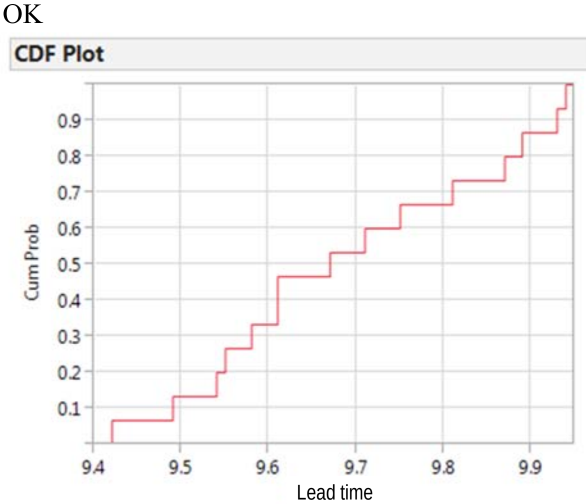
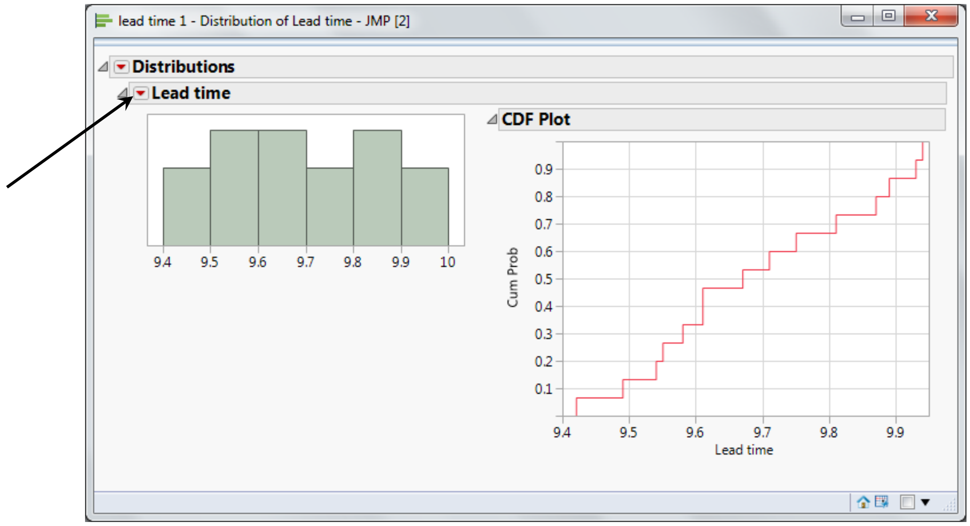



- Double click on a number on the Y axis → change *Increment* to 0.1 → check *Major Gridlines* → uncheck *Minor Tickmark* → OK
- Double click on a number on the X axis → check *Major Gridlines* → uncheck *Minor Tickmark* → OK



## Calculating percentages






- Suppose we want to know the percentage of data points exceeding 9.8
- Click the *Lead time* red triangle → select *Capability Analysis* → enter 9.8 for the *Upper Spec Limit* → click OK

## Percentages (cont'd)



**Capability Analysis**

| Specification | Value | Portion | % Actual |
|---|---|---|---|
| Lower Spec Limit | . | Below LSL | . |
| Spec Target | . | Above USL | 33.3333 |
| Upper Spec Limit | 9.8 | Total Outside | 33.3333 |

- Ignore the *Long Term Sigma* section of the output
- It gives predicted percentages based on the Normal (bell shaped) distribution curve
- We will cover distribution fitting in the next section

## Plotting data in time sequence

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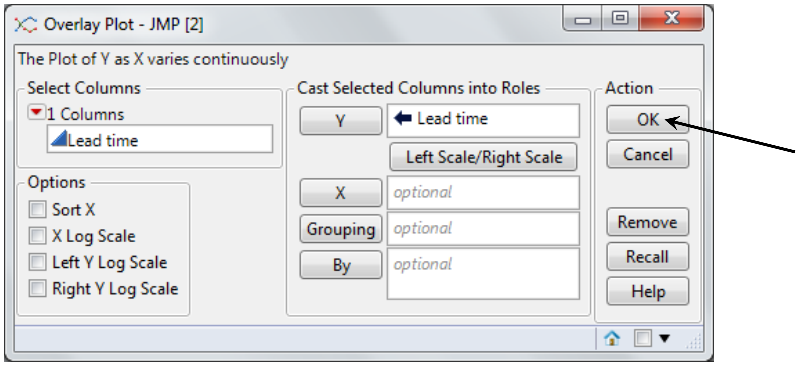
Graph → Overlay Plot




- You can have different left and right scales for plotting multiple Y variables
- A date, time, or other sequencing variable would go in the X slot
- Putting a variable in the *Grouping* or *By* slot will produce one plot for each value of that variable (the output formats differ slightly)

## Overlay plot (cont'd)

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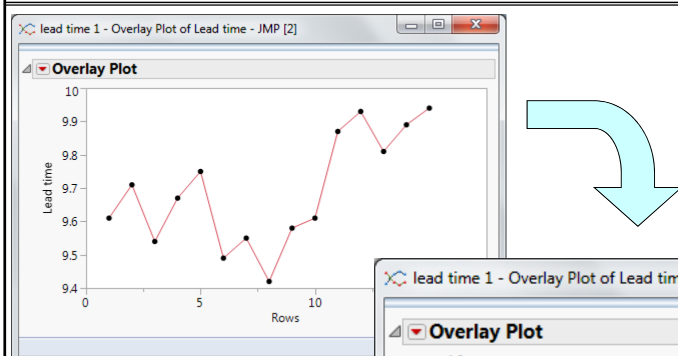


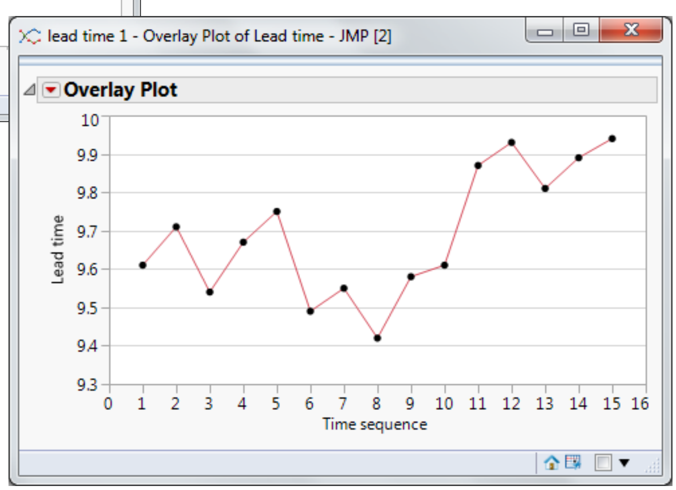


- Modify the default plot as shown
- Good way to look for assignable cause patterns other than outliers
- Same as a line chart in Excel

## Saving your analyses and data table

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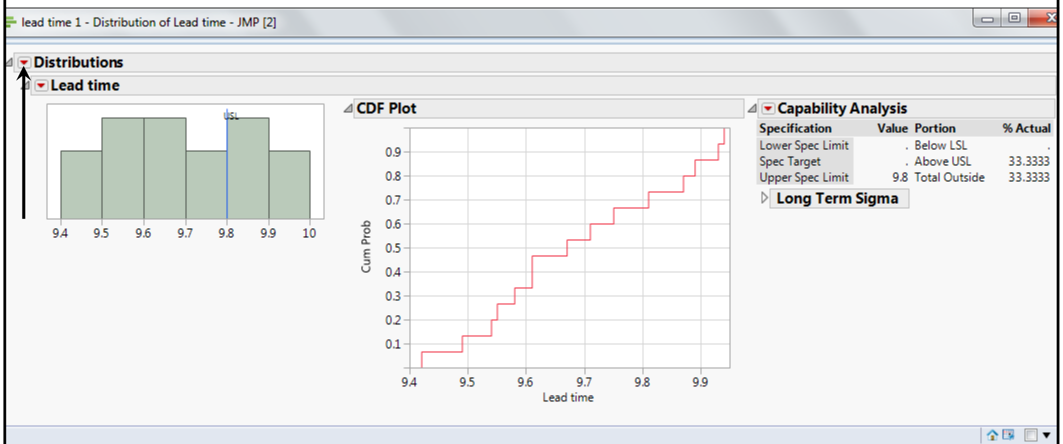


- Click on the thumbnail for your distribution analysis
- Click the red triangle next to *Distributions*
- Select *Script* → *Save Script to Data Table*

## Saving things (cont'd)

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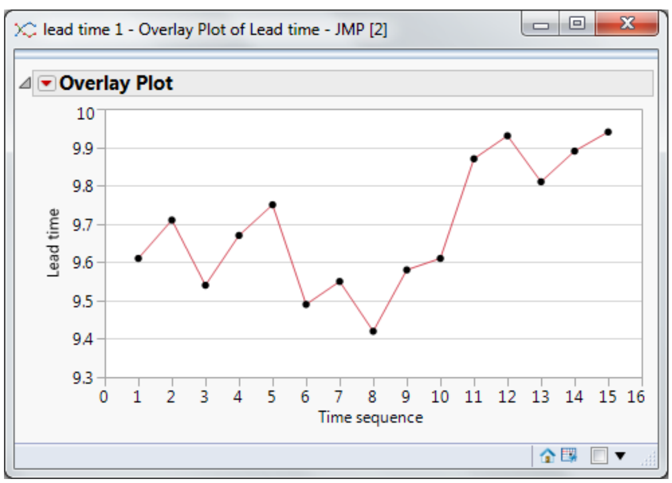
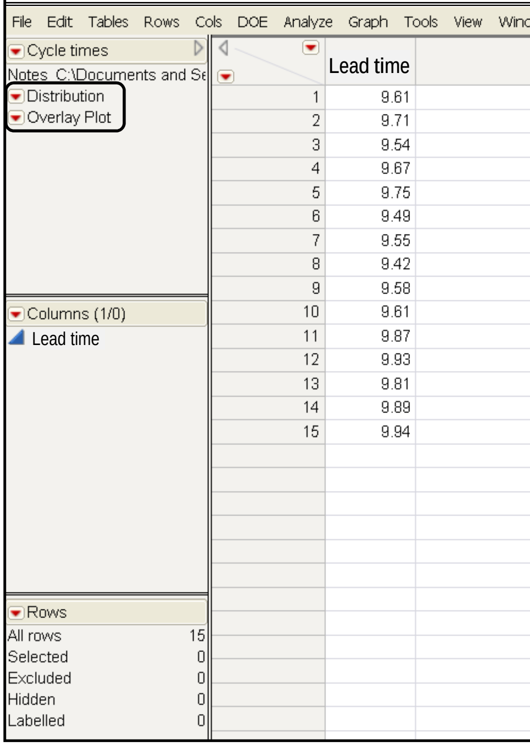



- Click on the thumbnail for your overlay plot
- Click the red triangle next to *Overlay Plot*
- Select *Script* → *Save Script to Data Table*
- Go to your data table

# Saving things (cont'd)






- Two scripts have been added to the left panel
- If you save the file (as JMP), the scripts will be saved with it
- The next time you open the file, you can run the scripts to recreate the analyses exactly as you left them
- Close and save your data table now*

*Use **Save As** to make sure you can find the file next time you want to open it

# Notes

## Exercise 2.1



Open *LSSV2 data sets \ quotation process* (in JMP). Perform the following data analysis tasks for the variable *TAT* (turnaround time).

(a) Run a distribution analysis. Note the large number of points plotted separately on the outlier box plot. This pattern is common with asymmetric "ski slope" distributions that pile up near zero. These points are *not* assignable causes, so don't exclude them. (If you exclude them and run the analysis again, a new set will crop up. If there were points far to the right of the main group, *they* might be assignable causes and, upon investigation, might need to be excluded.)

(b) Record the average, standard deviation, sample size, minimum, and maximum.

(c) Turn off the outlier box plot.

(d) Find the % of data points exceeding 3.

(e) Save your analysis script. Close and save the data table.

## Exercise 2.2



Open *LSSV2 data sets \ DI water* (in JMP). Perform the following data analysis tasks for the variable *Resistivity*.

(a) Create an overlay plot. You should see something that suggests bad data (stretch the graph if necessary). Use your mouse to draw a box around the suspicious data points. Right click in an uninhabited area of the plot, select *Row Hide and Exclude*.

(b) Run a distribution analysis. Record the average, standard deviation, sample size, minimum, and maximum.

(c) Turn off the outlier box plot.

(d) Find the % of data points falling below 1500.

(e) Save your analysis scripts. Close and save the data table.

# 3 Fitting and Using Distributions

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- Distribution curves
- Checking goodness of fit
- JMP examples
- Fitting and using the Normal distribution
- Fitting and using the Lognormal distribution
- Finding the best fitting distribution(s)
- Using the best fitting distributions(s)

# Frequency histogram

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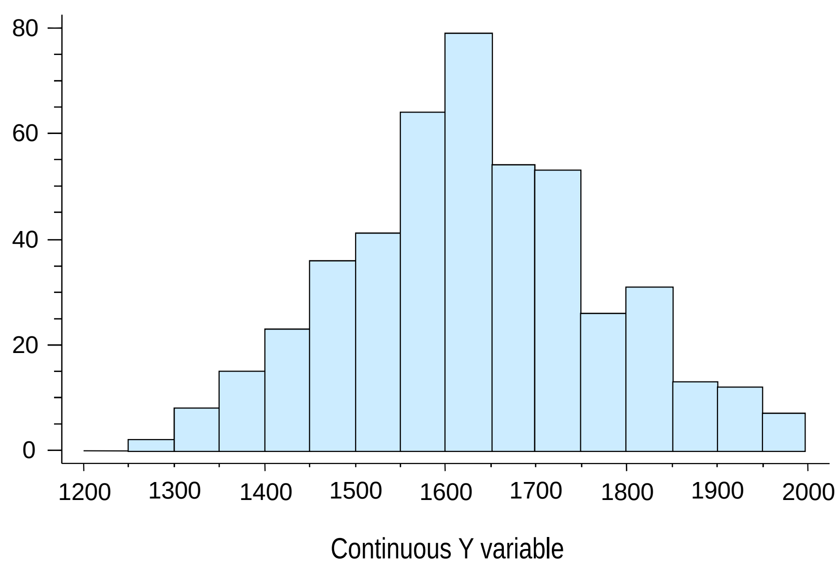
*A description of the data*



## Distribution curves

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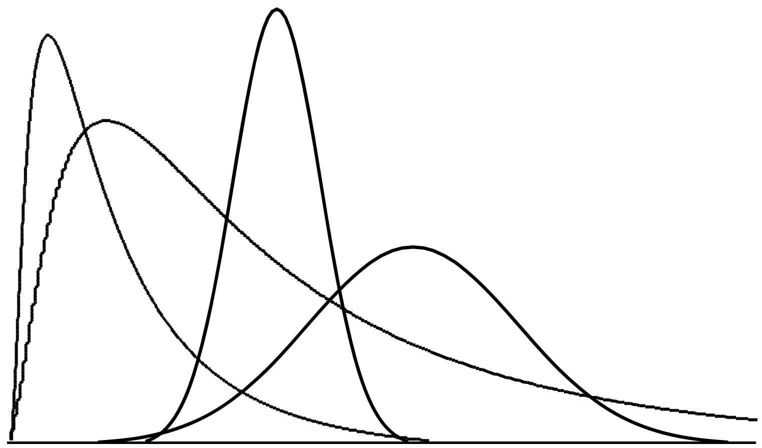
*Possible descriptions of the* population



Continuous Y variable

## Distribution curves (cont'd)

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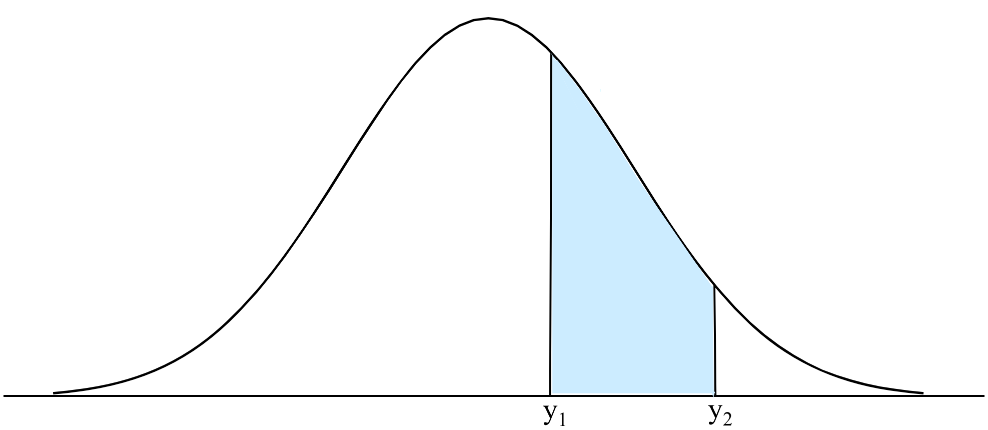
*Area under the curve between* $y_1$ *and* $y_2$

$=$ *% of the population with* $y_1 < Y \leq y_2$




Continuous Y variable

## Distribution curves (cont'd)

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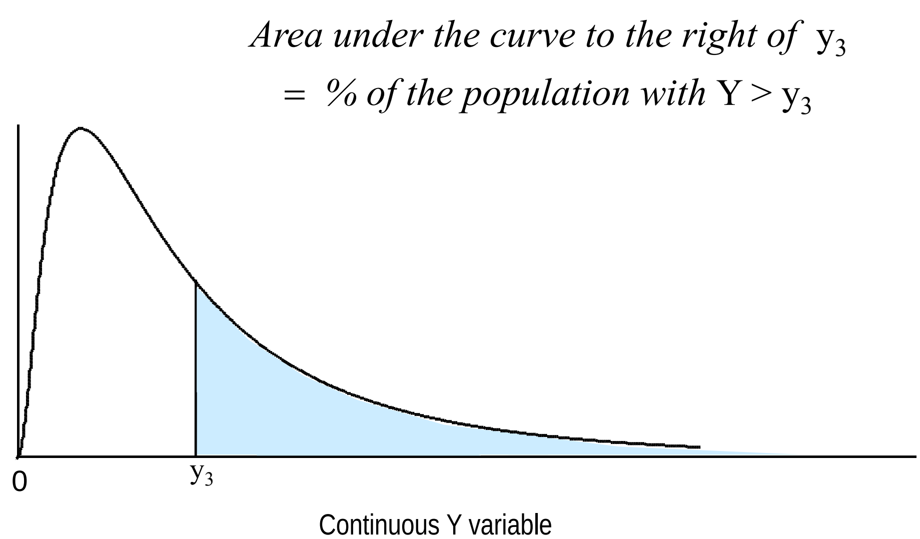


## Fitting a distribution curve to the data

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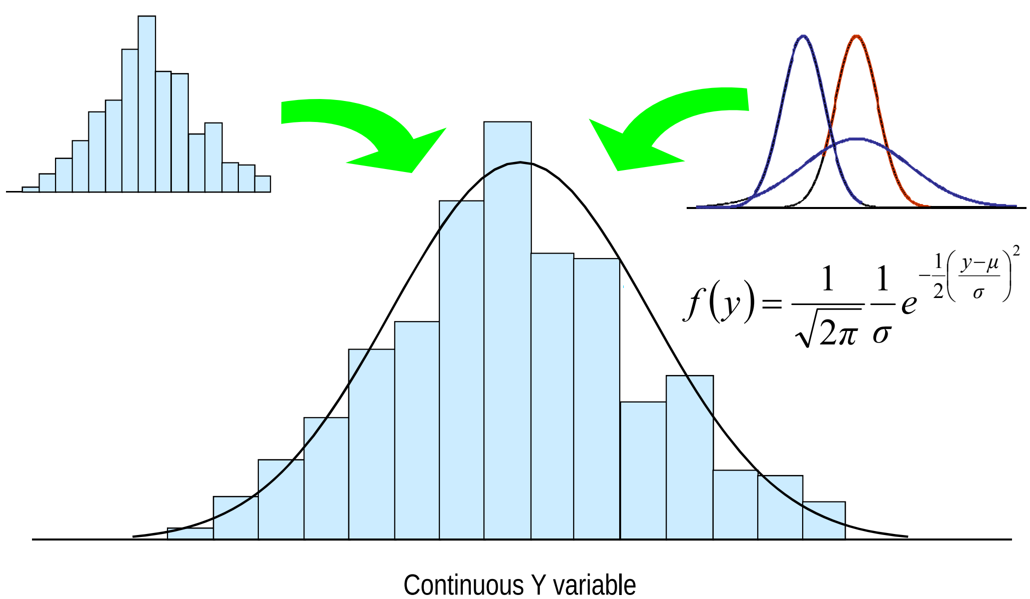


- The Normal curve depends only on $\mu$ and $\sigma$ (population mean and std. dev.)
- Plug the sample mean and std. dev. into the formula in place of $\mu$ and $\sigma$

## Distribution curves allow us to extrapolate . . .

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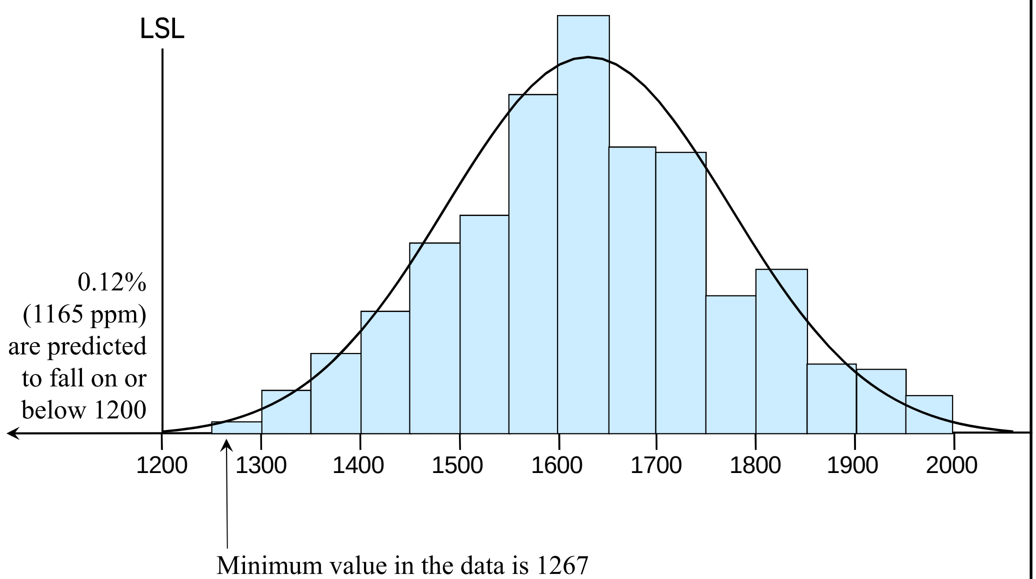


## . . . but only if the distribution matches the data!

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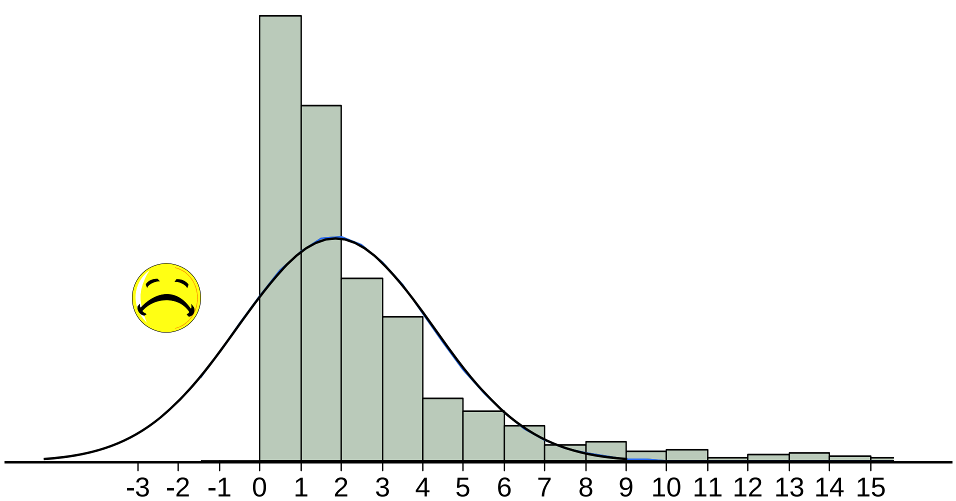

## Checking goodness of fit

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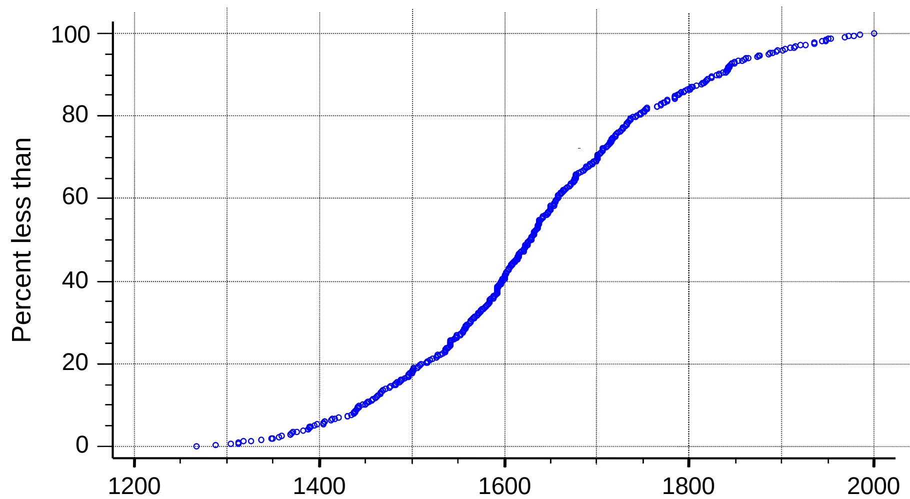
*Data CDF*




## Checking goodness of fit (cont'd)

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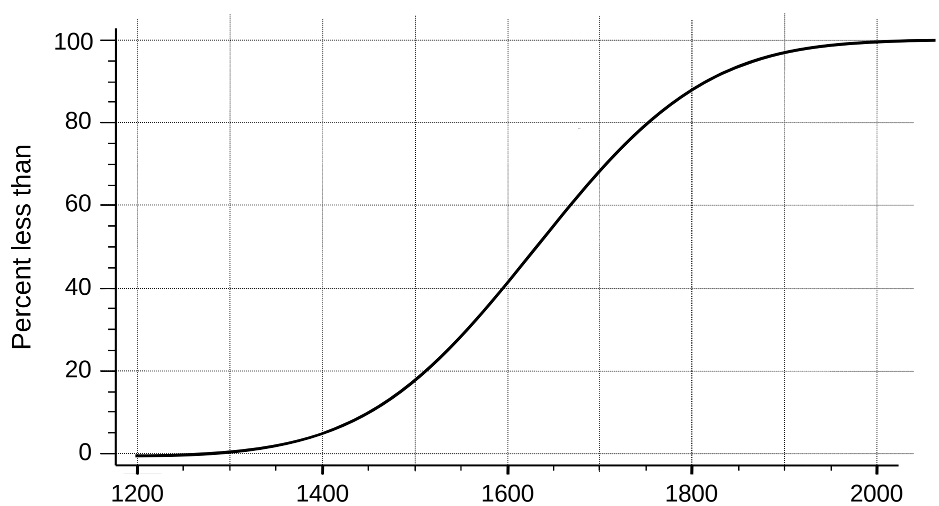
*Best fitting population CDF (assuming Normal)*



## Checking goodness of fit (cont'd)

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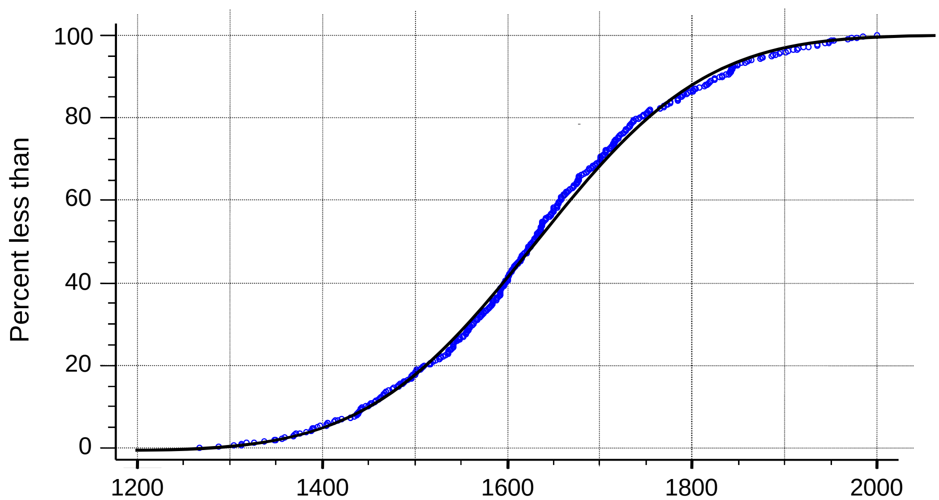
*Data and population CDFs should match*




## Quantile plot (also known as probability plot)

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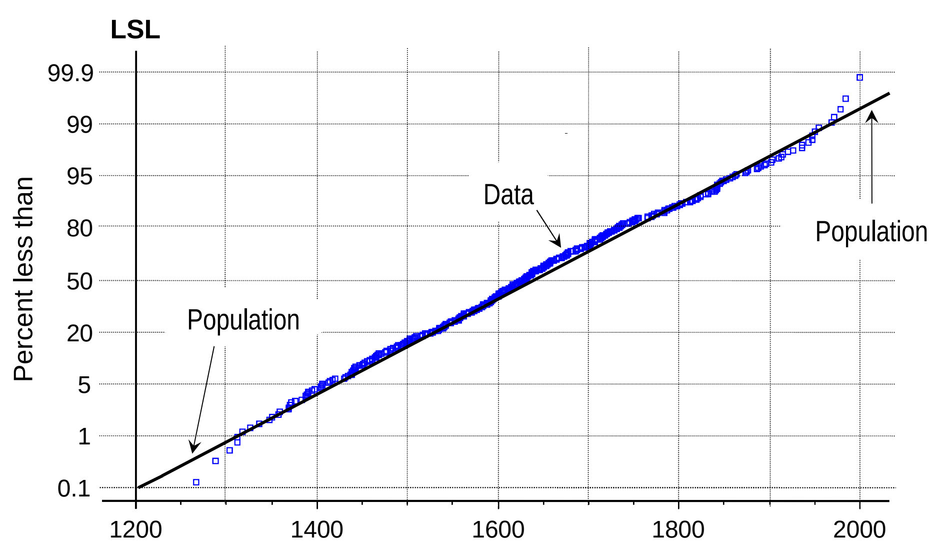
*CDFs plotted on a Normal distribution scale*



## JMP example: Normal data

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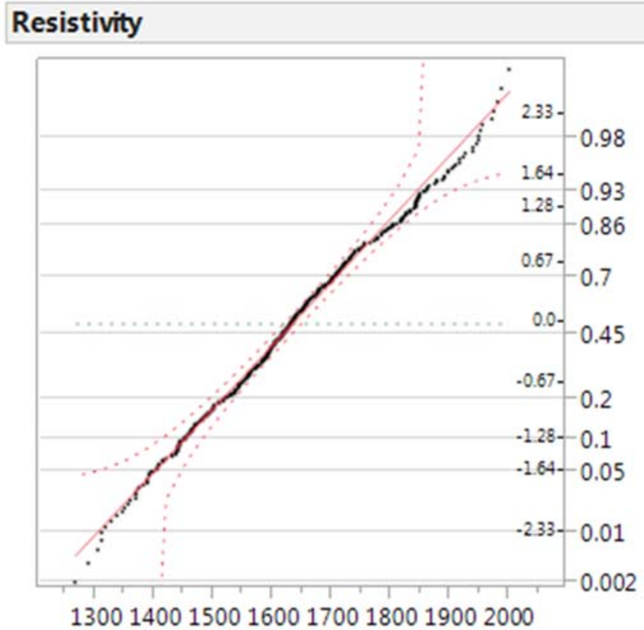
- Data table *DI water.jmp* Variable *Resistivity*
- Distribution → *Resistivity* red triangle → Normal Quantile Plot
- Fit is good — the points fall along the line and stay inside the hyperbolic band
- Leave the data table open




- To create images like this, use *Tools* → *Selection* to grab the portion of the output that you want, then copy and paste.

## JMP example: non-Normal data

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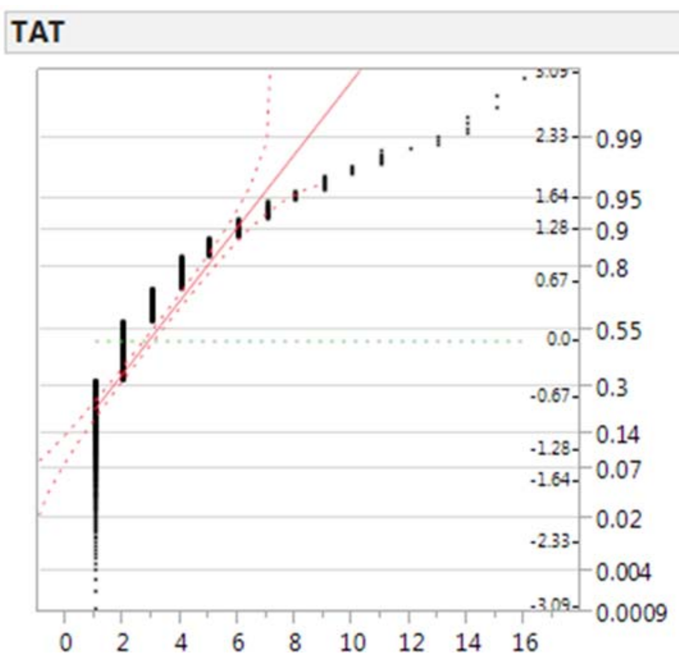
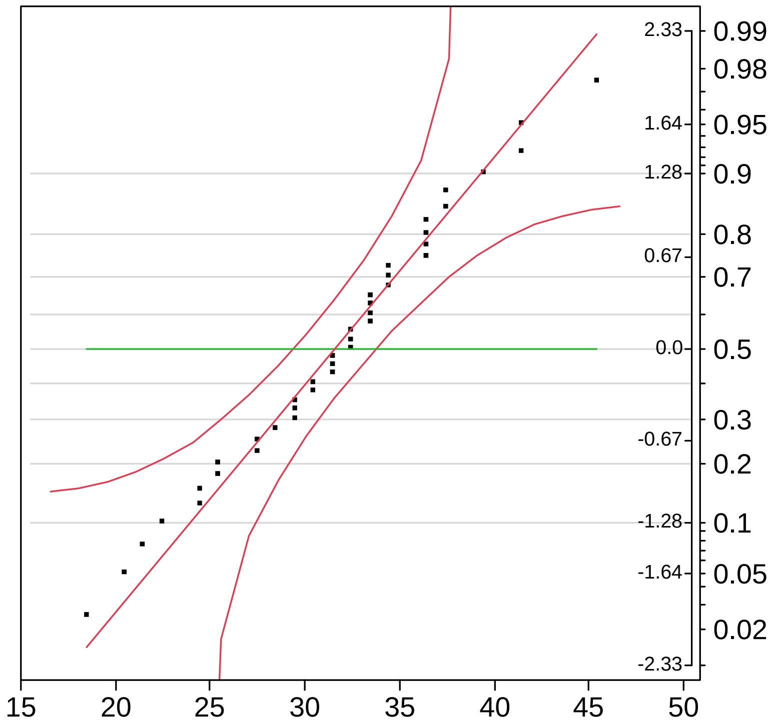
- Data table *quotation process.jmp*, variable *TAT*
- Distribution → *TAT* red triangle → Normal Quantile Plot
- Fit is bad — the points do not fall along the line, and do not stay inside the hyperbolic bands
- Leave the data table open



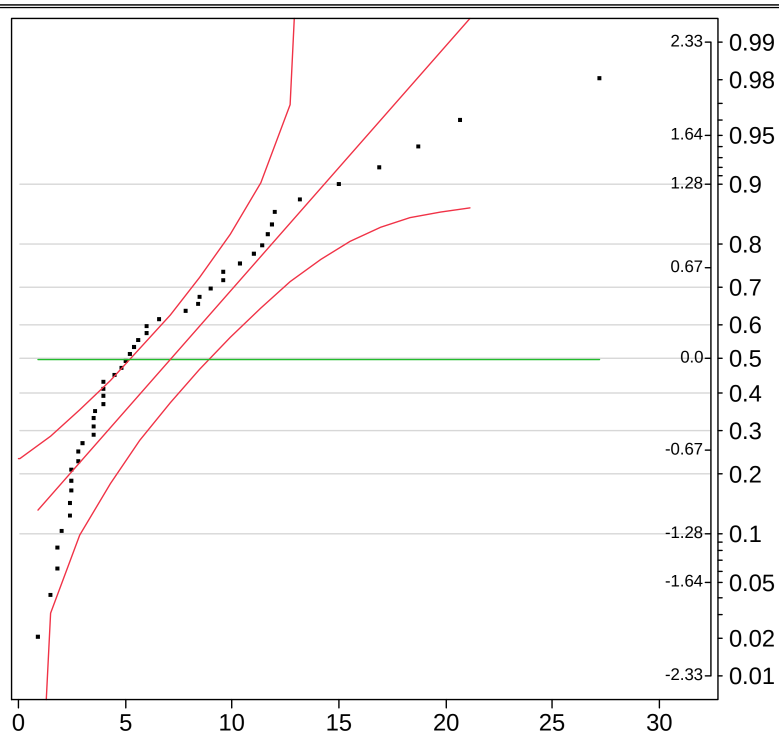
## Is this data Normal?

49




## Is this data Normal?

50



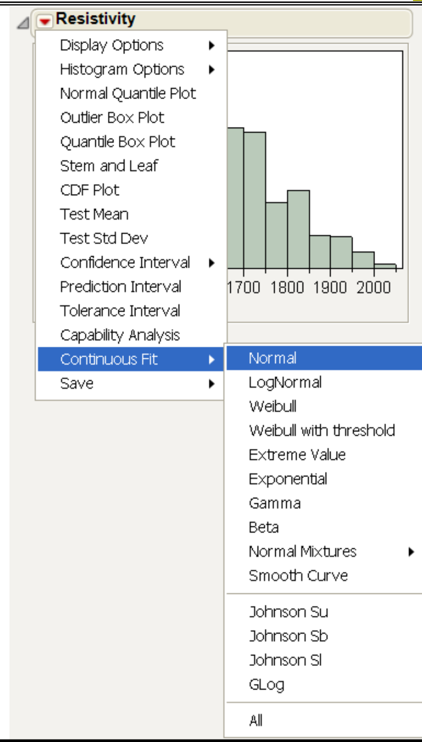
## Fitting and using the Normal distribution

51

- Data table: *DI water.jmp*

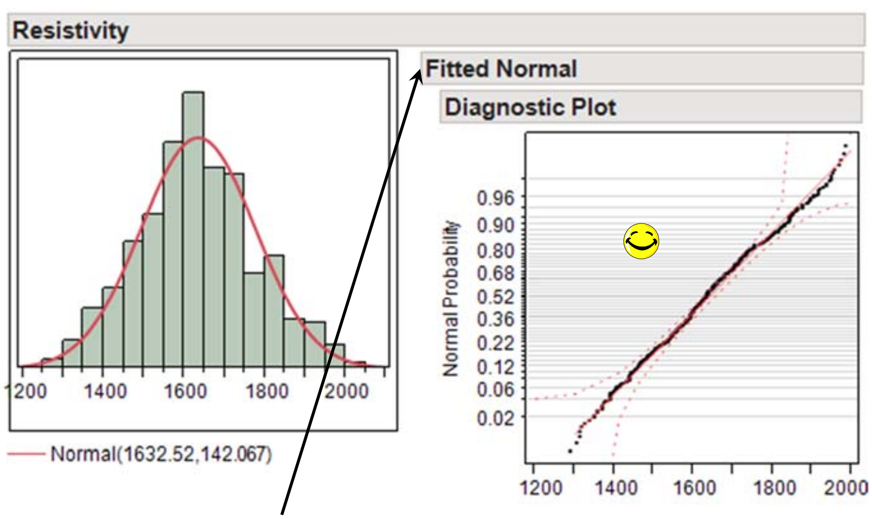
- Analyze
  - → Distribution
  - → *Resistivity*
  - → N should be 464

- Red triangle (no Alt key)
  - → Continuous Fit
  - → Normal




## Normal distribution (cont'd)

52




Click on the *Fitted Normal* red triangle

→ Select *Spec Limits*

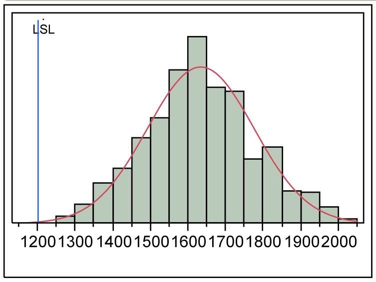
→ Enter 1200 for *Lower Spec Limit*

→ OK

## Normal distribution (cont'd)



**Resistivity**



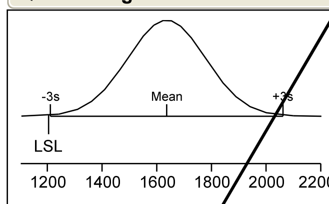

Normal(1632.52,142.067)

**Fitted Normal**

**Capability Analysis**

| Specification | Value | Portion | % Actual |
|---|---|---|---|
| Lower Spec Limit | 1200 | Below LSL | 0.0000 |
| Spec Target | . | Above USL | . |
| Upper Spec Limit | . | Total Outside | 0.0000 |

**Quantile Sigma**




| Capability | Index |
|---|---|
| CP | . |
| CPK | 1.015 |
| CPM | . |
| CPL | 1.015 |
| CPU | . |

| Portion | Percent | PPM |
|---|---|---|
| Below LSL | 0.1165 | 1165.3025 |
| Above USL | . | . |
| Total Outside | 0.1165 | 1165.3025 |

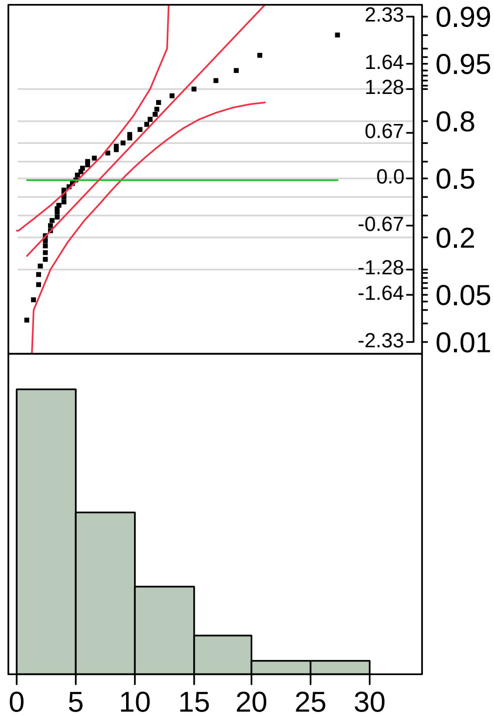
- None of the measurements in the data set are less than 1200
- 0.12% (1165 ppm) are predicted to fall below 1200 in the population (future production)
- Save script, close and save data table

## Notes

## Fitting and using the Lognormal distribution

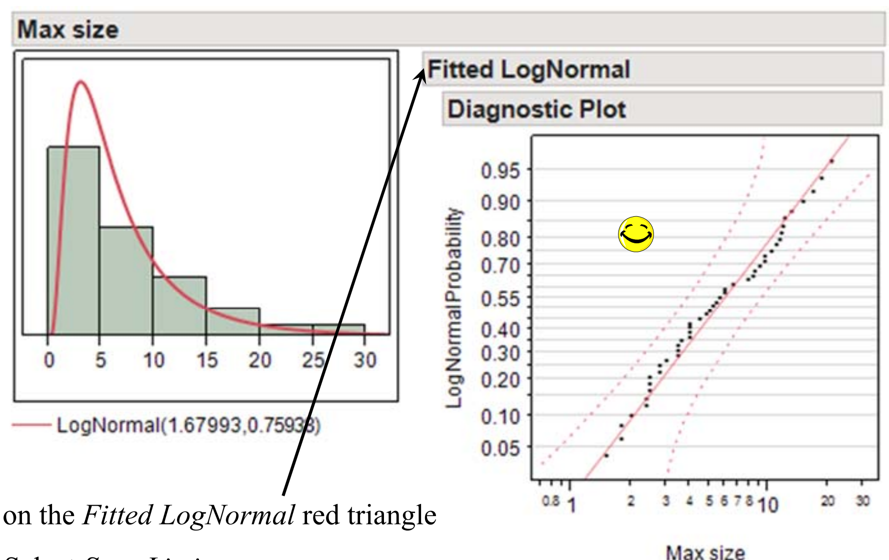


- Open *number & size of defects* (in JMP)
- Analyze → Distribution → *Max size*
- *Max size* is not Normal
- The *LogNormal* distribution is the most common alternative
- Red triangle → Continuous Fit → LogNormal




## Lognormal distribution (cont'd)






Click on the *Fitted LogNormal* red triangle

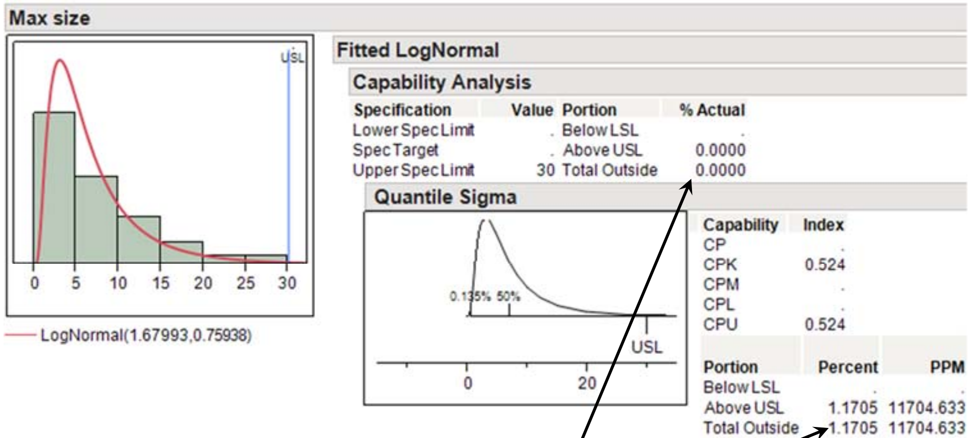
→ Select *Spec Limits*

→ Enter 30 for *Upper Spec Limit*

→ OK

# Lognormal distribution (cont'd)






- None of the measurements in the data set are greater than 30
- 1.17% are predicted to exceed 30 in the population (future production)
- Save script, close and save data table

Notes

## Finding the best-fitting distribution(s)

59

4/0 Cols
678/0 Rows

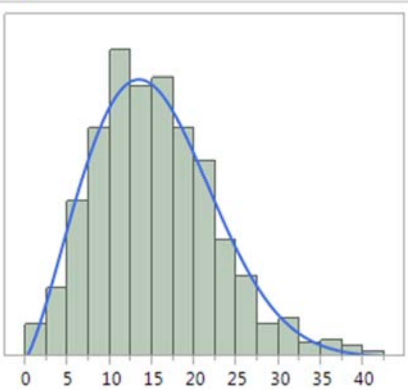
| | Aligner | X dev | Y dev | R dev |
|---|---|---|---|---|
| 1 | 1 | -17 | 4 | 17.464249197 |
| 2 | 2 | -7 | 6 | 9.2195444573 |
| 3 | 3 | -10 | -21 | 23.259406699 |
| 4 | 2 | 0 | -1 | 1 |
| 5 | 2 | -10 | 5 | 11.180339887 |
| 6 | 2 | -7 | 0 | 7 |
| 7 | 3 | -14 | -15 | 20.518284529 |
| 8 | 2 | -3 | -17 | 17.262676502 |
| 9 | 2 | -8 | 3 | 8.5440037453 |
| 10 | 2 | -7 | -8 | 10.630145813 |
| 11 | 1 | -11 | -6 | 12.529964086 |
| 12 | 2 | -6 | 0 | 6 |
| 13 | 2 | -7 | 5 | 8.602325267 |
| 14 | 3 | -10 | -5 | 11.180339887 |
| 15 | 2 | -3 | 1 | 3.1622776602 |
| 16 | 2 | -8 | 4 | 8.94427191 |
| 17 | 3 | -16 | -12 | 20 |
| 18 | 3 | -16 | -15 | 21.931712199 |
| 19 | 1 | -14 | 3 | 14.317821063 |
| 20 | 2 | -8 | -8 | 11.313708499 |
| 21 | 3 | -23 | -2 | 23.086792761 |
| 22 | 3 | -19 | -15 | 24.207436874 |
| 23 | 2 | -7 | 9 | 11.401754251 |
| 24 | 2 | -10 | 0 | 10 |
| 25 | 2 | -9 | -5 | 10.295630141 |
| 26 | 1 | -8 | -11 | 13.601470509 |
| 27 | 2 | -8 | -3 | 8.5440037453 |
| 28 | 3 | -16 | 0 | 16 |
| 29 | 1 | -13 | -21 | 24.69817807 |
| 30 | 3 | -8 | -4 | 8.94427191 |

- Open *alignment process.jmp*
- Analyze → Distribution → *R dev*
- Remove:
  - ✓ Summary Statistics
  - ✓ Outlier Box Plot
- Red triangle → Continuous Fit → All
- See next slide

## Best-fitting distributions (cont'd)

60

**R dev**




**Compare Distributions**

| Show | Distribution | Number of Parameters | -2*LogLikelihood | AICc |
|---|---|---|---|---|
| ☑ | Weibull | 2 | 4539.3202 | 4543.3 |
| ☐ | Extreme Value | 2 | 4539.3202 | 4543.3 |
| ☐ | Johnson Sl | 3 | 4541.07649 | 4547.1 |
| ☐ | Johnson Su | 4 | 4541.07649 | 4549.1 |
| ☐ | Normal 3 Mixture | 8 | 4539.81762 | 4556.0 |
| ☐ | Normal 2 Mixture | 5 | 4549.79406 | 4559.9 |
| ☐ | Gamma | 2 | 4567.15519 | 4571.2 |
| ☐ | Normal | 2 | 4580.47707 | 4584.5 |
| ☐ | LogNormal | 2 | 4672.7545 | 4676.8 |
| ☐ | GLog | 3 | 4672.7545 | 4678.8 |
| ☐ | Exponential | 1 | 5053.88325 | 5055.9 |

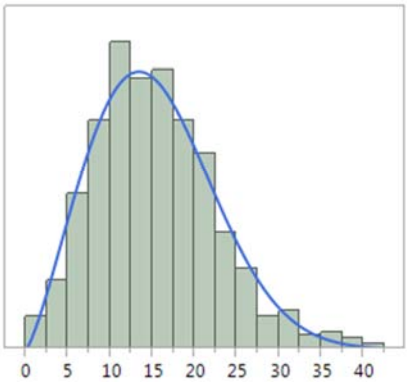
Double click
↓
Column Numeric Format
↓
Fixed Decimal (1)
↓
OK

- Distributions are ranked by AICc ("Akaike Information Criterion corrected" — will call it AIC from now on)
- AIC measures *lack* of fit → smaller values are better

## Best-fitting distributions (cont'd)



**R dev**




**Compare Distributions**

| Show | Distribution | Number of Parameters | -2*LogLikelihood | AICc |
|---|---|---|---|---|
| ☑ | Weibull | 2 | 4539.3202 | 4543.3 |
| ☐ | Extreme Value | 2 | 4539.3202 | 4543.3 |
| ☐ | Johnson Sl | 3 | 4541.07649 | 4547.1 |
| ☐ | Johnson Su | 4 | 4541.07649 | 4549.1 |
| ☐ | Normal 3 Mixtur | 8 | 4539.81762 | 4556.0 |
| ☐ | Normal 2 Mixtur | 5 | 4549.79406 | 4559.9 |
| ☐ | Gamma | 2 | 4567.15519 | 4571.2 |
| ☐ | Normal | 2 | 4580.47707 | 4584.5 |
| ☐ | LogNormal | 2 | 4672.7545 | 4676.8 |
| ☐ | GLog | 3 | 4672.7545 | 4678.8 |
| ☐ | Exponential | 1 | 5053.88325 | 5055.9 |

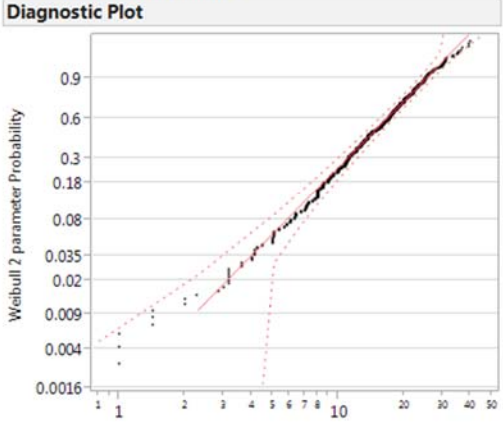
- Distributions with the same AIC (rounded to the nearest tenth) have the same lack of fit (equivalently, the same goodness of fit)
- In this example, the *Weibull* and *Extreme value* distributions are tied for best fit
- Distributions with equivalent goodness of fit may give different predictions
- If there are multiple best fitting distributions, give predictions for all of them

## Best-fitting distribution (cont'd)



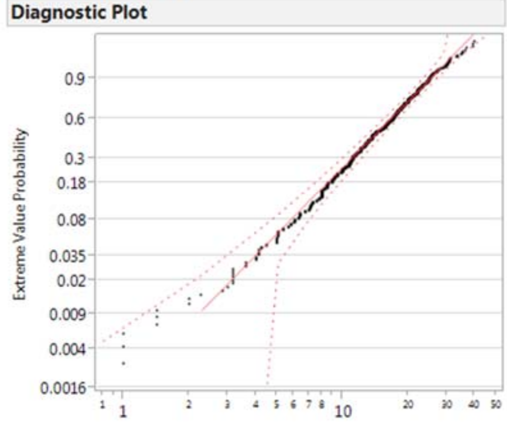
**Fitted 2 parameter Weibull**

**Diagnostic Plot**




**Fitted Extreme Value**

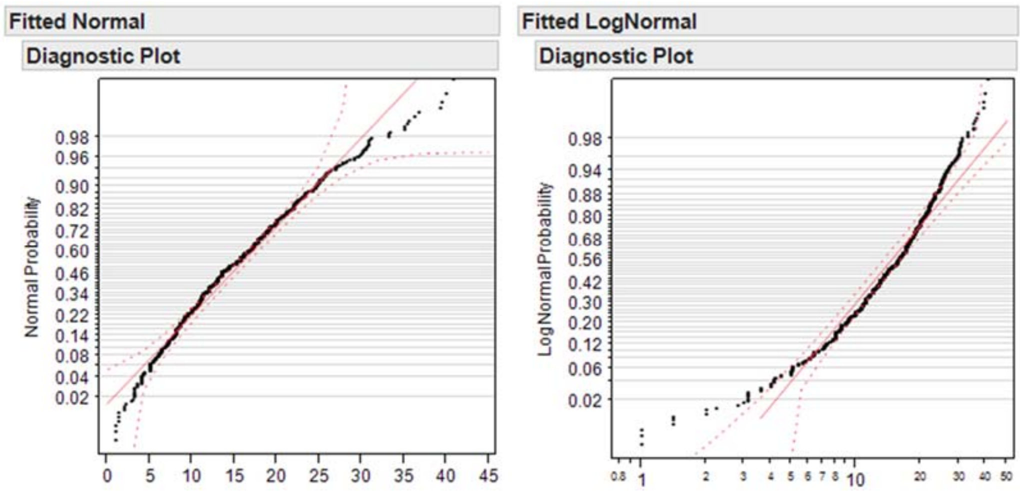
**Diagnostic Plot**




- The diagnostic plots for *Weibull* and *Extreme value* both look good
- Note that the plots are identical
- Fun fact: this always happens with these two distributions

## Best-fitting distribution (cont'd)






Examples of bad diagnostic plots (but by no means the worst)

## Additional guidelines



- Regardless of AIC values, don't use distributions with bad diagnostic plots
- Regardless of AIC values, don't use distributions that predict 0 percent or PPM

## Using the best-fitting distributions: (a) Weibull



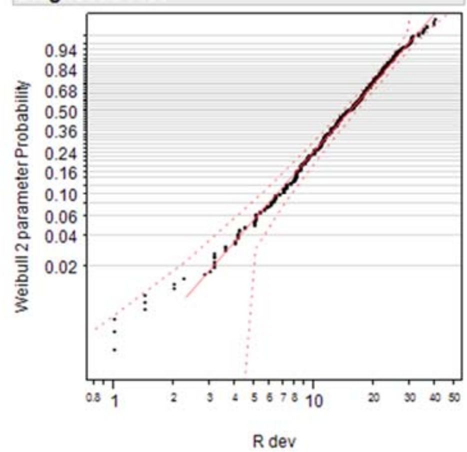
What % of future parts will have *R dev* > 40?

**Compare Distributions**

| Show | Distribution | Number of Parameters | -2*LogLikelihood | AICc |
|---|---|---|---|---|
| ☑ | Weibull | 2 | 4539 | 4543 |
| ☐ | Extreme Value | 2 | 4539 | 4543 |
| ☐ | Johnson Sl | 3 | 4541 | 4547 |
| ☐ | Johnson Su | 4 | 4541 | 4549 |
| ☐ | Normal 3 Mixture | 8 | 4540 | 4556 |
| ☐ | Normal 2 Mixture | 5 | 4550 | 4560 |
| ☐ | Gamma | 2 | 4567 | 4571 |
| ☐ | Normal | 2 | 4580 | 4584 |
| ☐ | LogNormal | 2 | 4673 | 4677 |
| ☐ | GLog | 3 | 4673 | 4679 |
| ☐ | Exponential | 1 | 5054 | 5056 |

**Fitted 2 parameter Weibull**

**Diagnostic Plot**




- Click on the red triangle here
- Select *Spec Limits*
- Enter 40 for *Upper Spec Limit* → OK

## Weibull fit (cont'd)

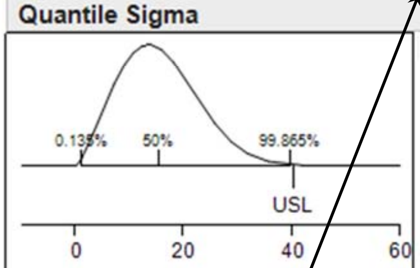


**Fitted 2 parameter Weibull**

**Capability Analysis**

| Specification | Value | Portion | % Actual |
|---|---|---|---|
| Lower Spec Limit | . | Below LSL | . |
| Spec Target | . | Above USL | 0.1475 |
| Upper Spec Limit | 40 | Total Outside | 0.1475 |

**Quantile Sigma**




| Capability | Index |
|---|---|
| CP | . |
| CPK | 1.016 |
| CPM | . |
| CPL | . |
| CPU | 1.016 |

| Portion | Percent | PPM | ~~Sigma Quality~~ |
|---|---|---|---|
| Below LSL | . | . | ~~.~~ |
| Above USL | 0.1158 | 1158.3796 | ~~4.546~~ |
| Total Outside | 0.1158 | 1158.3796 | ~~4.546~~ |

- 0.15% of the data values exceed 40
- 0.12% are predicted to exceed 40 in the population (future production)

# Using the best-fitting distributions: (b) Extreme value

67

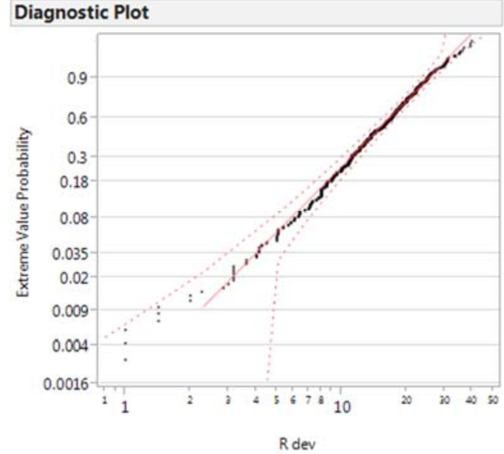
What % of future parts will have *R dev* > 40?

**Compare Distributions**

| Show | Distribution | Number of Parameters | -2*LogLikelihood | AICc |
|---|---|---|---|---|
| ☐ | Weibull | 2 | 4539.3202 | 4543.3 |
| ☑ | Extreme Value | 2 | 4539.3202 | 4543.3 |
| ☐ | Johnson Sl | 3 | 4541.07649 | 4547.1 |
| ☐ | Johnson Su | 4 | 4541.07649 | 4549.1 |
| ☐ | Normal 3 Mixtur | 8 | 4539.81762 | 4556.0 |
| ☐ | Normal 2 Mixtur | 5 | 4549.79406 | 4559.9 |
| ☐ | Gamma | 2 | 4567.15519 | 4571.2 |
| ☐ | Normal | 2 | 4580.47707 | 4584.5 |
| ☐ | LogNormal | 2 | 4672.7545 | 4676.8 |
| ☐ | GLog | 3 | 4672.7545 | 4678.8 |
| ☐ | Exponential | 1 | 5053.88325 | 5055.9 |

**Fitted Extreme Value**

**Diagnostic Plot**




- Click on the red triangle here
- Select *Spec Limits*
- Enter 40 for *Upper Spec Limit* → OK

# Extreme value fit (cont'd)

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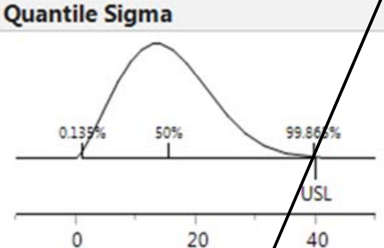
**Fitted Extreme Value**

**Capability Analysis**

| Specification | Value | Portion | % Actual |
|---|---|---|---|
| Lower Spec Limit | . | Below LSL | . |
| Spec Target | . | Above USL | 0.1475 |
| Upper Spec Limit | 40 | Total Outside | 0.1475 |

**Quantile Sigma**




| Capability | Index |
|---|---|
| CP | . |
| CPK | 1.016 |
| CPM | . |
| CPL | . |
| CPU | 1.016 |

| Portion | Percent | PPM | ~~Sigma Quality~~ |
|---|---|---|---|
| Below LSL | . | . | ~~.~~ |
| Above USL | 0.1158 | 1158.3796 | ~~4.546~~ |
| Total Outside | 0.1158 | 1158.3796 | ~~4.546~~ |

- 0.15% of the data values exceed 40
- 0.12% are predicted to exceed 40 in the population (future production)
- The results for *Weibull* and *Extreme Value* are identical (always happens with these two distributions)

## Exercise 3.1



Answer both questions in each case below. For the second question, find the best fitting distribution(s) and use it (them) to find answer(s). Save the analysis scripts, close and save the data tables.

a) Data table *quotation process,* variable TAT. What % of RFQs in the data set have TAT > 15? What % (or PPM) of future RFQs will have TAT > 15?

b) Data set *solution properties,* variable *SG coded*. What % of solution vials in the data set have *SG coded* > 50? What % of future solution vials (or PPM) will have *SG coded* > 50?

## Exercise 3.1 (cont'd)



c) Data table *number and size of defects*, variable *# Defects*. What % of castings in the data set have more than 50 defects? What % of future castings (or PPM) will have more than 50 defects?

d) Data table *casting dimensions*, variable *Length*. What % of castings in the data set have length outside the interval [598, 602]? What % of future castings (or PPM) will have lengths outside this interval?

## Exercise 3.1 (cont'd)



e) Data table *casting dimensions*, variable *Diam*. What % of castings in the data set have diameters outside the interval [49, 51]? What % of future castings (or PPM) will have diameters outside this interval?

Notes

## 4 Introduction to Life Data



Life = elapsed time until the occurrence of some event

- Failure of an item on test
- Planned end of test
- Unplanned end of test
- Failure of an item in service
- Scheduled downtime

Definitions of "time"

- Seconds, minutes, hours
- Days, weeks, months
- Usage cycles, number of moves, distance

## Life data (cont'd)



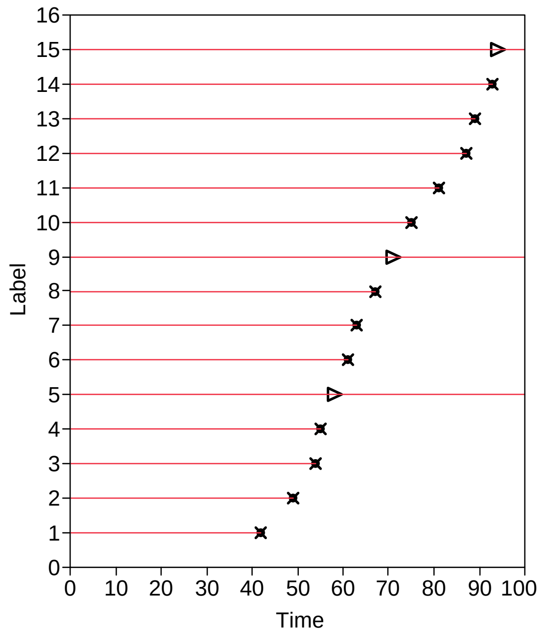
Usually there is one event of primary interest

- Usually, failure of an item

Other events may preempt the event of primary interest

- Planned end of test
- Unplanned end of test
- These are called "suspensions"
- We say that the time to failure is "censored"

# Example: *failures and suspensions.jmp*




- 15 items were tested
- 12 failures (×)
- 3 suspensions ( ▷·······)
- This "event plot" distinguishes suspensions from failures and shows the event times
- If we don't distinguish suspensions from failures, the calculated failure probabilities will be biased upwards
- This will make our reliability look worse than it really is



# Notes

# Cumulative distribution function (CDF)

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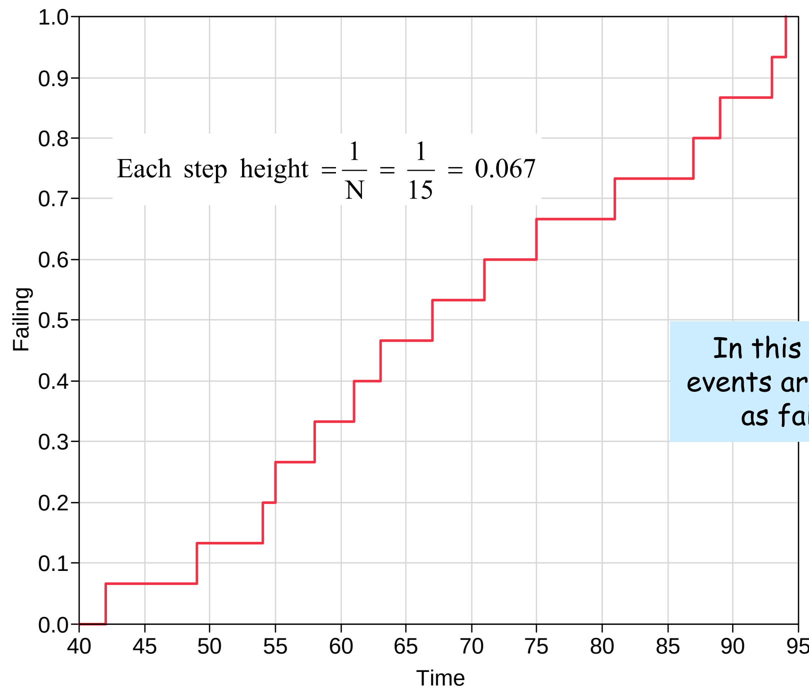


In this plot, all events are treated as failures

# CDF distinguishing suspensions from failures

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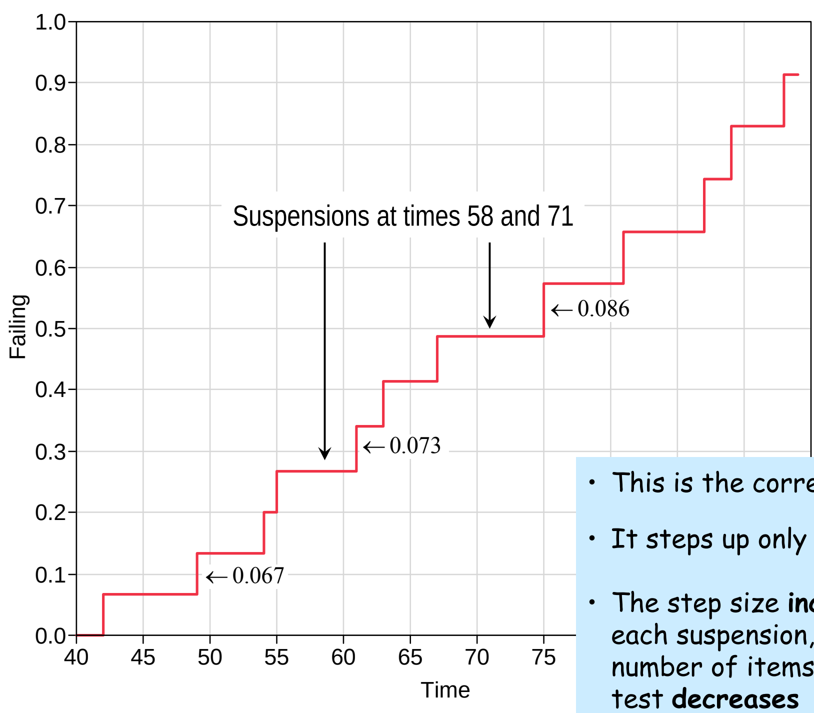
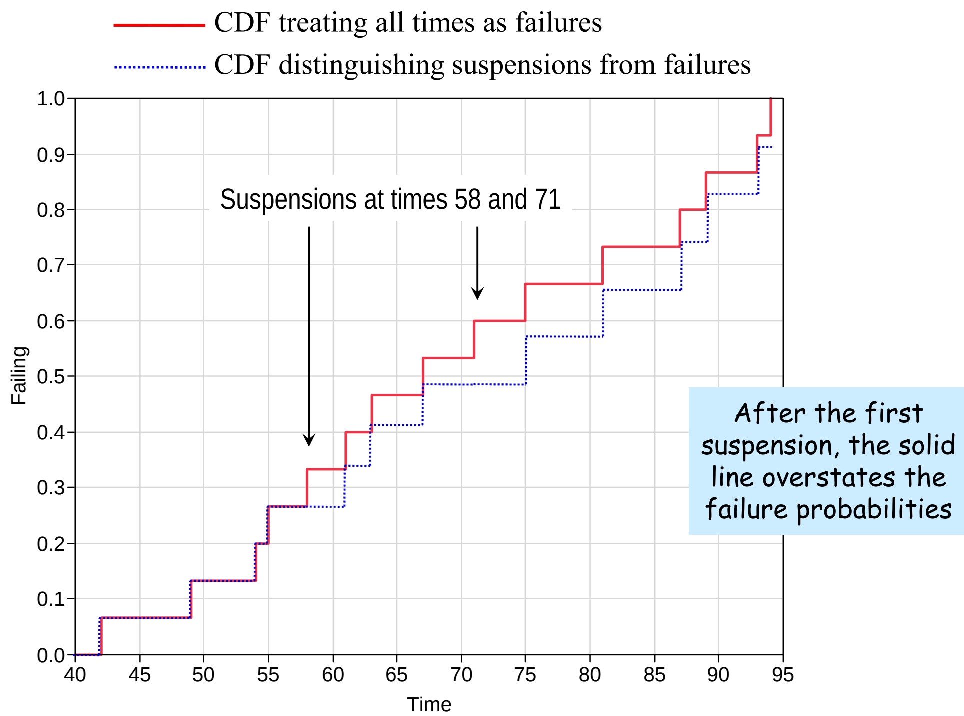



- This is the correct plot
- It steps up only at failure times
- The step size **increases** after each suspension, because the number of items remaining on test **decreases**

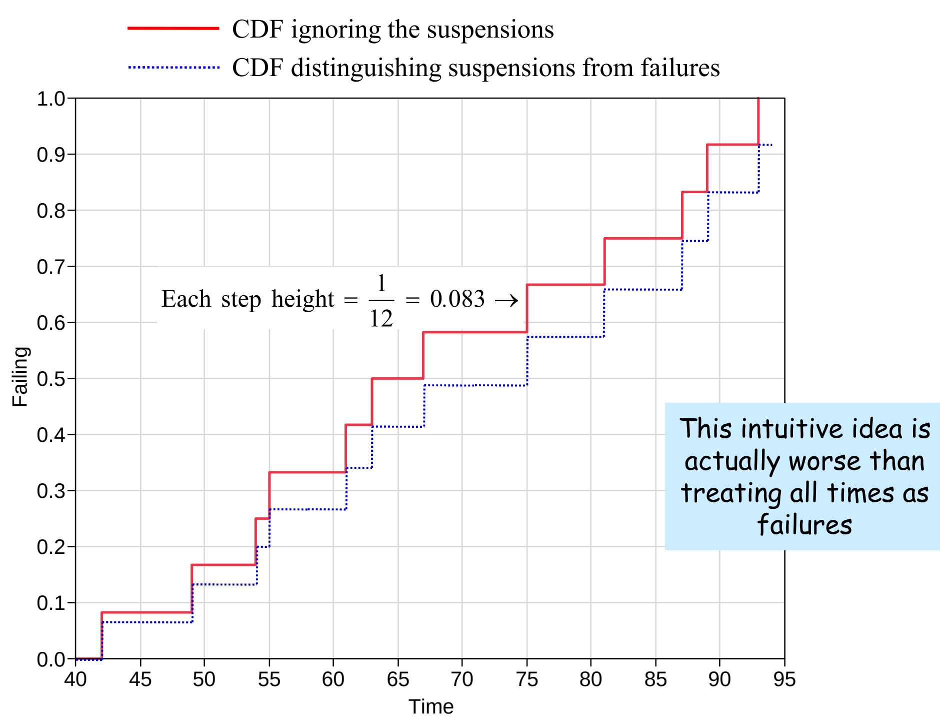
## Overlay of CDFs






## Can't we just ignore the suspensions?





# 5 Analyzing Life Data

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- The Exponential distribution
- The Weibull distribution
- Fitting life distributions in JMP
- Finding and using the best fitting life distribution

Notes

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## Failure curves for the Exponential distribution

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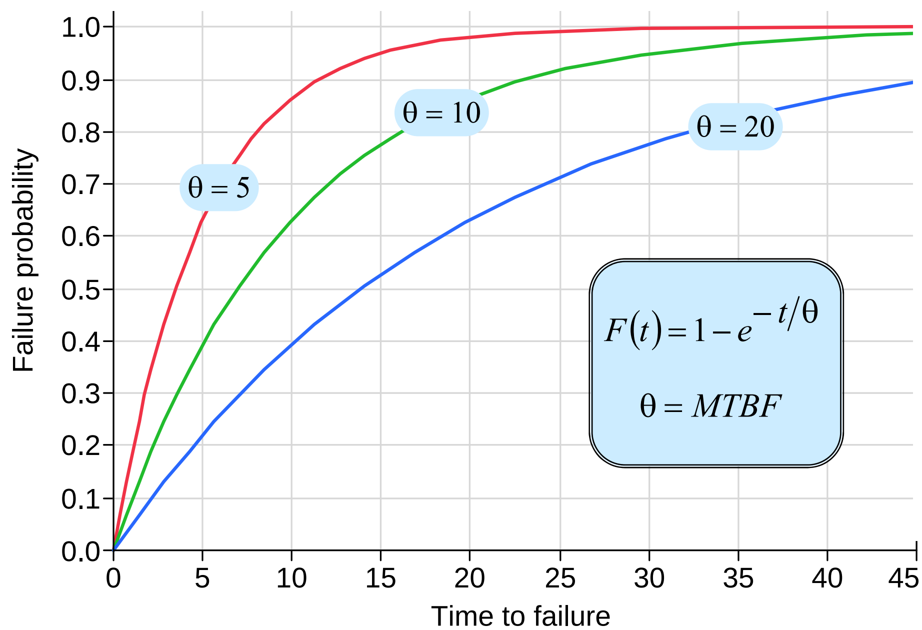


## Notes

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The Exponential distribution is the simplest life distribution. It has only one parameter: the mean time between/before failure (MTBF). The Greek letter θ (theta) is often used to denote the population value of the MTBF.

Shown above are the *failure functions* $F(t)$ for three different Exponential distributions. $F(t)$ is the probability that an item will fail before time $t$.

The *reliability function* is defined as $R(t) = 1 - F(t)$. $R(t)$ is the probability that an item will survive beyond time $t$. The Exponential reliability function is given by $R(t) = \exp(-t/\theta)$.

## Failure curves for the Weibull distribution

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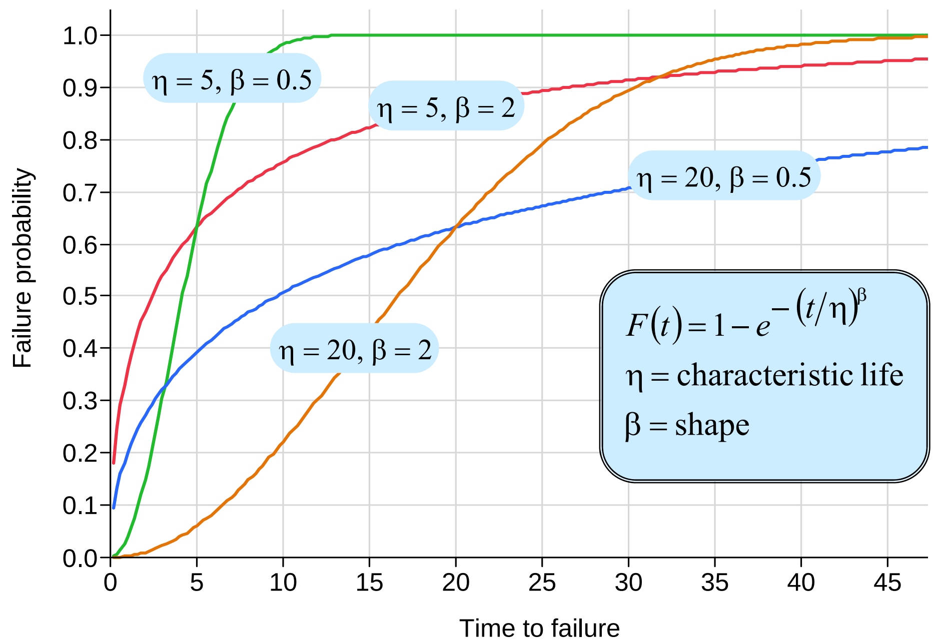


## Notes

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The Weibull distribution was introduced to the reliability engineering community in the 1950s by a man named Waloddi Weibull. (What were his parents thinking?) Prior to that, most reliability work was based on the Exponential distribution. Due to its greater flexibility, the Weibull has become one of the most widely-used life distributions.

The Weibull distribution has two parameters: the *characteristic life* η (eta), and the *shape* β (beta). The characteristic life (η) has the same qualitative interpretation as the MTBF (θ). The shape parameter (β) determines which of two distinct failure modes are represented. When $\beta < 1$, we have a *burn-in* or *infant-mortality* failure mode. When $\beta > 1$, we have a *wear-out* failure mode. A Weibull distribution with $\beta = 1$ is identical to an Exponential distribution with $\theta = \eta$.

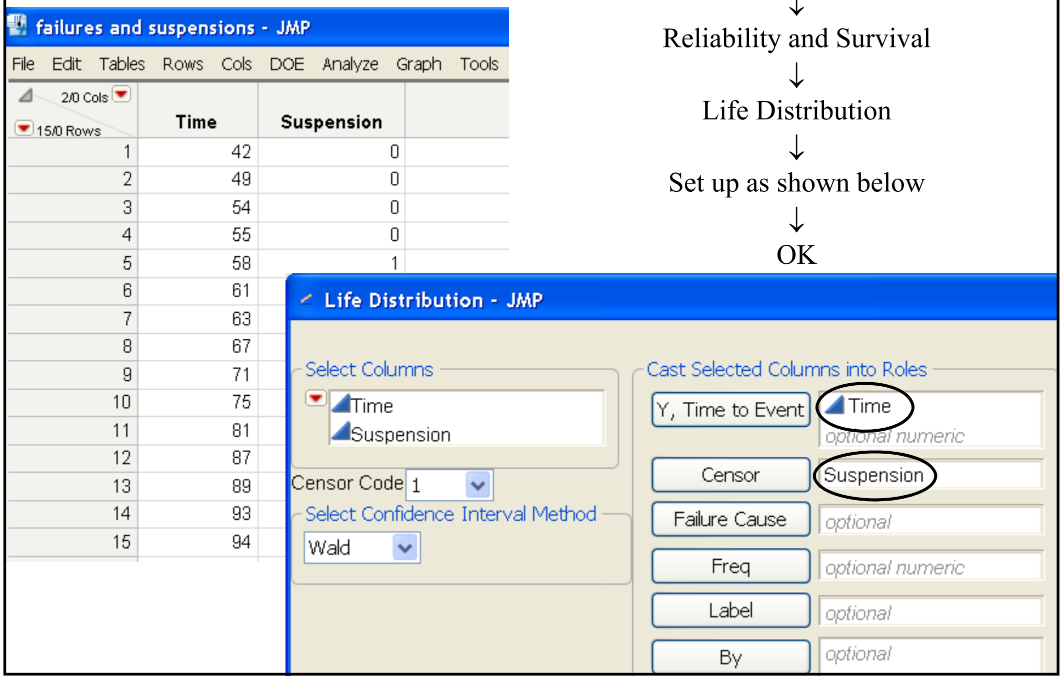
Shown above are failure functions $F(t)$ for four different Weibull distributions. $F(t)$ is the probability that an item will fail before time $t$.

The Weibull reliability function (probability that an item will survive beyond time $t$) is given by $R(t) = \exp[-(t/\eta)^{\beta}]$.

# Fitting life distributions in JMP

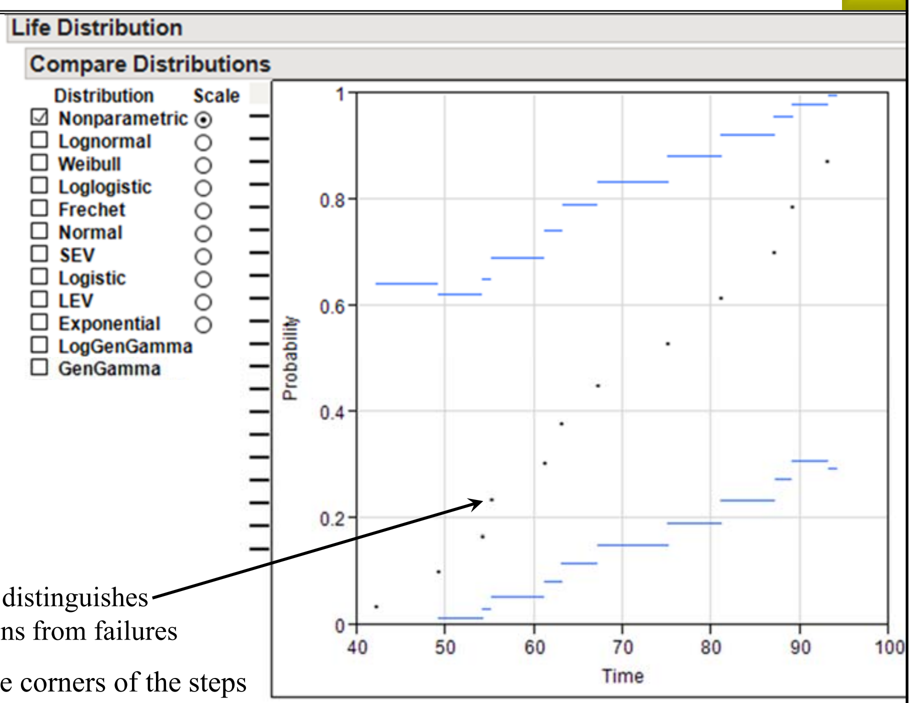
Open data table *failures and suspensions*

Analyze
↓
Reliability and Survival
↓
Life Distribution
↓
Set up as shown below
↓
OK






# Fitting life distributions (cont'd)




- CDF that distinguishes suspensions from failures
- Shows the corners of the steps but not the "staircase"

## Fitting life distributions (cont'd)

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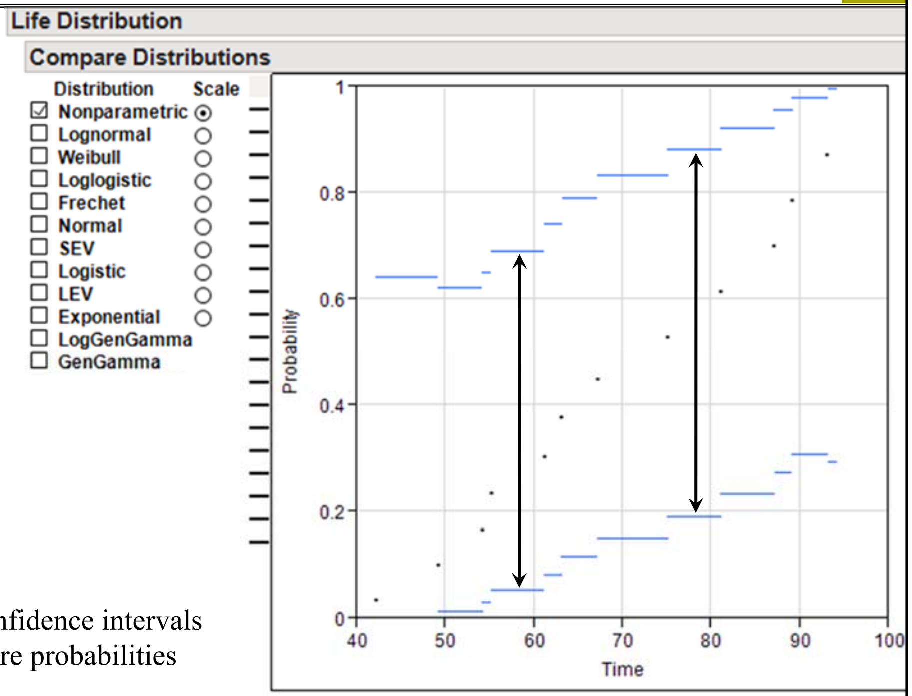


- ↕ 95% confidence intervals for failure probabilities
- ↕ These are "nonparametric"

## Notes

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This analysis is referred to as *nonparametric*, meaning that it is not based on a statistical model (such as the ones listed on the left.) This is a good thing, because statistical models can be wrong. However, there are drawbacks:

a) The nonparametric CDF is discontinuous.

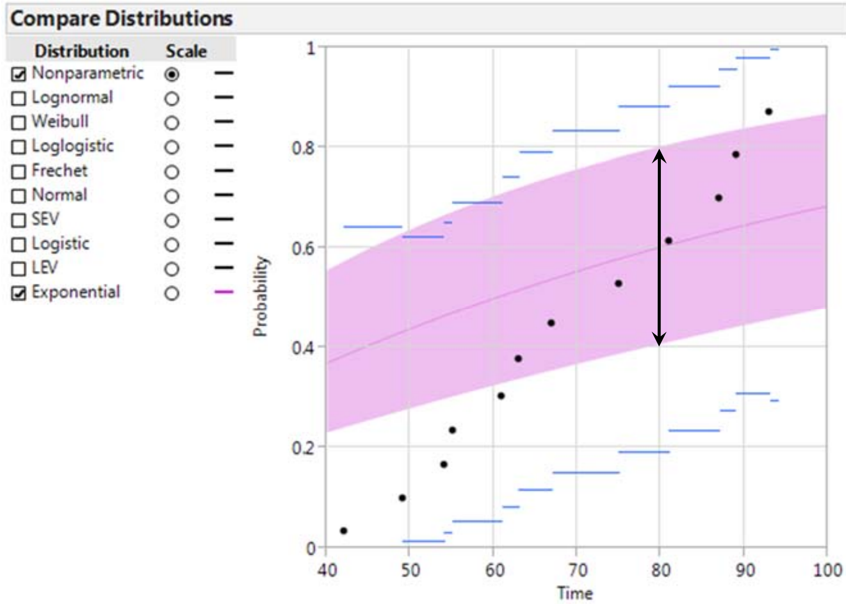
b) Large numbers of failures are required to get margins of error small enough to be useful.

In practice, it is preferable to use a statistical model that fits the data well. This provides a continuous estimate of the failure function and smaller margins of error.

You can change the confidence level by selecting *Change Confidence Level* on the menu produced by the red triangle next to *Life Distribution*.

## Exponential fit — linear probability scale



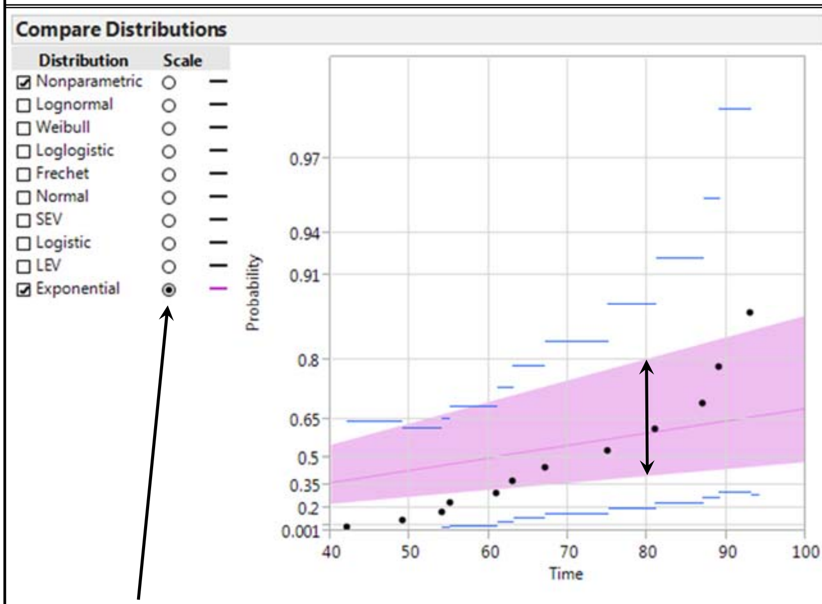



95% confidence interval for $F(80)$ based on the Exponential model

Bad fit — the Exponential failure curve doesn't match the data

## Exponential fit — Exponential probability scale






95% confidence interval for $F(80)$ based on the Exponential model

The *Scale* button modifies the vertical axis so that the failure curve for the chosen distribution plots as a straight line

## Weibull fit — linear probability scale

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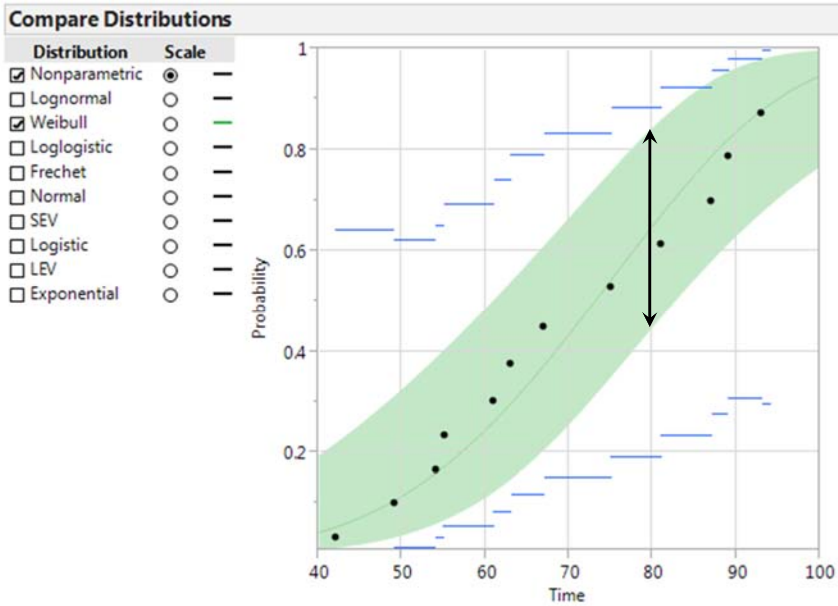
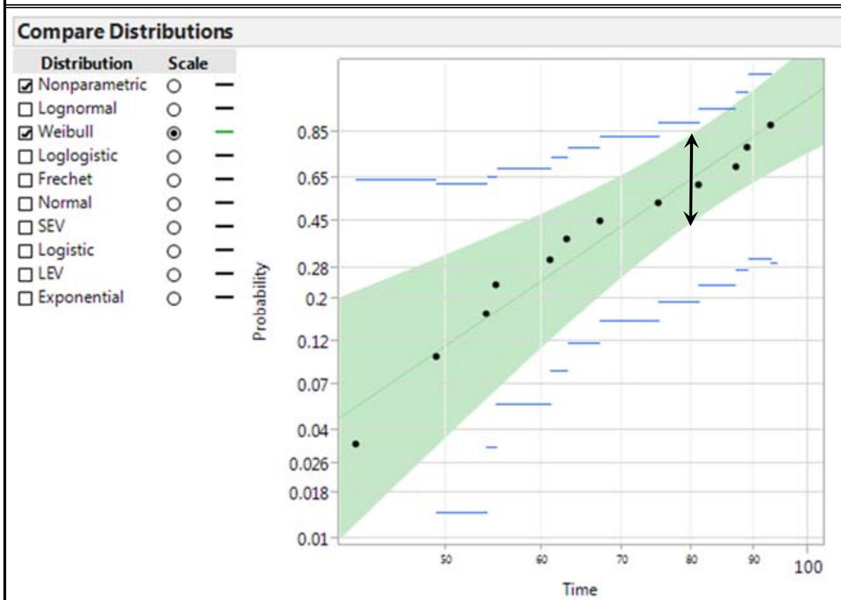



95% confidence interval for $F(80)$ based on the Weibull model

A better fit

## Weibull fit — Weibull probability scale

94




95% confidence interval for $F(80)$ based on the Weibull model

Better is good . . . but what about *best*?

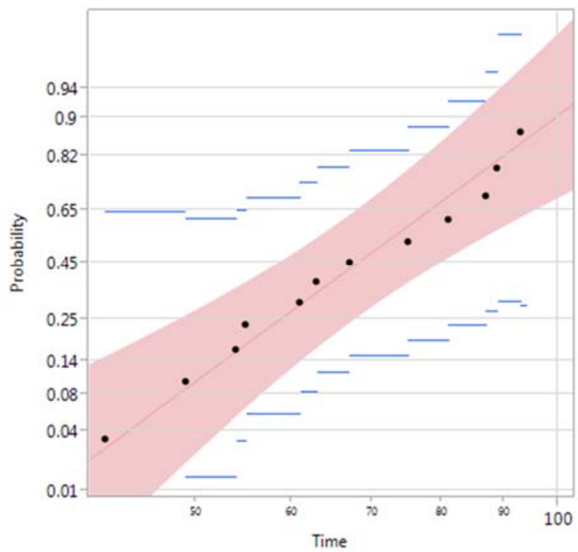
# Finding and using the best fitting distribution



Click the *Life Distribution* red triangle → Fit All Nonnegative

**Compare Distributions**

| Distribution | Scale |
|---|---|
| ☑ Nonparametric | ○ |
| ☑ Lognormal | ◉ |
| ☐ Weibull | ○ |
| ☐ Loglogistic | ○ |
| ☐ Frechet | ○ |
| ☐ Normal | ○ |
| ☐ SEV | ○ |
| ☐ Logistic | ○ |
| ☐ LEV | ○ |
| ☐ Exponential | ○ |




- JMP plots the best fitting model on the corresponding probability scale
- In this case, *Lognormal* appears to give the best fit

*You can't have a negative time to failure!

# Best fitting distribution (cont'd)



**Statistics**

**Model Comparisons**

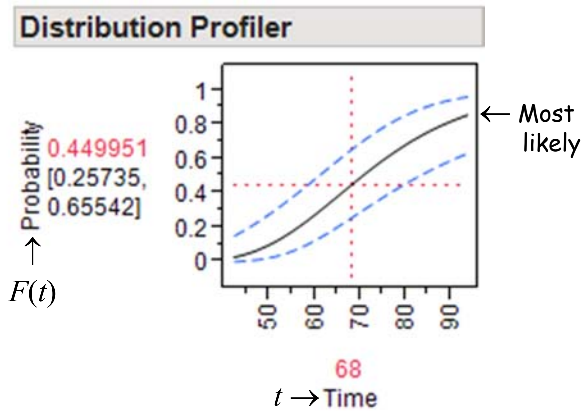
| Distribution | AICc | -2Loglikelihood | BIC |
|---|---|---|---|
| Lognormal | 112.6 | 107.57926 | 112.99536 |
| Weibull | 112.8 | 107.81732 | 113.23342 |
| Loglogistic | 113.3 | 108.33193 | 113.74804 |
| Frechet | 113.8 | 108.75681 | 114.17291 |
| Exponential | 133.4 | 131.06658 | 133.77463 |

- As before, models are ranked by AIC (smaller is better)
- As before, round the AIC values to the nearest tenth
- *Lognormal* gives the best fit

## The distribution profiler

97

- Plots the probability $F(t)$ that an item from this population will fail at or before time $t$
- The solid curve is the *most likely* value of $F(t)$
- For example, the most likely value of $F(68)$ is 0.45 (45%) (shown in red on the left side of the profiler)

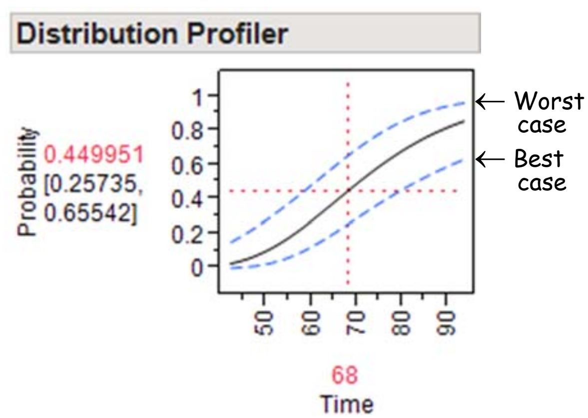



- The corresponding *reliability* $R(t)$ is defined as $1 - F(t)$
- $R(t)$ is the probability that an item from this population will not fail until *after* time $t$
- For example, $R(68) = 0.55$ (55%)

## Distribution profiler (cont'd)

98

- The dashed curves give 95% confidence intervals for $F(t)$
- The upper dashed curve gives the *worst case* value of $F(t)$*
- For example, the worst case value of $F(68)$ is 0.655 (65.5%)

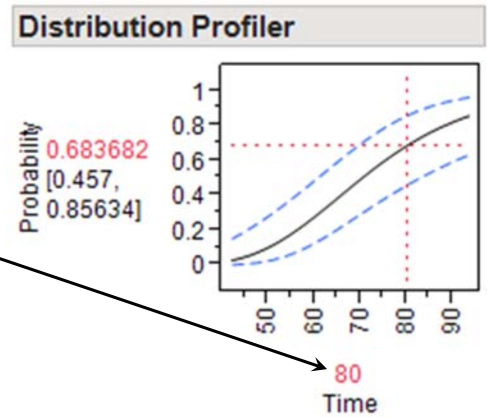



- The lower dashed curve gives the *best case* value of $F(t)$**
- For example, the best case value of $F(68) = 0.275$ (27.5%)

*For Engineering. **For Sales.

## Distribution profiler (cont'd)






- Suppose we are interested in $F(80)$
- Change the value 68 to 80 (click and edit)
- The most likely value of $F(80)$ is 68.4%
- The worst case value of $F(t)$ is 85.6%
- The best case value of $F(80)$ is 45.7%

## Notes

## Exercise 5.1



Open *LSSV2 data sets \ print life* (in JMP). The “time” to failure is *Pages*.

a) Identify the best fitting non-negative distribution(s). Use that (those) distribution(s) to answer the following questions.

b) What is the most likely value of *F*(10,000)?

c) With 95% confidence, what is the worst-case value of *F*(10,000)?

d) Save the analysis script, close and save the data table.

## Exercise 5.2



Open *LSSV2 data sets \ probe reliability* (in JMP). The “time” to failure is *Hits*.

a) Identify the best fitting non-negative distribution(s). Use that (those) distribution(s) to answer the following questions.

b) What is the most likely value of *F*(200)?

c) With 95% confidence, what is the worst-case value of *F*(200)?

d) Save the analysis script, close and save the data table.

## Exercise 5.3



Open *LSSV2 data sets \ field reliability* (in JMP). The time to failure is *Calendar Days*.

a) Identify the best fitting non-negative distribution(s). Use that (those) distribution(s) to answer the following questions.

b) What is the most likely value of *F*(365)?

c) With 95% confidence, what is the worst-case value of *F*(365)?

d) Save the analysis script, close and save the data table.

## Notes

## 6 Categorical MSA Without Standards

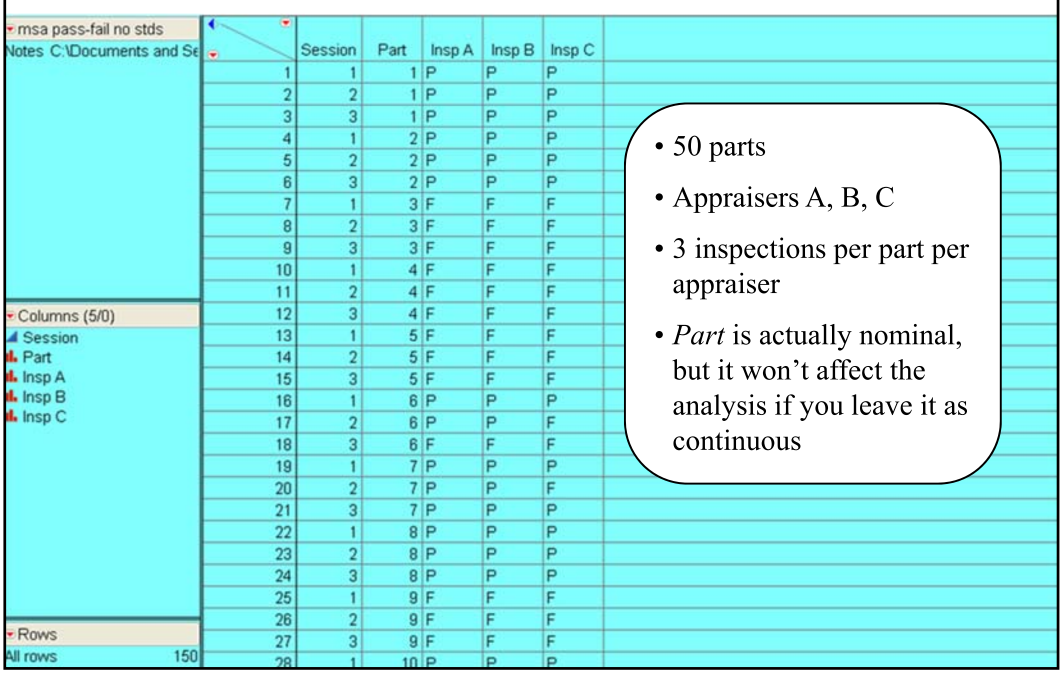
105

- It is preferable to base nominal MSA on a set of items whose true status is known (standards)
- With standards, we can determine the probabilities of passing bad items and failing good ones
- Creating standards can be difficult and time consuming
- Lacking standards, "% agreement within and between appraisers" can serve as a proxy for "% agreement with standard"

## Example 1

106

Open *LSSV2 data sets \ pass-fail no stds* (in JMP)




| | Session | Part | Insp A | Insp B | Insp C |
|---|---|---|---|---|---|
| 1 | 1 | 1 | P | P | P |
| 2 | 2 | 1 | P | P | P |
| 3 | 3 | 1 | P | P | P |
| 4 | 1 | 2 | P | P | P |
| 5 | 2 | 2 | P | P | P |
| 6 | 3 | 2 | P | P | P |
| 7 | 1 | 3 | F | F | F |
| 8 | 2 | 3 | F | F | F |
| 9 | 3 | 3 | F | F | F |
| 10 | 1 | 4 | F | F | F |
| 11 | 2 | 4 | F | F | F |
| 12 | 3 | 4 | F | F | F |
| 13 | 1 | 5 | F | F | F |
| 14 | 2 | 5 | F | F | F |
| 15 | 3 | 5 | F | F | F |
| 16 | 1 | 6 | P | P | P |
| 17 | 2 | 6 | P | P | F |
| 18 | 3 | 6 | F | F | F |
| 19 | 1 | 7 | P | P | P |
| 20 | 2 | 7 | P | P | F |
| 21 | 3 | 7 | P | P | P |
| 22 | 1 | 8 | P | P | P |
| 23 | 2 | 8 | P | P | P |
| 24 | 3 | 8 | P | P | P |
| 25 | 1 | 9 | F | F | F |
| 26 | 2 | 9 | F | F | F |
| 27 | 3 | 9 | F | F | F |
| 28 | 1 | 10 | P | P | P |

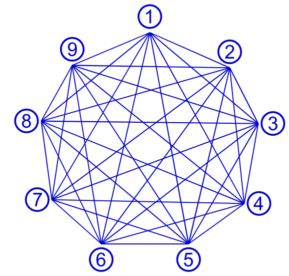
- 50 parts
- Appraisers A, B, C
- 3 inspections per part per appraiser
- *Part* is actually nominal, but it won't affect the analysis if you leave it as continuous

# Agreement within & between appraisers



| | Session | Part | Insp A | Insp B | Insp C |
|---|---|---|---|---|---|
| 1 | 1 | 1 | P | P | P |
| 2 | 2 | 1 | P | P | P |
| 3 | 3 | 1 | P | P | P |
| 4 | 1 | 2 | P | P | P |
| 5 | 2 | 2 | P | P | P |
| 6 | 3 | 2 | P | P | P |
| 7 | 1 | 3 | F | F | F |
| 8 | 2 | 3 | F | F | F |
| 9 | 3 | 3 | F | F | F |
| 10 | 1 | 4 | F | F | F |
| 11 | 2 | 4 | F | F | F |
| 12 | 3 | 4 | F | F | F |
| 13 | 1 | 5 | F | F | F |
| 14 | 2 | 5 | F | F | F |
| 15 | 3 | 5 | F | F | F |
| 16 | 1 | 6 | P | P | P |
| 17 | 2 | 6 | P | P | F |
| 18 | 3 | 6 | F | F | F |
| 19 | 1 | 7 | P | P | P |
| 20 | 2 | 7 | P | P | F |
| 21 | 3 | 7 | P | P | P |
| 22 | 1 | 8 | P | P | P |
| 23 | 2 | 8 | P | P | P |
| 24 | 3 | 8 | P | P | P |
| 25 | 1 | 9 | F | F | F |
| 26 | 2 | 9 | F | F | F |
| 27 | 3 | 9 | F | F | F |
| 28 | 1 | 10 | P | P | P |
| 29 | 2 | 10 | P | P | P |
| 30 | 3 | 10 | P | P | P |
| 31 | 1 | 11 | P | P | P |
| 32 | 2 | 11 | P | P | P |
| 33 | 3 | 11 | P | P | P |
| 34 | 1 | 12 | F | F | F |
| 35 | 2 | 12 | F | F | P |
| 36 | 3 | 12 | F | F | F |

- 100% agreement



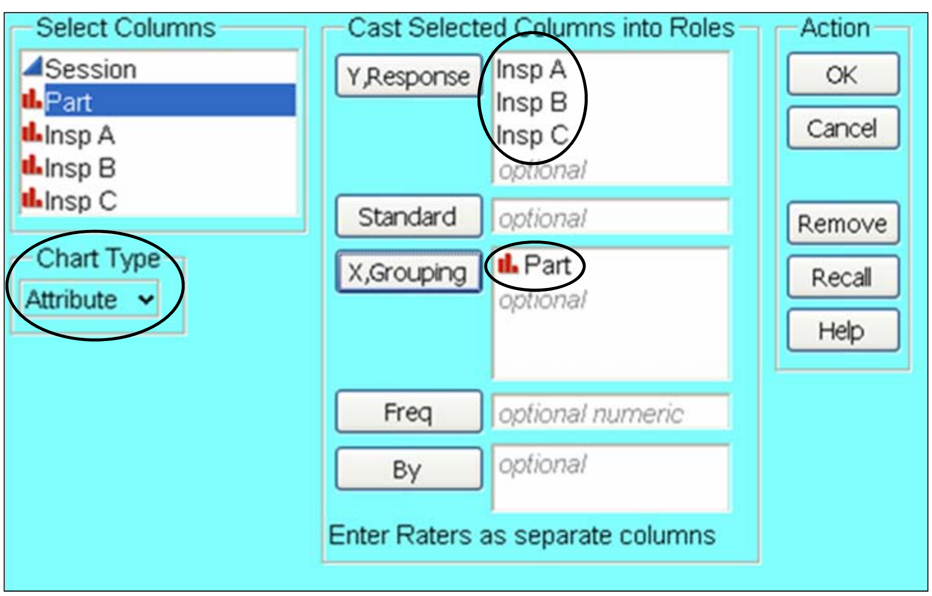

- 36 opportunities for pairwise agreement
- 16 pairwise agreements
- Agreement = 16/36 = 0.444

- 36 opportunities for pairwise agreement
- 8 pairwise disagreements
- Agreement = 28/36 = 0.778

# Analyzing a categorical MSA without standards

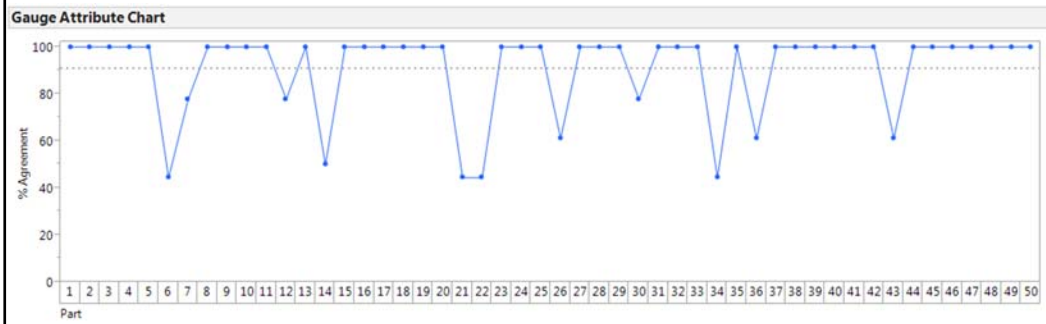


*Analyze → Quality and Process → Variability / Attribute Gauge Chart*



# Agreement report

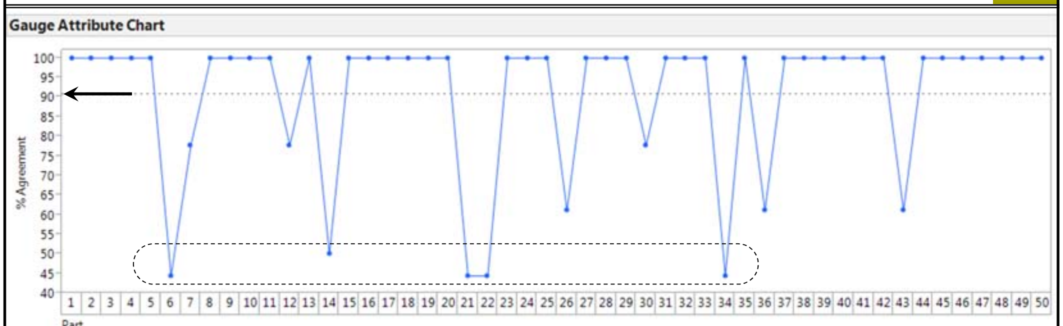





- Plot of the agreement percentages for the items in the study
- It is helpful to rescale the vertical axis
- See next slide

# Agreement report (cont'd)

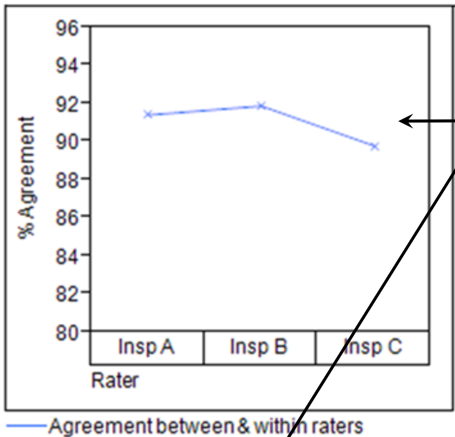





- The horizontal dotted line marks the "agreement grand mean"
- In this example, the agreement grand mean is a little over 90
- Nowhere in the report is this number printed — bad JMP!
- If the agreement grand mean is too low, follow-up should focus on the items with the lowest % agreement
- There are no recognized standards for the agreement grand mean. A lower bound of 95% is fairly common. 99% is often used in applications involving safety.

# Agreement report (cont'd)






- These are the agreement percentages for each appraiser
- The appraiser with the lowest percentage represents the greatest opportunity for improvement
- Sometimes the smallest % agreement among the appraisers is used as the metric

**Agreement Report**

| Rater | % Agreement | 95% Lower CI | 95% Upper CI |
|---|---|---|---|
| Insp A | 91.4286 | 89.5082 | 93.0248 |
| Insp B | 91.9048 | 90.0502 | 93.4388 |
| Insp C | 89.8095 | 87.6057 | 91.6588 |

| Number Inspected | Number Matched | ~~% Agreement~~ | 95% Lower CI |
|---|---|---|---|
| 50 | 39 | ~~78.000~~ | 64.758 |

- Percentage of items for which agreement was 100%
- This should not be used as a metric

# Notes

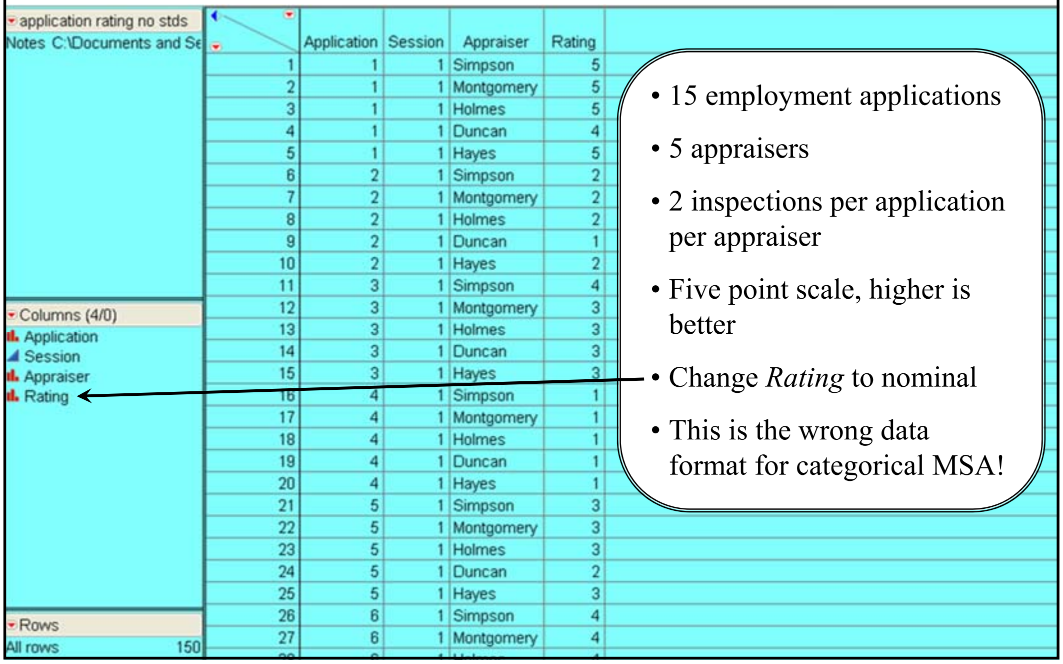


Save the script, close and save the data table.

# Example 2



Open *LSSV2 data sets \ application rating no stds* (in JMP)



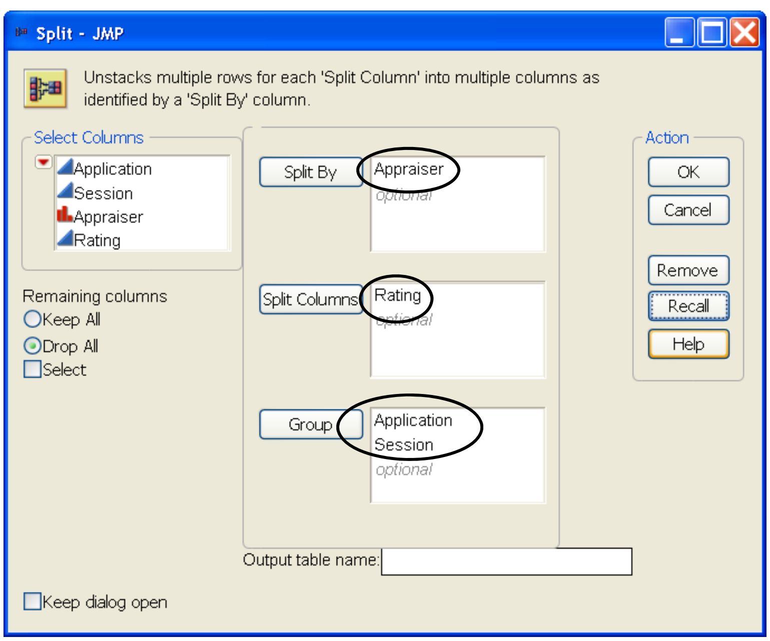

| | Application | Session | Appraiser | Rating |
|---|---|---|---|---|
| 1 | 1 | 1 | Simpson | 5 |
| 2 | 1 | 1 | Montgomery | 5 |
| 3 | 1 | 1 | Holmes | 5 |
| 4 | 1 | 1 | Duncan | 4 |
| 5 | 1 | 1 | Hayes | 5 |
| 6 | 2 | 1 | Simpson | 2 |
| 7 | 2 | 1 | Montgomery | 2 |
| 8 | 2 | 1 | Holmes | 2 |
| 9 | 2 | 1 | Duncan | 1 |
| 10 | 2 | 1 | Hayes | 2 |
| 11 | 3 | 1 | Simpson | 4 |
| 12 | 3 | 1 | Montgomery | 3 |
| 13 | 3 | 1 | Holmes | 3 |
| 14 | 3 | 1 | Duncan | 3 |
| 15 | 3 | 1 | Hayes | 3 |
| 16 | 4 | 1 | Simpson | 1 |
| 17 | 4 | 1 | Montgomery | 1 |
| 18 | 4 | 1 | Holmes | 1 |
| 19 | 4 | 1 | Duncan | 1 |
| 20 | 4 | 1 | Hayes | 1 |
| 21 | 5 | 1 | Simpson | 3 |
| 22 | 5 | 1 | Montgomery | 3 |
| 23 | 5 | 1 | Holmes | 3 |
| 24 | 5 | 1 | Duncan | 2 |
| 25 | 5 | 1 | Hayes | 3 |
| 26 | 6 | 1 | Simpson | 4 |
| 27 | 6 | 1 | Montgomery | 4 |

- 15 employment applications
- 5 appraisers
- 2 inspections per application per appraiser
- Five point scale, higher is better
- Change *Rating* to nominal
- This is the wrong data format for categorical MSA!

# Unstacking a data table



*Tables → Split*



# Example 2 in required format



Untitled 12
Source

Columns (7/0)
Application
Session
Duncan
Hayes
Holmes
Montgomery
Simpson

Rows
All rows 30

| | Application | Session | Duncan | Hayes | Holmes | Montgomery | Simpson |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 4 | 5 | 5 | 5 | 5 |
| 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 |
| 3 | 3 | 1 | 3 | 3 | 3 | 3 | 4 |
| 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 5 | 1 | 2 | 3 | 3 | 3 | 3 |
| 6 | 6 | 1 | 4 | 4 | 4 | 4 | 4 |
| 7 | 7 | 1 | 4 | 5 | 5 | 5 | 5 |
| 8 | 8 | 1 | 3 | 3 | 3 | 3 | 3 |
| 9 | 9 | 1 | 1 | 2 | 2 | 2 | 2 |
| 10 | 10 | 1 | 3 | 5 | 4 | 4 | 4 |
| 11 | 11 | 1 | 1 | 2 | 1 | 1 | 1 |
| 12 | 12 | 1 | 2 | 3 | 3 | 3 | 3 |
| 13 | 13 | 1 | 5 | 5 | 5 | 5 | 5 |
| 14 | 14 | 1 | 2 | 2 | 2 | 2 | 2 |
| 15 | 15 | 1 | 4 | 4 | 4 | 4 | 4 |
| 16 | 1 | 2 | 4 | 5 | 5 | 5 | 4 |
| 17 | 2 | 2 | 1 | 2 | 2 | 2 | 2 |
| 18 | 3 | 2 | 3 | 3 | 4 | 4 | 4 |
| 19 | 4 | 2 | 1 | 1 | 1 | 1 | 1 |
| 20 | 5 | 2 | 2 | 3 | 3 | 3 | 3 |
| 21 | 6 | 2 | 4 | 4 | 4 | 5 | 5 |
| 22 | 7 | 2 | 4 | 5 | 5 | 5 | 5 |
| 23 | 8 | 2 | 3 | 4 | 3 | 3 | 3 |
| 24 | 9 | 2 | 1 | 2 | 2 | 2 | 2 |
| 25 | 10 | 2 | 3 | 5 | 4 | 4 | 4 |
| 26 | 11 | 2 | 1 | 2 | 1 | 1 | 1 |
| 27 | 12 | 2 | 2 | 3 | 3 | 3 | 3 |
| 28 | 13 | 2 | 5 | 5 | 5 | 5 | 5 |

# Example 2 (cont'd)



*Analyze → Quality and Process → Variability / Attribute Gauge Chart*

# Example 2 (cont’d)

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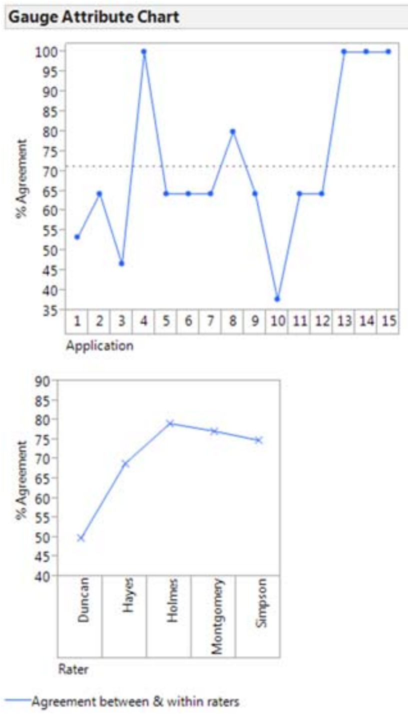


- The agreement grand mean is about 71 — way too low
- Follow-up: focus on application 10, and maybe 1 and 3 as well
- *Greatest opportunity for improvement:* further training of Duncan, and maybe Hayes as well

**Agreement Report**

| Rater | % Agreement | 95% Lower CI | 95% Upper CI |
|---|---|---|---|
| Duncan | 49.8039 | 27.2673 | 72.4205 |
| Hayes | 69.0196 | 43.9053 | 86.3784 |
| Holmes | 79.2157 | 53.9935 | 92.5247 |
| Montgomery | 77.2549 | 51.9716 | 91.4246 |
| Simpson | 74.9020 | 49.5997 | 90.0500 |

| Number Inspected | Number Matched | ~~% Agreement~~ | 95% Lower CI | 95% Upper CI |
|---|---|---|---|---|
| 15 | 4 | ~~26.667~~ | 10.897 | 51.950 |

# Notes

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Save the analysis script to the data table, close and save the data table as:

*application rating no stds unstacked.jmp*

## Exercise 6.1



Open *LSSV2 data sets \ print samples 1 no stds*. In this study 3 appraisers inspected 18 print samples 3 times each.

a) Reformat the file as needed and run the analysis.

b) Record the approximate agreement grand mean.

c) Which sample(s) would be most useful in follow-up?

d) Which appraiser has the lowest % agreement, and what is the % agreement?

e) Save the script, close and save the data table as *print samples 1 no stds unstacked.*

## Exercise 6.2



Open *LSSV2 data sets \ print samples 2 no stds*. This is the follow-up study after the appraisers received additional training.

a) Reformat the file as needed and run the analysis.

b) Record the approximate agreement grand mean.

c) Which appraiser has the lowest % agreement, and what is the % agreement?

d) Save the script, close and save the data table as *print samples 2 no stds unstacked.*

# 7 Comparing Populations — Continuous Y



- Example of comparing populations
- Analysis of variance (ANOVA) for comparing populations
- Interpreting P values
- Degrees of freedom for signal and noise
- ANOVA in JMP

Notes



Y variables are characteristics of parts or transactions that determine customer satisfaction, or lack thereof. The provide the data from which project metrics can be computed.

Comparison of statistical populations is equivalent to $Y = f(X)$ analysis where the X variable is categorical. The distinct values of the X variable define the populations or sub-populations to be compared.

JMP uses the term *continuous* for quantitative variables. Except in the DOE section, JMP uses the term *nominal* for categorical variables.

## Example of comparing populations

123

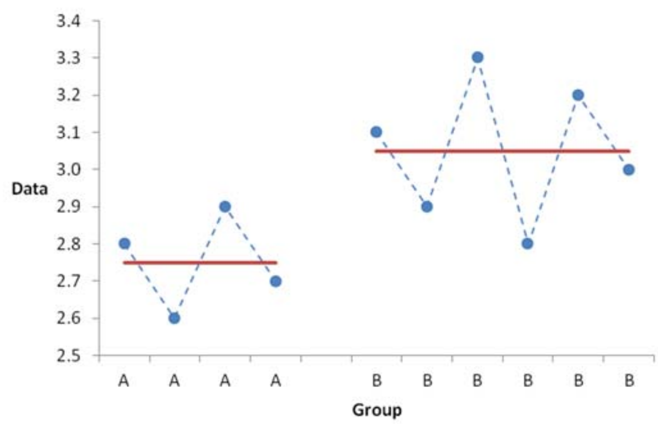
| Group | Data | Avg. | SD |
|---|---|---|---|
| A | 2.8 | 2.75 | 0.129 |
| A | 2.6 | | |
| A | 2.9 | | |
| A | 2.7 | | |
| B | 3.1 | 3.05 | 0.187 |
| B | 2.9 | | |
| B | 3.3 | | |
| B | 2.8 | | |
| B | 3.2 | | |
| B | 3.0 | | |

- We have two groups of data
- Could be a *before/after* comparison
- Could be a *stratification* analysis

- The sample means for the two groups are different
- Is this enough to conclude that the *population* means are different?

## Example (cont'd)

124




- Plotting the data is helpful, but doesn't give a definitive answer
- How far apart do the sample means have to be before we can say the population means are different?
- How do we take the *scatter* around the means into account?

# ANOVA for comparing populations (1 of 6)

125

*LSSV2 other stuff \ ANOVA two groups*

| B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Group** | | **Data** | | **Grand mean** | | **Variance** | | **Group** | | **Error** | |
| A | | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 | |
| A | | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 | |
| A | | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 | |
| A | | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 | |
| B | | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 | |
| B | | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 | |
| B | | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 | |
| B | | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 | |
| B | | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 | |
| B | | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 | |

# ANOVA (1 of 6, cont'd)

126

This worksheet shows all the calculations used to determine, based on the data, whether or not the population means are different.

The first step is to calculate the *Variance* column by subtracting the grand mean from the *Data* column. The *Variance* is then decomposed into *Group* (the "signal") plus *Error* (the "noise").

The *Group* column captures the portion of total variation caused by the difference between the sample means. The *Error* column captures the rest of the variation, variously called the *residual*, *unexplained*, or *noise* variation.

## ANOVA (2 of 6)

127

*LSSV2 other stuff \ ANOVA two groups*

| A | B | C | D | E | F | G | H | I | J | K | L | M |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Group** | | **Data** | | **Grand mean** | | **Variance** | | **Group** | | **Error** | |
| | A | | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 | |
| | A | | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 | |
| | A | | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 | |
| | A | | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 | |
| | B | | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 | |
| | B | | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 | |
| | B | | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 | |
| | B | | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 | |
| | B | | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 | |
| | B | | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 | |
| Degrees of freedom (DF) | | | 10 | − | 1 | = | 9 | = | 1 | + | 8 | |

## ANOVA (2 of 6, cont'd)

128

The *Data* column consists of 10 mathematically independent quantities. We describe this by saying it has 10 *degrees of freedom* (DF).

The *Grand mean* column consists of 10 values, but they are all identical. This column has 1 DF.

The *Variance* column contains 10 values, but they are mathematically constrained to sum to 0. This column contains only 9 independent quantities, so it has 9 DF.

The *Group* column inherits the zero-sum constraint from the *Variance* column, and it consists of only 2 distinct values. This column contains only one independent quantity, so it has 1 DF.

The *Error* column has 8 DF, because DF have to add up.

The DF for *Group* and *Error* play a role in determining whether or not the population means are different.

# ANOVA (3 of 6)



*LSSV2 other stuff \ ANOVA two groups*

| Group | Data | | Grand mean | | Variance | | Group | | Error |
|---|---|---|---|---|---|---|---|---|---|
| A | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 |
| A | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 |
| A | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 |
| A | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 |
| B | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 |
| B | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 |
| B | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 |
| B | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 |
| B | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 |
| B | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 |
| Degrees of freedom (DF) | 10 | − | 1 | = | 9 | = | 1 | + | 8 |
| Sum of squares (SS) | 86.29 | − | 85.85 | = | 0.441 | = | 0.216 | + | 0.225 |
| Mean square (MS) | *(SS / DF)* | | | | 0.049 | | 0.216 | | 0.028 |

# ANOVA (3 of 6, cont'd)



The sum of squares (SS) is a measure of the magnitude of each column. It is the sum of the squares of the values in a column.

The sums of squares for the *Variance*, *Group*, and *Error* columns are usually much smaller than those of the *Data* and *Grand mean* columns.

The mean square (MS) is the statistically normalized measure of the magnitude of each column. It is the SS for a column divided by the DF for that column.

The mean squares for the *Data* and *Grand mean* columns play no role in determining whether or not the population means are different, so the MS is usually calculated only for the *Variance*, *Group*, and *Error* columns.

## ANOVA (4 of 6)



*LSSV2 other stuff \ ANOVA two groups*

| Group | Data | | Grand mean | | Variance | | Group | | Error |
|---|---|---|---|---|---|---|---|---|---|
| A | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 |
| A | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 |
| A | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 |
| A | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 |
| B | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 |
| B | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 |
| B | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 |
| B | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 |
| B | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 |
| B | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 |
| Degrees of freedom (DF) | 10 | − | 1 | = | 9 | = | 1 | + | 8 |
| Sum of squares (SS) | 86.29 | − | 85.85 | = | 0.441 | = | 0.216 | + | 0.225 |
| Mean square (MS) | *(SS / DF)* | | | | 0.049 | | 0.216 | | 0.028 |
| F ratio | (*Group MS / Error MS*) | | | | | | | | 7.680 |

## ANOVA (4 of 6, cont'd)



The *Group* MS measures the magnitude of the variation caused by the difference between the sample means.

The *Error* MS measures the magnitude of the variation caused by everything *except* the difference between the sample means.

The *F ratio* is the *Group* MS divided by *Error* MS. It is a signal-to-noise ratio. The larger the F ratio, the stronger the evidence of a difference between the population means.

## ANOVA (5 of 6)



| Group | Data | | Grand mean | | Variance | | Group | | Error |
|---|---|---|---|---|---|---|---|---|---|
| A | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 |
| A | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 |
| A | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 |
| A | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 |
| B | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 |
| B | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 |
| B | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 |
| B | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 |
| B | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 |
| B | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 |
| Degrees of freedom (DF) | 10 | − | 1 | = | 9 | = | 1 | + | 8 |
| Sum of squares (SS) | 86.29 | − | 85.85 | = | 0.441 | = | 0.216 | + | 0.225 |
| Mean square (MS) | *(SS / DF)* | | | | 0.049 | | 0.216 | | 0.028 |
| F ratio | *(Group MS / Error MS)* | | | | | | | | 7.680 |
| P value | *(Probability of an F ratio this large by chance alone)* | | | | | | | | 0.0242 |

## ANOVA (5 of 6, cont'd)



The *P value* is a probability calculation based on the F ratio, the DF for the *Group* column, and the DF for the *Error* column.

The P value should be interpreted as the probability of *no difference* between the population means.

If there are 3 or more groups, it should be interpreted as the probability that the population means are *all the same*.

## Interpreting P values

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| P value | Evidence that populations are different or variables are correlated | Confidence level (CL) |
|---|---|---|
| 1.00 – 0.15 | None | None |
| 0.15 – 0.05 | Some | 85% ≤ CL < 95% |
| 0.05 – 0.01 | Strong | 95% ≤ CL < 99% |
| 0.01 – 0.0001 | Very strong | CL ≥ 99% |

## P values (cont'd)

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As shown above, the P value has fixed reference values for interpretation.

The P value is inversely related to the F ratio:

- The smaller the P value, the stronger the evidence of a difference in the population means.

If there are 3 or more groups, the interpretation is:

- The smaller the P value, the stronger the evidence of one or more differences among the population means.

## ANOVA (6 of 6)



| Group | Data | | Grand mean | | Variance | | Group | | Error |
|---|---|---|---|---|---|---|---|---|---|
| A | 2.8 | | 2.93 | | -0.13 | | -0.18 | | 0.05 |
| A | 2.6 | | 2.93 | | -0.33 | | -0.18 | | -0.15 |
| A | 2.9 | | 2.93 | | -0.03 | | -0.18 | | 0.15 |
| A | 2.7 | | 2.93 | | -0.23 | | -0.18 | | -0.05 |
| B | 3.1 | − | 2.93 | = | 0.17 | = | 0.12 | + | 0.05 |
| B | 2.9 | | 2.93 | | -0.03 | | 0.12 | | -0.15 |
| B | 3.3 | | 2.93 | | 0.37 | | 0.12 | | 0.25 |
| B | 2.8 | | 2.93 | | -0.13 | | 0.12 | | -0.25 |
| B | 3.2 | | 2.93 | | 0.27 | | 0.12 | | 0.15 |
| B | 3.0 | | 2.93 | | 0.07 | | 0.12 | | -0.05 |
| Degrees of freedom (DF) | 10 | − | 1 | = | 9 | = | 1 | + | 8 |
| Sum of squares (SS) | 86.29 | − | 85.85 | = | 0.441 | = | 0.216 | + | 0.225 |
| Mean square (MS) | *(SS / DF)* | | | | 0.049 | | 0.216 | | 0.028 |
| F ratio | *(Group MS / Error MS)* | | | | | | | | 7.680 |
| P value | *(Probability of an F ratio this large by chance alone)* | | | | | | | | 0.0242 |
| Root mean square (RMS) | *(Square root of MS)* | | | | 0.221 | | | | 0.168 |

## ANOVA (6 of 6, cont'd)



The *Root Mean Square* (RMS) for a column is the square root of the MS for that column.

The RMS for the *Variance* column (0.221) is equal to the usual standard deviation of the data (STDEV function in Excel).

The RMS for the *Error* column (0.168) is the standard deviation of the noise variation (error, residual, unexplained, etc.).

JMP uses the term *Root Mean Square Error* (RMSE) for the RMS of the *Error* column.*

*Given that Statistics is a body of knowledge dedicated to quantifying and reducing variation, the variation in statistical terminology is appalling.

## Degrees of freedom for comparing populations



N = total sample size

G = number of groups being compared

G – 1 = DF for the group column

N – G = DF for the error column

- The *Error* DF is more important than the *Group* DF
- It determines the accuracy of the predicted values
- Larger is better, 10 is OK, bare minimum is 5
- When DF is mentioned without a qualifier, it always means *Error* DF

## Exercise 7.1



Open *ANOVA three groups*. Enter the appropriate numbers and formulas into the white cells to produce an ANOVA for the data shown here.

| | Group | Data | | Grand mean | | Variance | | Group | | Error |
|---|---|---|---|---|---|---|---|---|---|---|
| | A | 2.7 | | | | | | | | |
| | A | 2.7 | | | | | | | | |
| | A | 2.8 | | | | | | | | |
| | A | 2.9 | | | | | | | | |
| | B | 3.1 | | | | | | | | |
| | B | 3.2 | – | | = | | = | | + | |
| | B | 3.3 | | | | | | | | |
| | B | 3.3 | | | | | | | | |
| | C | 2.6 | | | | | | | | |
| | C | 2.7 | | | | | | | | |
| | C | 2.7 | | | | | | | | |
| | C | 2.8 | | | | | | | | |
| Degrees of freedom (DF) | | | – | | = | | = | | + | |
| Sum of squares (SS) | | | – | | = | | = | | + | |
| Mean square (MS) | | *(SS / DF)* | | | | | | | | |
| F ratio | | *(Group MS / Error MS)* | | | | | | | | |
| P value | | *(Probability of getting an F ratio this large by chance alone)* | | | | | | | | |
| Root mean square (RMS) | | *(Square root of MS)* | | | | | | | | |

## ANOVA in JMP

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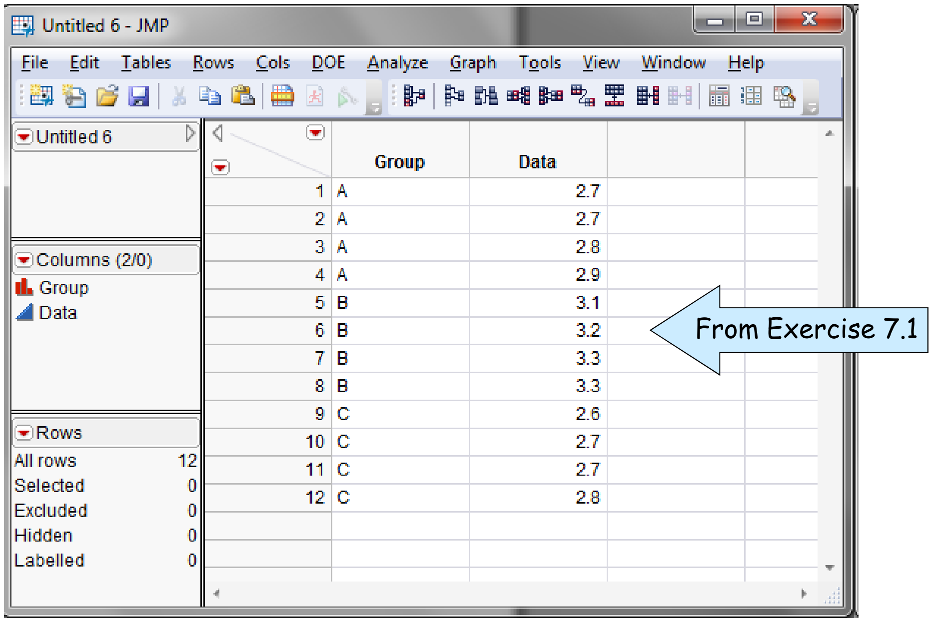
*File* → *New* → *Data Table* → Enter (or copy-paste) data as shown




## ANOVA in JMP (cont'd)

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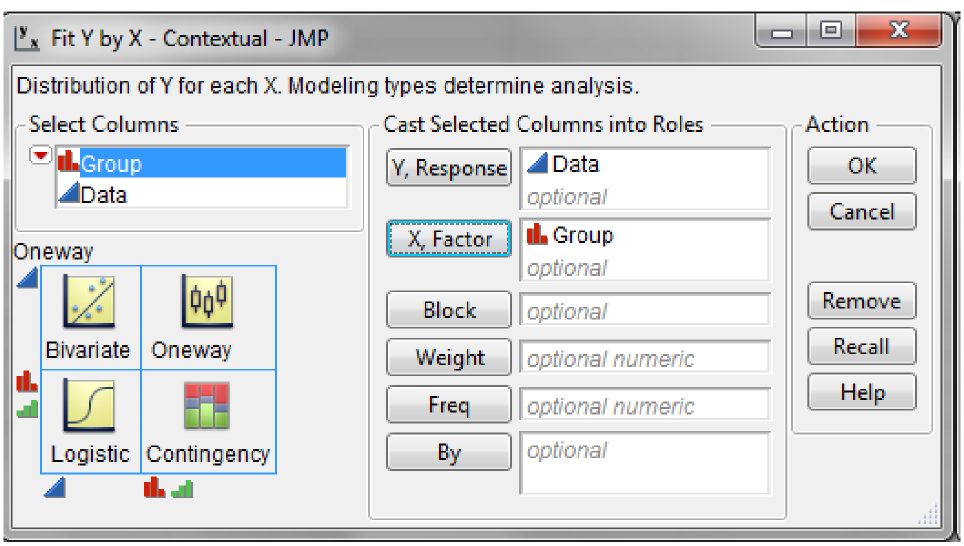
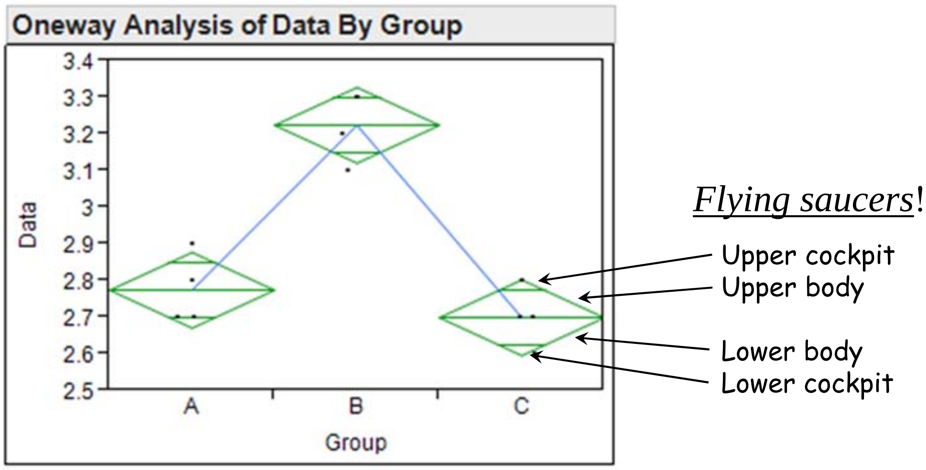
*Analyze* → *Fit Y by X* → Set up as shown → OK



## Explanation of “mean diamonds”

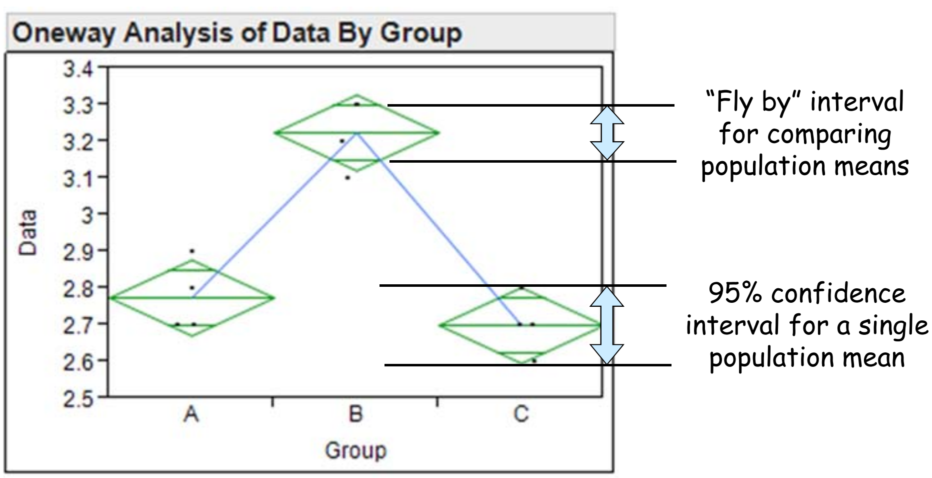





*Population means are different (with 95% confidence)* ⟺ *Saucers can fly horizontally past each other with no contact between their bodies*

## Mean diamonds (cont’d)






Approx. formula for “fly by” interval: $\text{Sample mean} \pm \sqrt{2}\left(\text{RMSE}/\sqrt{\text{N}}\right)$

Approx. formula for 95% confidence interval: $\text{Sample mean} \pm 2\left(\text{RMSE}/\sqrt{\text{N}}\right)$

N = sample size for each group

## Analysis details



### Oneway Anova

#### Summary of Fit

| | |
|---|---|
| Rsquare | 0.895833 |
| Adj Rsquare | 0.872685 |
| Root Mean Square Error | $\boxed{0.091287}$ |
| Mean of Response | 2.9 |
| Observations (or Sum Wgts) | 12 |

RMSE

#### Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Ratio | Prob > F |
|---|---|---|---|---|---|
| Group | 2 | 0.64500000 | 0.322500 | 38.7000 | <.0001* |
| Error | 9 | 0.07500000 | 0.008333 | | |
| C. Total | 11 | 0.72000000 | | | |

Signal

Noise

P value

- Standard deviation of the noise variation (error, residual, unexplained etc.)
- Smaller is better
- Has units of the Y variable

## Analysis details (cont'd)



### Oneway Anova

#### Summary of Fit

| | |
|---|---|
| Rsquare | 0.895833 |
| Adj Rsquare | $\boxed{0.872685}$ |
| Root Mean Square Error | 0.091287 |
| Mean of Response | 2.9 |
| Observations (or Sum Wgts) | 12 |

Adjusted $R^2$

#### Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Ratio | Prob > F |
|---|---|---|---|---|---|
| Group | 2 | 0.64500000 | 0.322500 | 38.7000 | <.0001* |
| Error | 9 | 0.07500000 | 0.008333 | | |
| C. Total | 11 | 0.72000000 | | | |

- Proportion of the total variation in Y that is caused by ("explained by") variation in X
- Larger is better
- Unitless

## How adjusted $R^2$ is calculated

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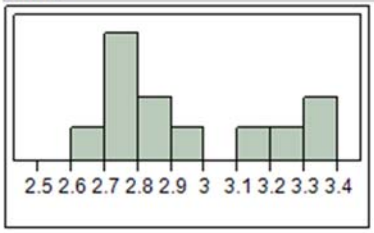
**Distributions**

**Data**




**Summary Statistics**

| | |
|---|---|
| Mean | 2.9 |
| Std Dev | $\boxed{0.2558409}$ |
| N | 12 |

STDEV

Total variation in the data

$$\text{Proportion of Y variation NOT caused by X} = \left(\frac{\text{RMSE}}{\text{STDEV}}\right)^2 = \left(\frac{0.091287}{0.2558409}\right)^2 = 0.127315$$

$$\text{Proportion of Y variation CAUSED by X} = 1 - \left(\frac{\text{RMSE}}{\text{STDEV}}\right)^2 = 0.872685 = \text{Adjusted } R^2$$

## Notes

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## Exercise 7.2



Data table: *number and size of defects*. *Max size* is the area in square centimeters of the largest contiguous weld repair area on each casting. Smaller *Max size* is better.

a) Test for a difference between welders A and B with respect to *Max size*. Give the P value and interpret the result. (Ignore the *t Test* section of the output.)

b) Which welder represents best practice? What follow-up action should be taken?

c) Give the value and the units of the RMSE in this example.

d) The RMSE is meaningful only if each group has roughly the same amount of variation. Is this true in this case?

e) Save your analysis script to the data table, close and save the data table.

## Exercise 7.3



Data table: *quotation process*. A supplier receives requests for quote (RFQs) from customers, then develops and submits quotes. TAT is the turnaround around time in days. BU is the business unit which prepares the quote.

a) Is the modeling type for BU correct? If not, change it to what it should be. Test for differences among the business units. Give the P value and interpret the result.

b) Which BU(s) represent best practice? What follow-up action should be taken?

c) Save your analysis script to the data table, close and save the data table.

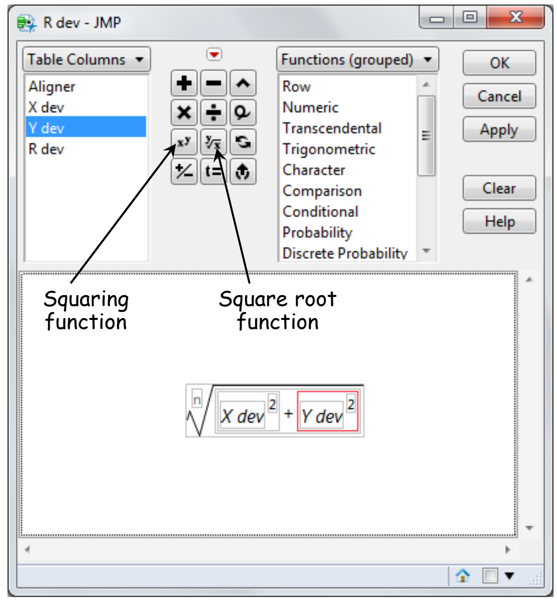
# Exercise 7.4



Data table: *alignment process*. This process attaches orifice plates to chips. Three similar aligners (alignment tools) are used in this process. *Y dev* and *X dev* are the vertical and horizontal deviations from target in mils.

The alignment specification applies to the radial deviation calculated from *X* and *Y*. Double click on the blank column header next to *Y dev*, click on *Column 4*, rename as *R dev*.

Right click on *R dev*, select *Formula*. Use your mouse and the keypad provided to create the formula for *R dev*.




# Exercise 7.4 (cont'd)



a) Is the modeling type for *Aligner* correct? If not, change it to what it should be. Test for differences among the three aligners with respect to *R dev*. Give the P value and interpret the result.

b) Which aligner represents best practice? (Smaller *R dev* is better.) What follow-up action should be taken?

c) Save your analysis script to the data table, close and save the data table.

## Exercise 7.5



Data table: *casting dimensions*. We want to reduce variation in the length of roughly cylindrical castings. The specification for *Length* is 600 ± 1.5. The wax patterns for these castings are molded on two machines A and B.

a) Test for differences between the molding machines with respect to *Length*. Give the P value and interpret the result.

b) Which machine represents best practice? (It is helpful to draw a reference line at the nominal value. Right click on one of the numbers on the vertical axis, select *Axis Settings*, use the *Reference Lines* tool. ) What follow-up action should be taken?

c) Save your analysis script to the data table, but don't close the data table.

## Exercise 7.5 (cont'd)



We also want to reduce variation in the diameter of the castings. The specification for *Diam* is 50 ± 0.75.

d) Test for differences between the molding machines with respect to *Diam*. Give the P value and interpret the result.

e) Which machine represents best practice? (It is helpful to draw a reference line at the nominal value.) What follow-up action should be taken?

f) For each of the variables *Length* and *Diam*, a certain proportion of the total variation is caused by the difference between the machines. For which variable is this proportion highest?

g) Save your analysis script to the data table, close and save the data table.

# 8 Comparing Populations — Pass/fail Y

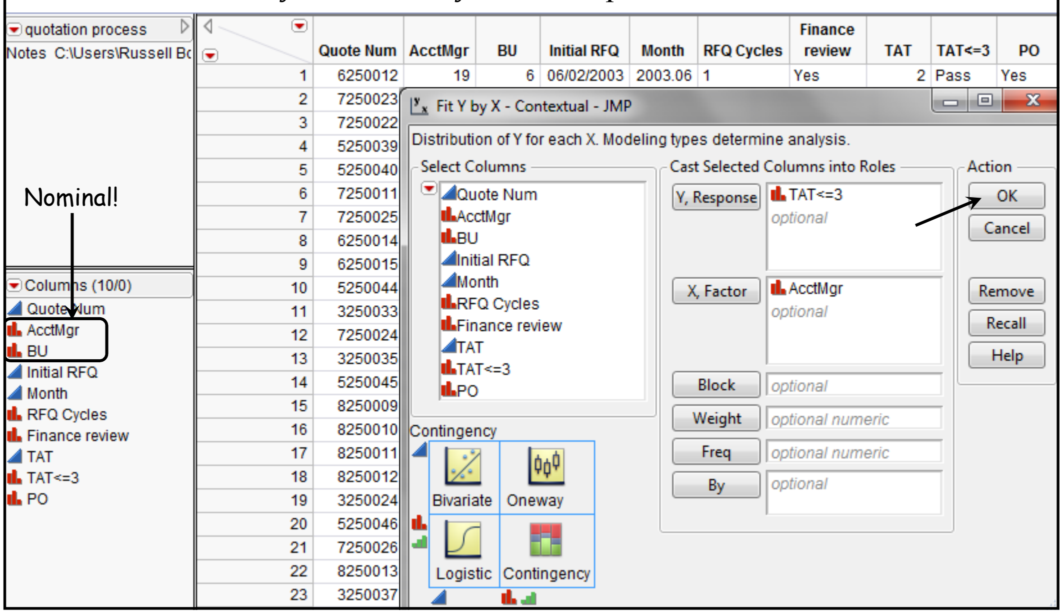


| | |
|---|---|
| Raw data | One part or transaction per row |
| Tabulated data | Multiple parts or transactions per row |

# Raw data example

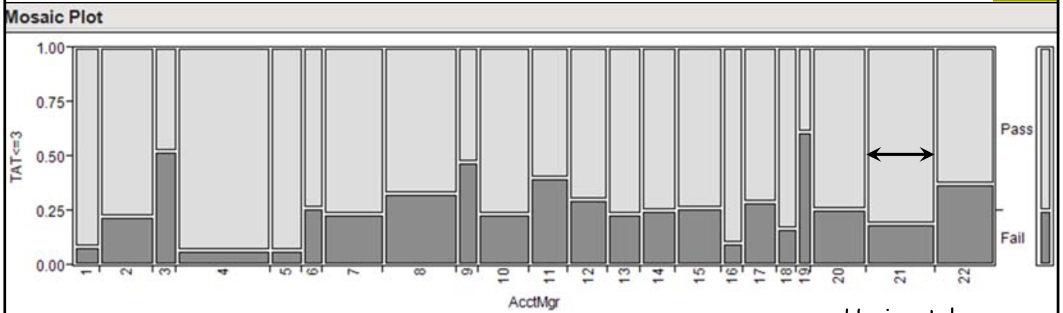


Data table: *quotation process.jmp*
Want to compare the account managers in terms of % late
*Analyze* → *Fit Y by X* → set up as shown → OK



# Raw data (cont'd)






Horizontal dimension is proportional to sample size

**Tests**

| N | DF | -LogLike | RSquare (U) |
|---|---|---|---|
| 837 | 21 | 32.411285 | 0.0687 |

| Test | ChiSquare | Prob>ChiSq |
|---|---|---|
| Likelihood Ratio | 64.823 | $\boxed{<.0001^*}$ |
| Pearson | 62.018 | <.0001* |

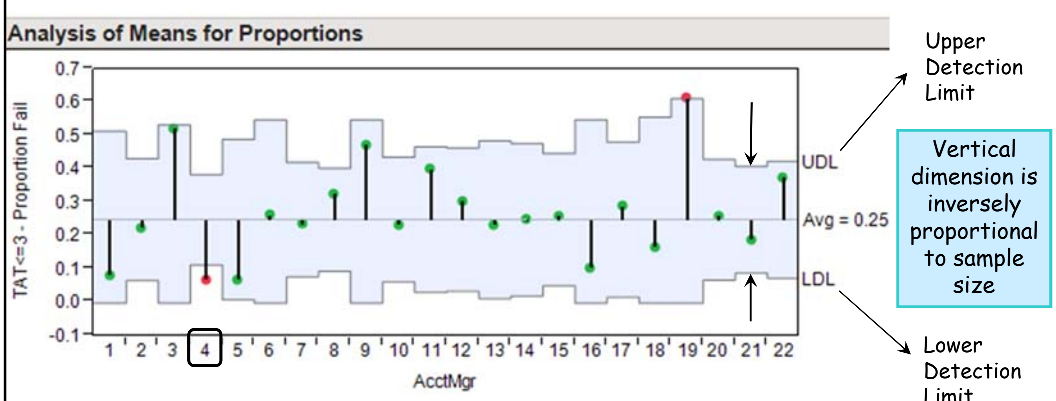
P value

- Very strong evidence of differences among account managers
- Who represents best practice?

# Raw data (cont'd)



- Red triangle → *Analysis of Means for Proportions*




- Not as useful as “flying saucers” (not available for pass/fail Y)
- Points above (below) the shaded region are significantly higher (lower) than points inside the shaded region
- Account manager 4 represents best practice → find out what that is, make it the standard
- Save your analysis script to the data table, but don’t close the data table.

## Exercise 8.1



a) Test for differences among the business units in terms of % late. Give the P value and interpret the result. Is there best practice? If so, where is it (based on this analysis only)?

b) Right click in the PO header, select *Column Properties* → *Value Ordering*, then reverse the default ordering. (This will make it easier to see differences in the PO hit rate.) Test for differences among the account managers with respect to PO hit rate. Give the P value and interpret the result. Is there best practice? If so, where is it?

c) Test for differences among the business units with respect to PO hit rate. Give the P value and interpret the result. Is there best practice? If so, where is it?

d) Save your scripts, close and save the data table.

## Exercise 8.2



Open *ATE Mar & Apr* (in JMP). If necessary, change the modeling types for *Model Number* and *Test Station*.

a) Test for a difference between the model numbers with respect to failure rate. Give the P value and interpret the result.

b) Test for differences among the test stations with respect to failure rate. Give the P value and interpret the result. If significant differences exist, describe them and suggest possible causes.

c) Save your scripts, close and save the data table.

## Tabulated pass/fail data

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- Pass/fail data often comes in tabulated form
- Each row may represent a
  - ✓ Production lot
  - ✓ Work order
  - ✓ Time period
  - ✓ Machine
  - ✓ Work center
  - ✓ Part number . . .
- This format is perfect for plotting % defective
- However, it is the wrong format for comparing populations in JMP

## Example

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- Open *LSSV2 data sets \ out-of-box failures* (in JMP)
- See next slide

## Plotting % defective

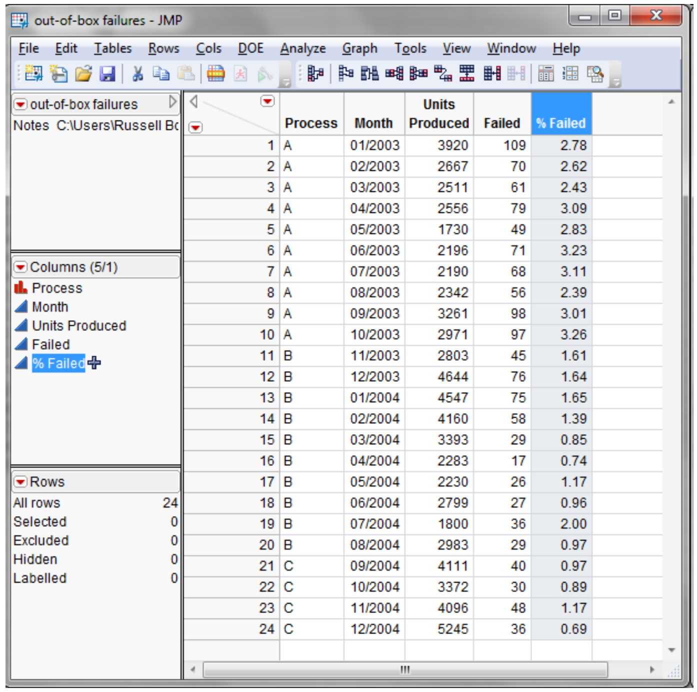


1. Create a new column called *% Failed*

2. Define it by the formula

$$\frac{\text{Failed}}{\text{Units Produced}} \times 100$$

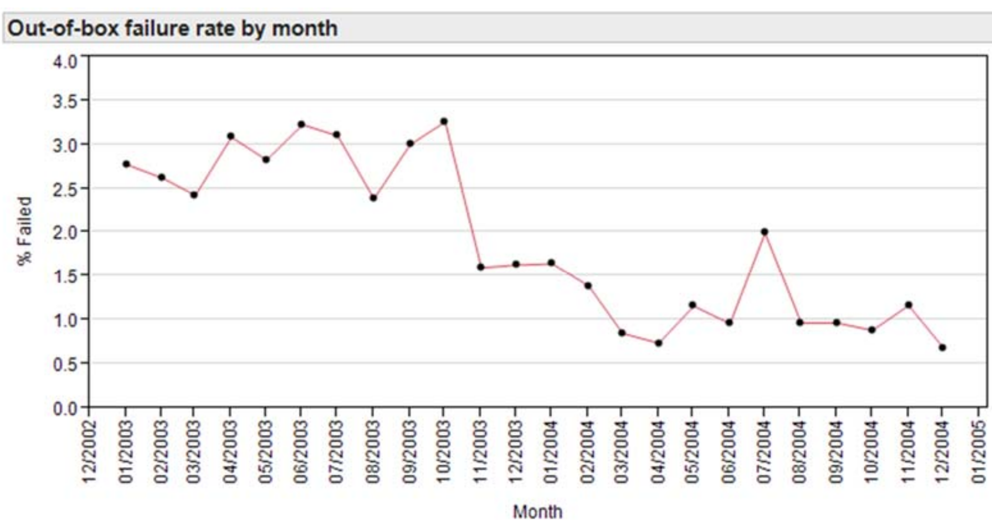
3. Use *Graph* → *Overlay Plot* to create the plot on the next slide




| | Process | Month | Units Produced | Failed | % Failed |
|---|---|---|---|---|---|
| 1 | A | 01/2003 | 3920 | 109 | 2.78 |
| 2 | A | 02/2003 | 2667 | 70 | 2.62 |
| 3 | A | 03/2003 | 2511 | 61 | 2.43 |
| 4 | A | 04/2003 | 2556 | 79 | 3.09 |
| 5 | A | 05/2003 | 1730 | 49 | 2.83 |
| 6 | A | 06/2003 | 2196 | 71 | 3.23 |
| 7 | A | 07/2003 | 2190 | 68 | 3.11 |
| 8 | A | 08/2003 | 2342 | 56 | 2.39 |
| 9 | A | 09/2003 | 3261 | 98 | 3.01 |
| 10 | A | 10/2003 | 2971 | 97 | 3.26 |
| 11 | B | 11/2003 | 2803 | 45 | 1.61 |
| 12 | B | 12/2003 | 4644 | 76 | 1.64 |
| 13 | B | 01/2004 | 4547 | 75 | 1.65 |
| 14 | B | 02/2004 | 4160 | 58 | 1.39 |
| 15 | B | 03/2004 | 3393 | 29 | 0.85 |
| 16 | B | 04/2004 | 2283 | 17 | 0.74 |
| 17 | B | 05/2004 | 2230 | 26 | 1.17 |
| 18 | B | 06/2004 | 2799 | 27 | 0.96 |
| 19 | B | 07/2004 | 1800 | 36 | 2.00 |
| 20 | B | 08/2004 | 2983 | 29 | 0.97 |
| 21 | C | 09/2004 | 4111 | 40 | 0.97 |
| 22 | C | 10/2004 | 3372 | 30 | 0.89 |
| 23 | C | 11/2004 | 4096 | 48 | 1.17 |
| 24 | C | 12/2004 | 5245 | 36 | 0.69 |

## Plotting % defective (cont'd)





## Reformatting for comparing populations



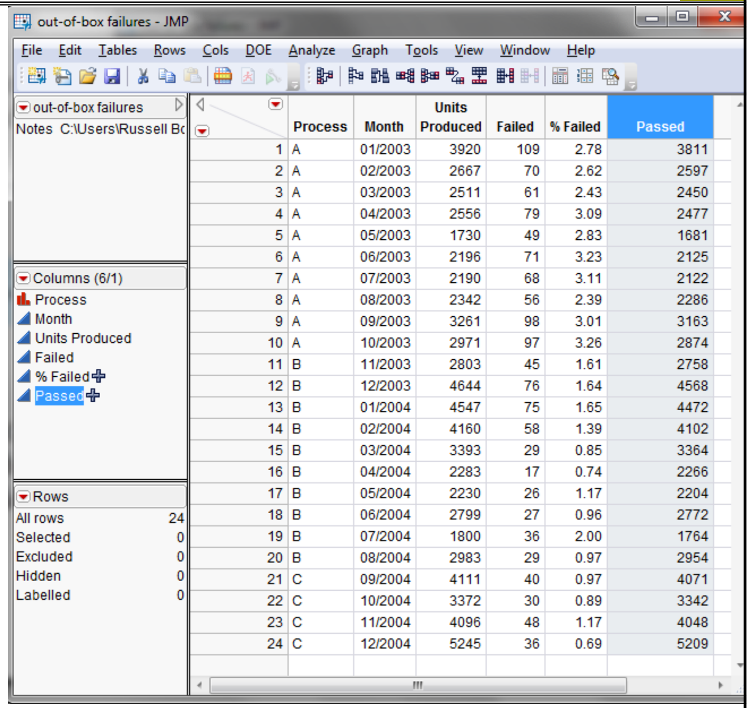
1. Create a new column called *Passed* defined by the formula

   Units Produced - Failed

2. Go to *Tables* → *Stack*

3. Use *Failed* and *Passed* as the *Stack Columns*

4. See next slide



| | Process | Month | Units Produced | Failed | % Failed | Passed |
|---|---|---|---|---|---|---|
| 1 | A | 01/2003 | 3920 | 109 | 2.78 | 3811 |
| 2 | A | 02/2003 | 2667 | 70 | 2.62 | 2597 |
| 3 | A | 03/2003 | 2511 | 61 | 2.43 | 2450 |
| 4 | A | 04/2003 | 2556 | 79 | 3.09 | 2477 |
| 5 | A | 05/2003 | 1730 | 49 | 2.83 | 1681 |
| 6 | A | 06/2003 | 2196 | 71 | 3.23 | 2125 |
| 7 | A | 07/2003 | 2190 | 68 | 3.11 | 2122 |
| 8 | A | 08/2003 | 2342 | 56 | 2.39 | 2286 |
| 9 | A | 09/2003 | 3261 | 98 | 3.01 | 3163 |
| 10 | A | 10/2003 | 2971 | 97 | 3.26 | 2874 |
| 11 | B | 11/2003 | 2803 | 45 | 1.61 | 2758 |
| 12 | B | 12/2003 | 4644 | 76 | 1.64 | 4568 |
| 13 | B | 01/2004 | 4547 | 75 | 1.65 | 4472 |
| 14 | B | 02/2004 | 4160 | 58 | 1.39 | 4102 |
| 15 | B | 03/2004 | 3393 | 29 | 0.85 | 3364 |
| 16 | B | 04/2004 | 2283 | 17 | 0.74 | 2266 |
| 17 | B | 05/2004 | 2230 | 26 | 1.17 | 2204 |
| 18 | B | 06/2004 | 2799 | 27 | 0.96 | 2772 |
| 19 | B | 07/2004 | 1800 | 36 | 2.00 | 1764 |
| 20 | B | 08/2004 | 2983 | 29 | 0.97 | 2954 |
| 21 | C | 09/2004 | 4111 | 40 | 0.97 | 4071 |
| 22 | C | 10/2004 | 3372 | 30 | 0.89 | 3342 |
| 23 | C | 11/2004 | 4096 | 48 | 1.17 | 4048 |
| 24 | C | 12/2004 | 5245 | 36 | 0.69 | 5209 |

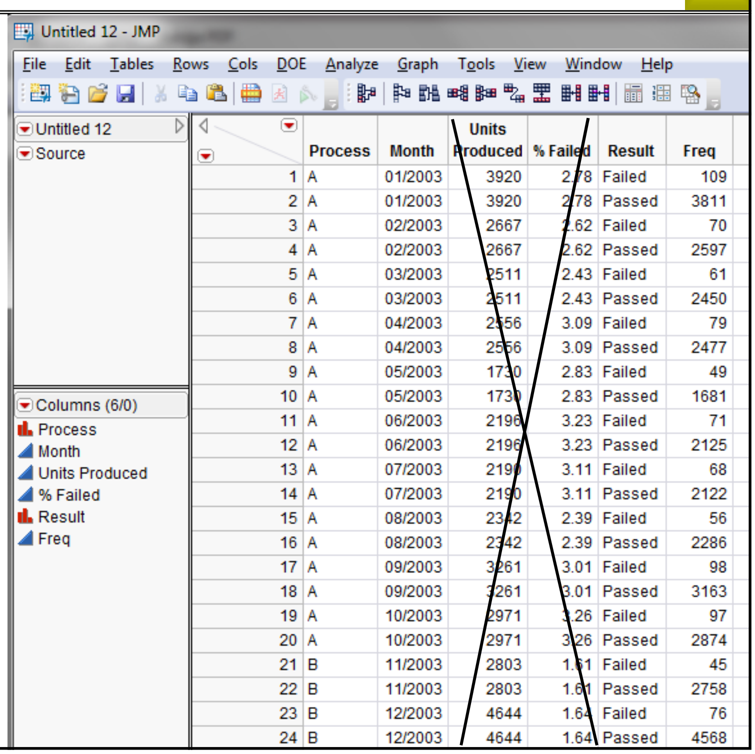
## Reformatting (cont'd)



6. Change the name of the *Data* column to *Freq* and the *Label* column to *Result*

7. There are now two rows for each month. The *Units Produced* and *% Failed* columns are no longer relevant, and may be deleted.

8. Save the new data table as *out of box failures stacked*



| | Process | Month | Units Produced | % Failed | Result | Freq |
|---|---|---|---|---|---|---|
| 1 | A | 01/2003 | 3920 | 2.78 | Failed | 109 |
| 2 | A | 01/2003 | 3920 | 2.78 | Passed | 3811 |
| 3 | A | 02/2003 | 2667 | 2.62 | Failed | 70 |
| 4 | A | 02/2003 | 2667 | 2.62 | Passed | 2597 |
| 5 | A | 03/2003 | 2511 | 2.43 | Failed | 61 |
| 6 | A | 03/2003 | 2511 | 2.43 | Passed | 2450 |
| 7 | A | 04/2003 | 2556 | 3.09 | Failed | 79 |
| 8 | A | 04/2003 | 2556 | 3.09 | Passed | 2477 |
| 9 | A | 05/2003 | 1730 | 2.83 | Failed | 49 |
| 10 | A | 05/2003 | 1730 | 2.83 | Passed | 1681 |
| 11 | A | 06/2003 | 2196 | 3.23 | Failed | 71 |
| 12 | A | 06/2003 | 2196 | 3.23 | Passed | 2125 |
| 13 | A | 07/2003 | 2190 | 3.11 | Failed | 68 |
| 14 | A | 07/2003 | 2190 | 3.11 | Passed | 2122 |
| 15 | A | 08/2003 | 2342 | 2.39 | Failed | 56 |
| 16 | A | 08/2003 | 2342 | 2.39 | Passed | 2286 |
| 17 | A | 09/2003 | 3261 | 3.01 | Failed | 98 |
| 18 | A | 09/2003 | 3261 | 3.01 | Passed | 3163 |
| 19 | A | 10/2003 | 2971 | 3.26 | Failed | 97 |
| 20 | A | 10/2003 | 2971 | 3.26 | Passed | 2874 |
| 21 | B | 11/2003 | 2803 | 1.61 | Failed | 45 |
| 22 | B | 11/2003 | 2803 | 1.61 | Passed | 2758 |
| 23 | B | 12/2003 | 4644 | 1.64 | Failed | 76 |
| 24 | B | 12/2003 | 4644 | 1.64 | Passed | 4568 |

## Analyzing the data

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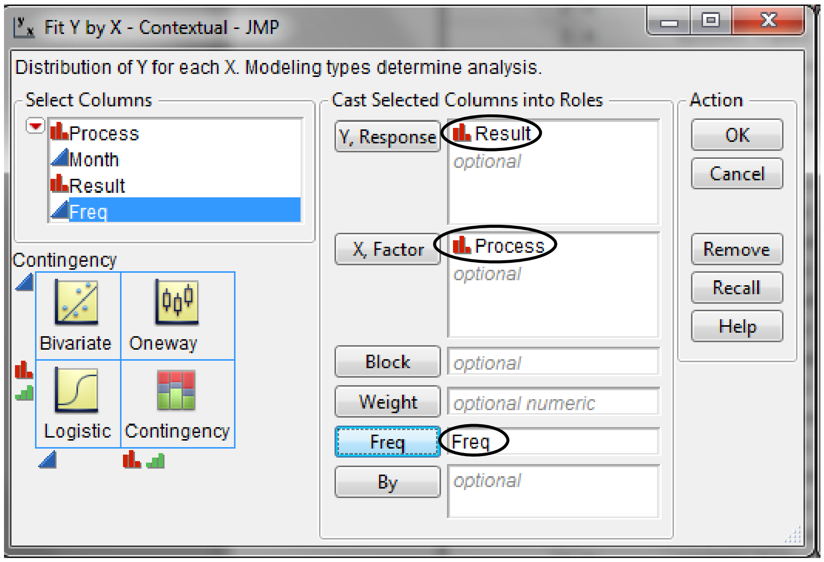
*Analyze* → *Fit Y by X* → set up as shown → OK




## Data analysis (cont'd)

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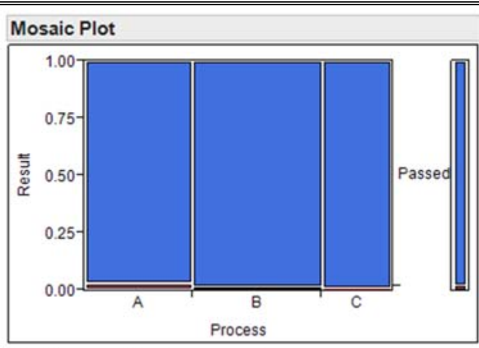



**Contingency Table**

Result

| Process | Count / Row % | Failed | Passed | |
|---|---|---|---|---|
| A | Count | 758 | 25586 | 26344 |
| | Row % | 2.88 | 97.12 | |
| B | Count | 418 | 31224 | 31642 |
| | Row % | 1.32 | 98.68 | |
| C | Count | 154 | 16670 | 16824 |
| | Row % | 0.92 | 99.08 | |
| | | 1330 | 73480 | 74810 |

**Tests**

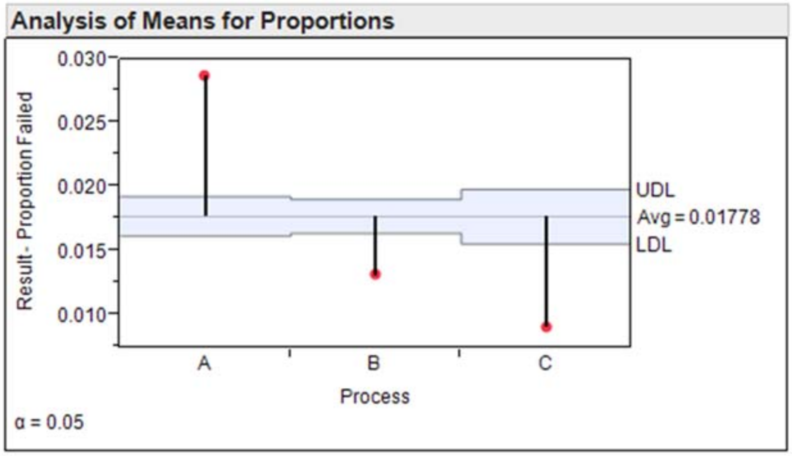
| N | DF | -LogLike | RSquare (U) |
|---|---|---|---|
| 74810 | 2 | 141.17363 | 0.0211 |

| Test | ChiSquare | Prob>ChiSq |
|---|---|---|
| Likelihood Ratio | 282.347 | <.0001* |
| Pearson | 291.850 | <.0001* |

- Very strong evidence that processes A, B, and C do not all have the same failure rate
- The mosaic plot does not help us determine where the differences are
- Click on the red triangle at the top of the analysis window
- Select *Analysis of Means for Proportions*
- See next slide

## Data analysis (cont'd)






- This plot shows that Processes B and C are significant improvements over Process A
- It does not tell us whether or not C is a significant improvement over B
- Save your script, but don't close the data table.

## Exercise 8.3



a) Exclude the rows for process A.

b) Test for a difference between C and B. Give the P value and interpret the result.

c) Close and save the data table. (No need to save the script again.)

## Exercise 8.4



Open *molding process - stratification* (in JMP).

a) Did JMP assign the correct modeling type for *Machine*?

b) Go to *Tables* → *Summary* → use *PN* as the *Group* variable → use *Machine* as the *Subgroup* variable → OK.

| 10/0 | PN | N Rows | N(1) | N(10) | N(11) | N(13) | N(14) | N(15) | N(2) | N(3) | N(9) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | GV0098 | 43 | 0 | 0 | 0 | 0 | 11 | 32 | 0 | 0 | 0 |
| 2 | GV0101 | 31 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 30 |
| 3 | GV0119 | 42 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 39 | 0 |
| 4 | GV0129 | 89 | 0 | 0 | 0 | 0 | 88 | 1 | 0 | 0 | 0 |
| 5 | GV0132 | 64 | 0 | 0 | 0 | 0 | 0 | 0 | 64 | 0 | 0 |
| 6 | GY0251 | 37 | 0 | 0 | 17 | 20 | 0 | 0 | 0 | 0 | 0 |
| 7 | GY0298 | 31 | 0 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 24 |
| 8 | GY0306 | 53 | 0 | 0 | 27 | 26 | 0 | 0 | 0 | 0 | 0 |
| 9 | GY0325 | 36 | 1 | 0 | 34 | 1 | 0 | 0 | 0 | 0 | 0 |
| 10 | KU0041 | 84 | 83 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |

c) Note that each part number runs on only one or two of the machines. A comparison of the part numbers could be biased by differences among the machines. A comparison of the machines could be biased by differences among the part numbers. Because of this, we should use the concatenated variable *PN-Machine* as the X variable in the analysis.

## Exercise 8.4 (cont'd)



e) Reformat the data for comparing populations (follow steps 1 – 7 in the worked example).

f) Test for significant differences among the combinations of part number and machine with respect to fraction defective. Give the P value and interpret the results.

g) Based on fraction defective, which three combinations of part number and machine would be the best focus for an improvement project?

h) Save your script, save the data table as *molding process - stratification*, then close it.

# 9 Simple Regression



- Terminology
- Purposes of regression analysis
- "Simple" regression
- The line of best fit
- Regression in JMP
- Regression through the origin

## Terminology



- The term *correlation* is often used any time we speak of relating one variable to another
- Technically, *correlation* should be used only if both variables are quantitative
- An input/output relationship between the two variables is not required (for example, two variables measured at the same point in a process)
- *Regression* is a special case of correlation where there *is* an input-output relationship Y = *f* (X), so X is a possible cause of variation in Y
- In regression we want to test for correlation, but beyond that we want to quantify the relationship and use it for some purpose

## Purposes of regression analysis



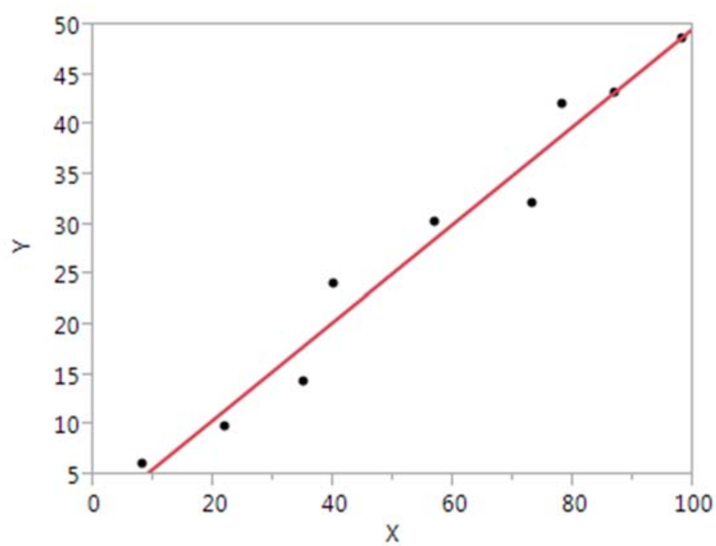
- Predict Y from X
- Determine best setting for X (optimization)
- Reduce variation in Y by controlling X

## “Simple” regression



Intercept ↓ Slope ↓

$$Y = 0.8387 + 0.4891\,X + \text{“Error”}$$



## The line of best fit

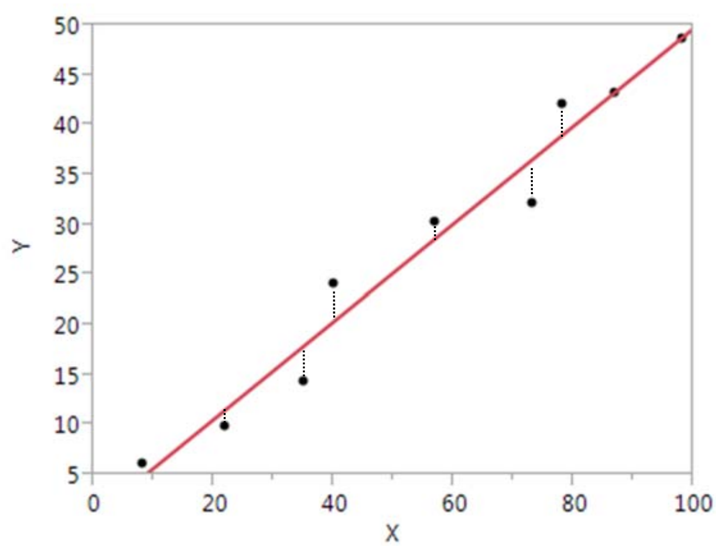
177

- There are many ways to the measure the distance from an (X, Y) data point to a line in the XY plane
- For example, the shortest distance is along a path perpendicular to the line
- The only distance relevant in regression is the "error" — the difference between a Y data value and its predicted value based on the line
- The line of best fit is the one that minimizes the sum of the squared errors
- This is an example of *least-squares model fitting*

## The line of best fit (cont'd)

178

The best fitting line is the one that minimizes the sum of the squared "errors"



## Finding the line of best fit

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Open *LSSV2 other stuff \ ANOVA linear fit*

Worksheet *Prediction & error 1*

| | A | B | E | F | G | H | I | J–P |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | X data | Y data | | Prediction | | Error | Y = 27.9033 + 0.0000 X |
| 3 | | 8 | 6.16 | | 27.90 | | -21.74 | |
| 4 | | 22 | 9.88 | | 27.90 | | -18.02 | |
| 5 | | 35 | 14.35 | | 27.90 | | -13.55 | |
| 6 | | 40 | 24.06 | | 27.90 | | -3.84 | |
| 7 | | 57 | 30.34 | = | 27.90 | + | 2.44 | |
| 8 | | 73 | 32.17 | | 27.90 | | 4.27 | |
| 9 | | 78 | 42.18 | | 27.90 | | 14.28 | |
| 10 | | 87 | 43.23 | | 27.90 | | 15.33 | |
| 11 | | 98 | 48.76 | | 27.90 | | 20.86 | |
| 12 | Sum of squares (SS) | | 8901.3 | = | 7007.4 | + | 1893.9 | |
| 13 | Degrees of freedom (DF) | | 9 | = | 1 | + | 8 | |
| 14 | Root mean square error (RMSE) | | | | | | 15.39 | |
| 15 | | | | | | | | |
| 16 | | Average Y | 27.90 | | | | | |
| 17 | | STDEV of Y | 15.39 | | | | | |

## Finding the line of best fit (cont'd)

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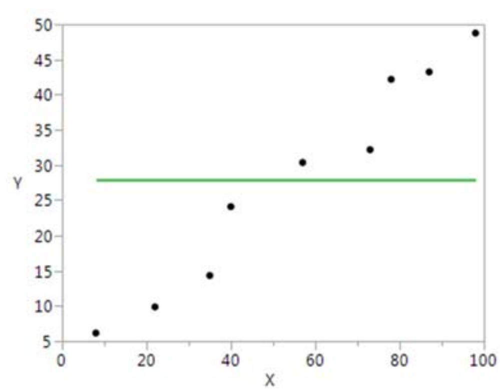
In this worksheet we ignore the X variable completely, and use the average value of Y as the prediction. This is just the calculation of the mean and standard deviation of the Y variable. (The values in cells I14 and E17 are the same.)




The sum of the squared errors (cell I12) can be dramatically reduced by using the X variable to "explain" more of the variation in the Y variable.

## Finding the line of best fit (cont'd) 181

Worksheet *Prediction & error 2*

| | X data | Y data | | Prediction | | Error |
|---|---|---|---|---|---|---|
| | 8 | 6.16 | | 4.75 | | 1.41 |
| | 22 | 9.88 | | 11.60 | | -1.72 |
| | 35 | 14.35 | | 17.96 | | -3.61 |
| | 40 | 24.06 | | 20.40 | | 3.66 |
| | 57 | 30.34 | = | 28.72 | + | 1.62 |
| | 73 | 32.17 | | 36.54 | | -4.37 |
| | 78 | 42.18 | | 38.99 | | 3.19 |
| | 87 | 43.23 | | 43.39 | | -0.16 |
| | 98 | 48.76 | | 48.77 | | -0.01 |
| Sum of squares (SS) | | 8901.3 | = | 8838.0 | + | 63.3 |
| Degrees of freedom (DF) | | 9 | = | 2 | + | 7 |
| Root mean square error (RMSE) | | | | | | 3.007 |

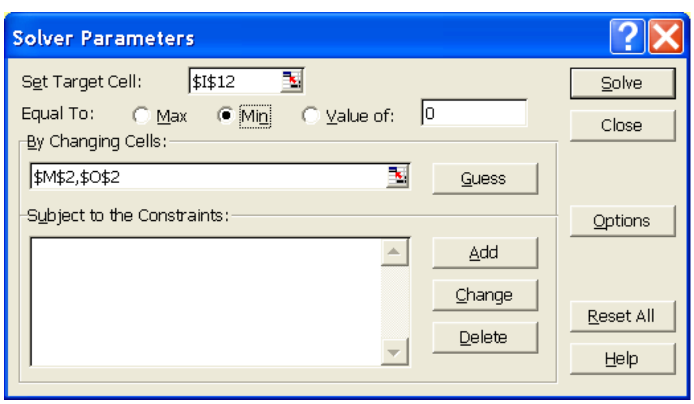
Y = 0.8387 + 0.4891 X

| | |
|---|---|
| Average Y | 27.90 |
| STDEV of Y | 15.39 |
| Adjusted R square | 0.962 |

Proportion of total Y variation caused by ("explained by") X variation

## Finding the line of best fit (cont'd) 182

In this worksheet we used the Excel *Solver* tool to find the line of best fit, using the setup shown below. The *Error* column now contains the deviations from that line. The sum of the squared errors has been reduced from 1893.9 to 63.3 (cell I12).




The distribution of degrees of freedom has also changed. The *Prediction* column is now completely determined by the intercept and slope in cells M2 and O2, so it has only 2 degrees of freedom. The *Error* column has 7 degrees of freedom because DFs have to add up.

## Degrees of freedom for regression



N = total sample size

G = number of parameters in the equation
= DF for the prediction column

N – G = DF for the error column

- The *Error* DF is more important than the *Prediction* DF
- It determines the accuracy of the predicted values
- Larger is better, 10 is OK, bare minimum is 5
- When DF is mentioned without a qualifier, it always means *Error* DF

## Notes

## Regression in JMP

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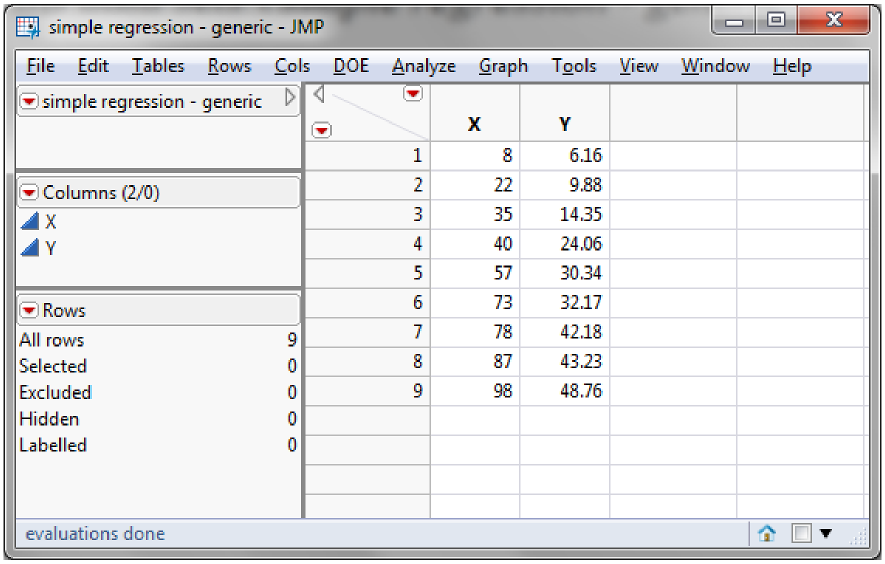
Open *LSSV2 data sets \ simple regression - generic*




## Regression in JMP (cont'd)

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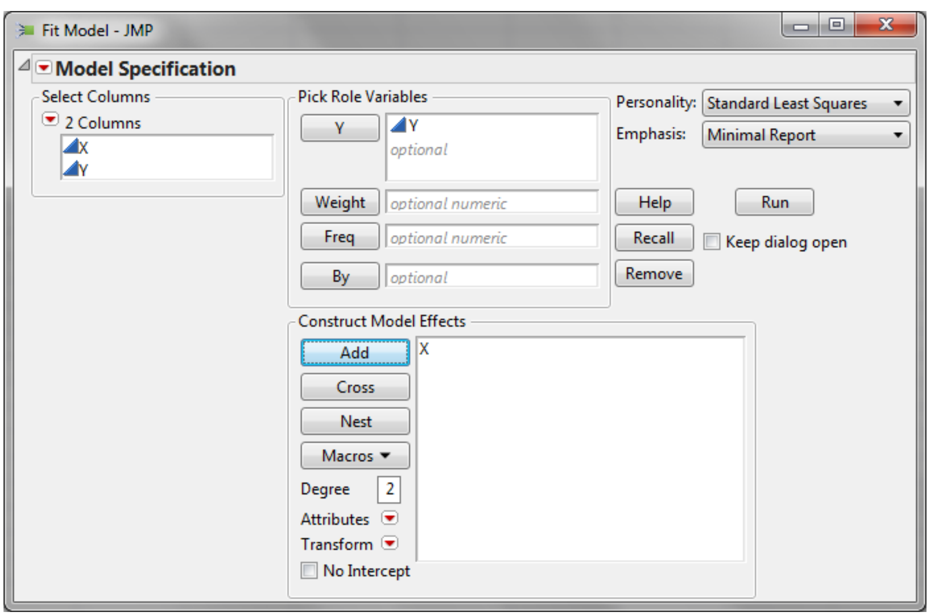
*Analyze* → *Fit Model* → Set up as shown → Run



## Analysis details

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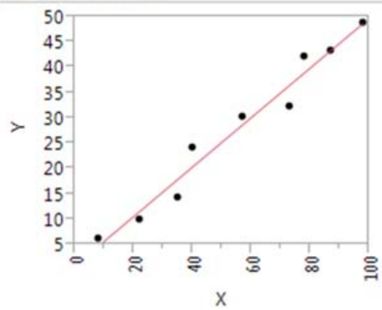
**Response Y**

**Regression Plot**




**Summary of Fit**

| | |
|---|---|
| RSquare | 0.966581 |
| RSquare Adj | 0.961807 |
| Root Mean Square Error | 3.006984 |
| Mean of Response | 27.90333 |
| Observations (or Sum Wgts) | 9 |

**Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Model | 1 | 1830.6557 | 1830.66 | 202.4624 |
| Error | 7 | 63.2937 | 9.04 | Prob > F |
| C. Total | 8 | 1893.9494 | | <.0001* |

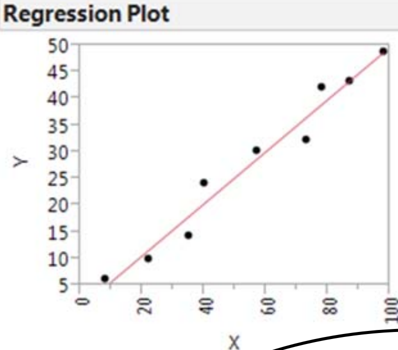
- Root Mean Square Error (RMSE)
- Standard deviation of Y variation caused by factors other than X
- “Error” standard deviation
- Also called “residual” standard deviation
- Smaller is better

P value

## Analysis details (cont’d)

188

**Regression Plot**




- Proportion of Y variation caused by (“explained by”) X variation
- Larger is better

**Summary of Fit**

| | |
|---|---|
| RSquare | 0.966581 |
| RSquare Adj | 0.961807 |
| Root Mean Square Error | 3.006984 |
| Mean of Response | 27.90333 |
| Observations (or Sum Wgts) | 9 |

$$= 1 - \left(\frac{\text{RMSE}}{\text{STDEV}}\right)^2 = 1 - \left(\frac{3.007}{15.386}\right)^2$$

(STDEV of Y is 15.386)

**Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Model | 1 | 1830.6557 | 1830.66 | 202.4624 |
| Error | 7 | 63.2937 | 9.04 | Prob > F |
| C. Total | 8 | 1893.9494 | | <.0001* |

## The *Parameter Estimates* option

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Red triangle next to *Response Y* → *Regression Reports* → *Parameter Estimates*

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 0.8386661 | 2.150023 | 0.39 | 0.7081 |
| X | 0.4891205 | 0.034375 | 14.23 | <.0001* |

- Estimates of the slope and intercept
- Line of best fit is Y = 0.8387 + 0.4891 X

- P values for the slope and intercept
- In simple regression, the *Analysis of Variance* P value is the P value for the slope
- In this example, the slope of the line is significantly different from 0, very strong evidence of a correlation between Y and X.
- The intercept is *not* significantly different from 0. This term should be removed from the model equation.

## Notes

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## Removing the intercept from the model equation

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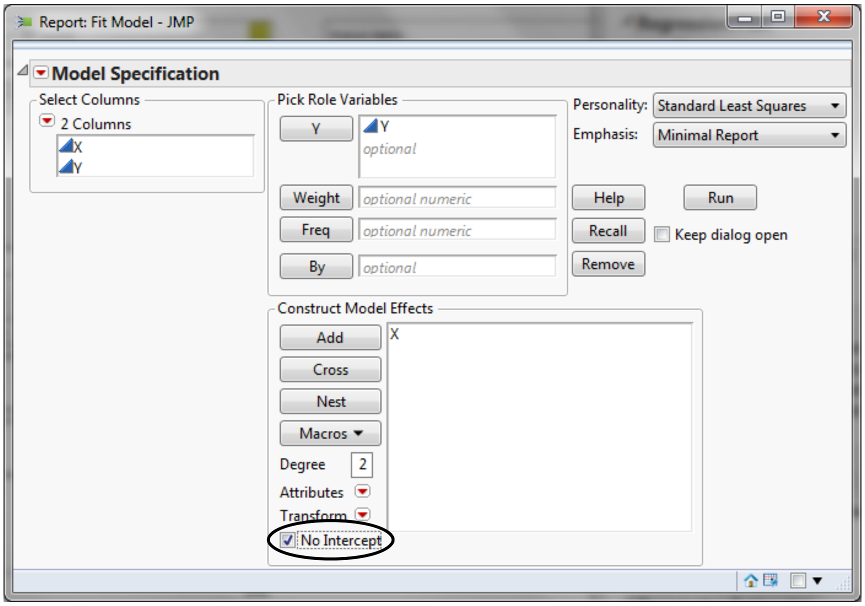
Red triangle next to *Response Y* → *Model Dialog* → check *No Intercept* → Run



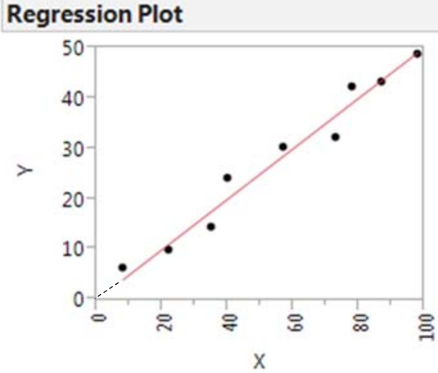

## This is called "regression through the origin"

192

- Now the line of best fit is Y = 0.5010X
- When X = 0, Y = 0
- This makes physical sense in some situations

**Response Y**

**Regression Plot**




**Summary of Fit**

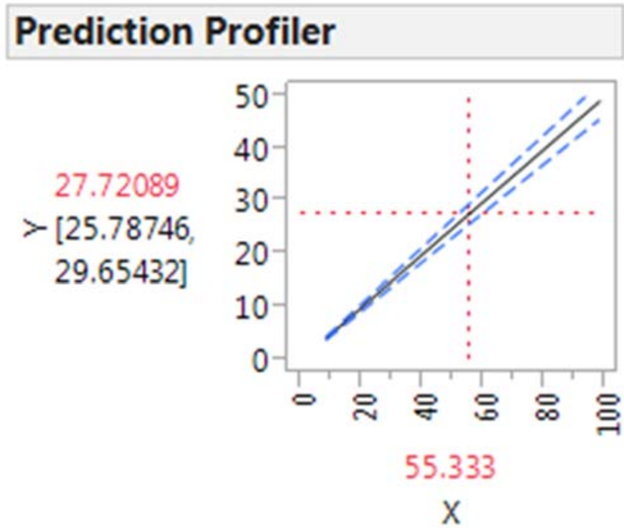
| | |
|---|---|
| RSquare | . |
| RSquare Adj | . |
| Root Mean Square Error | 2.843182 |
| Mean of Response | 27.90333 |
| Observations (or Sum Wgts) | 9 |

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| X | 0.500983 | 0.015152 | 33.06 | <.0001* |

## The prediction profiler



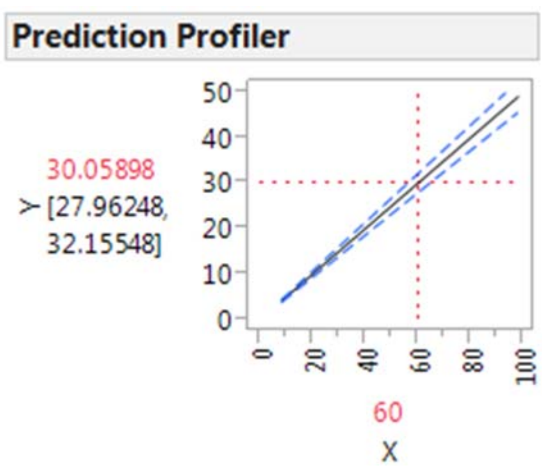



- Calculates predicted mean Y as a function of X
- Calculates confidence intervals for predicted means

## Simple example of prediction



- Suppose we are interested in the predicted mean Y for X = 60
- Click on the 55.333, change it to 60

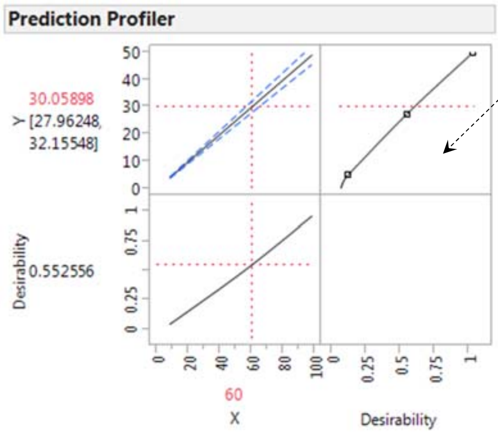



- Predicted mean Y (based on the data) is 30.06
- With 95% confidence, the population mean lies between 27.96 and 32.16

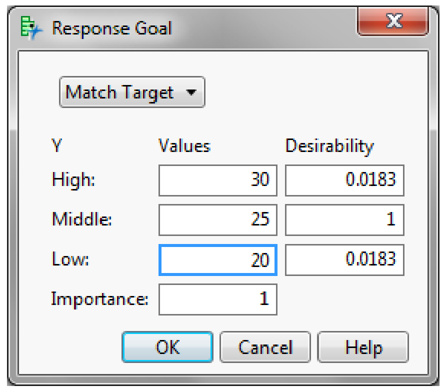
## Simple example of optimization

195

- Suppose we want to find the X value that predicts a mean Y value of 25
- Red triangle next to *Prediction Profiler* → *Desirability Functions*



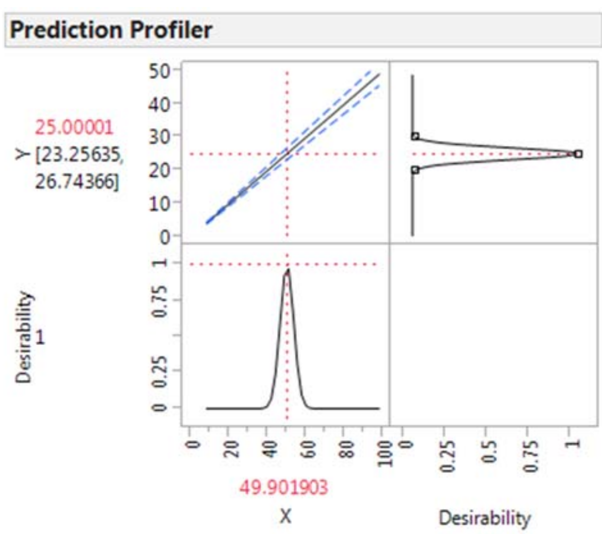

- Double click in here (don't touch the line plot)
- Modify the **Response Goal** dialog as shown below
- Click OK




## Optimization (cont'd)

196

- Red triangle next to *Prediction Profiler* → *Maximize Desirability*




- Predicted mean Y of 25 is achieved at X = 49.9
- With 95% confidence, this population mean lies between 23.3 and 26.7

## Exercise 9.1



a) Find the X value that predicts a mean Y value of 35. Give the confidence limits for the predicted mean.

b) The overall standard deviation of Y is 15.4. The RMSE from the regression is 2.84. Which of these would be the standard deviation of Y if we controlled X to a constant value?

c) Save your script, close and save the data table.

## Exercise 9.2



Open *LSSV2 data sets \ production vs capacity* (in JMP).

(a) Fit a regression for *Production qty* as a function of *Capacity utilized* (%). Is there a correlation? Give the appropriate P value and strength of evidence.

(b) Decide whether or not to remove the intercept from the model equation. Support your decision with the appropriate P value.

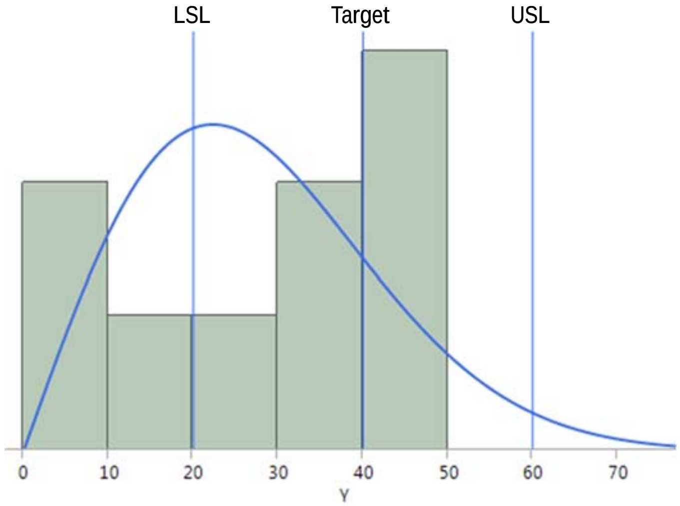
(c) Use your model from (b) to find the capacity utilization level that predicts a mean daily production quantity of 3500. Give the confidence limits.

(d) The overall standard deviation of *Production qty* is 733.5. The RMSE from the analysis in (c) is 406.06. Which of these would be the standard deviation if capacity utilization never changed?

(e) Save your scripts, close and save the data table.

## 10 Using the RMSE






Suppose we are not happy with our current process capability

Mean = 27.9, Std dev = 15.4

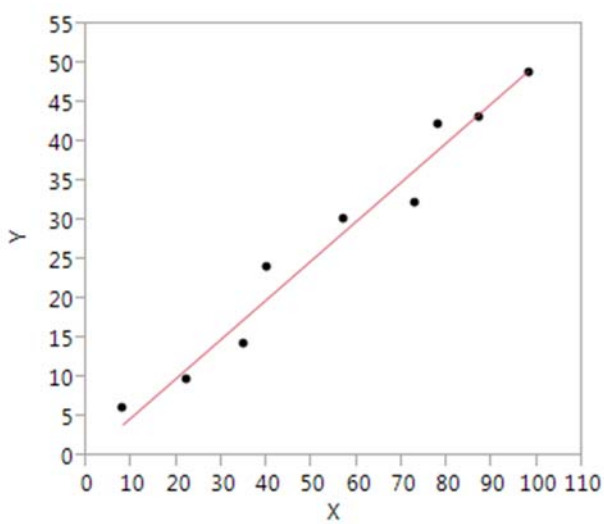
Defective in the data: 33.3%

Predicted from distribution curve: 35.8%

## RMSE (cont'd)



Suppose Y is correlated with a controllable X variable

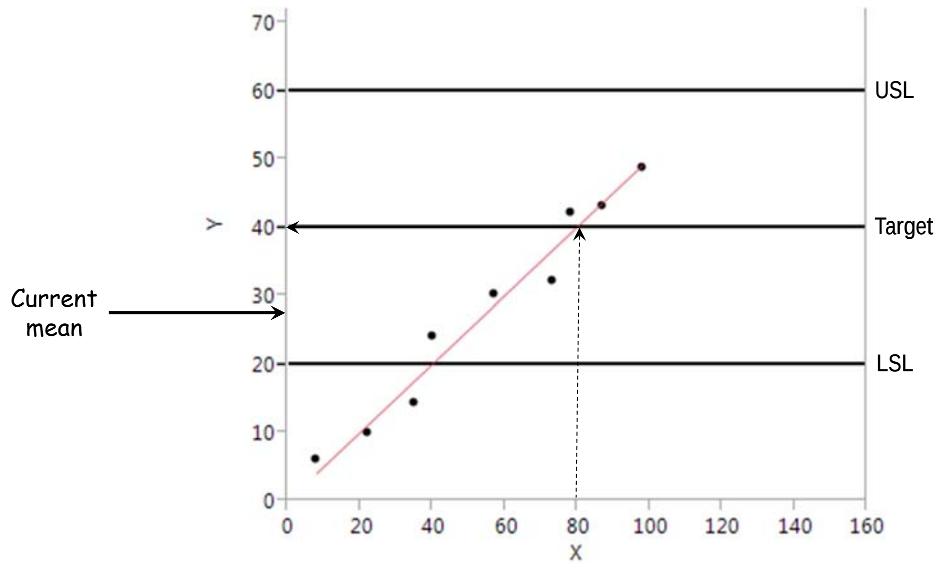



How can we use the regression to improve the Y capability?

## RMSE (cont'd)

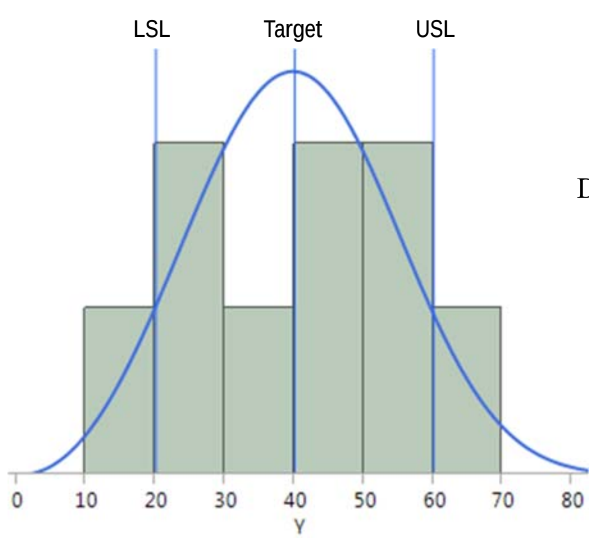
201

If we control X at 80, the mean will change from 27.9 to 40




## RMSE (cont'd)

202



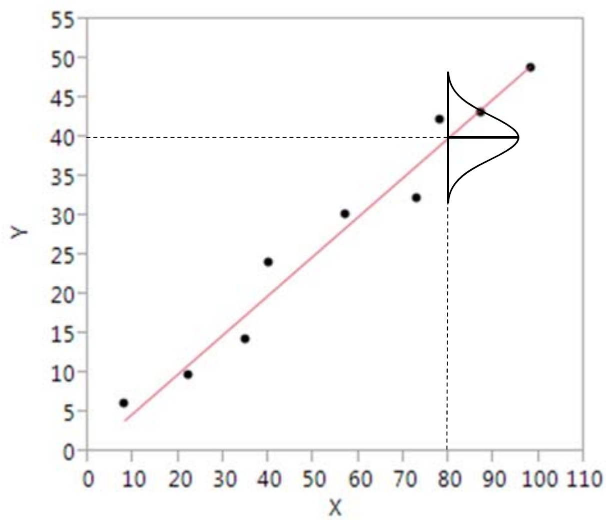

Mean = 40.0
Std dev = 15.4
Defective in the data: 22.2%
Distribution curve: 15.9%

- Moving mean Y to the center of the spec range does reduce % defective
- But is the mean the only thing that changes when we control X at 80?

## RMSE (cont'd)

203

By definition, RMSE is the standard deviation of Y that would result from eliminating the variation in X




$\sigma$ = RMSE
= 2.84

## RMSE (cont'd)

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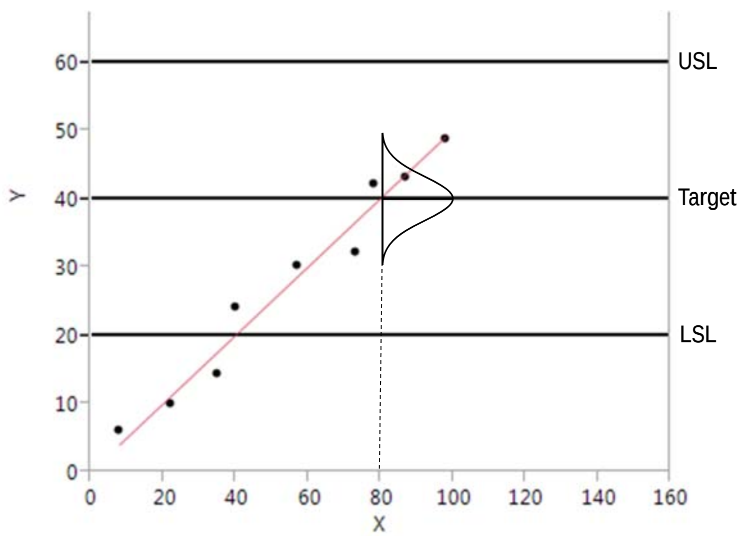
When we control X at 80, we don't just move the mean from 27.9 to 40 — we also reduce the standard deviation from 15.4 to 2.84 !




Hmm . . . how do we calculate the improved % defective?

## Open *LSSV2 other stuff \ t distribution calculator*

205

1. Enter the quantities in the YELLOW cells.
2. The other values are calculated for you.

| | |
|---|---|
| LSL | 20 |
| USL | 60 |
| Mean | 40 |
| Standard deviation | 2.843182 |
| Degrees of freedom | 8 |

| | LSL | USL | Total |
|---|---|---|---|
| Population % out of spec | 0.005 | 0.005 | 0.011 |
| Population PPM out of spec | 54.4 | 54.4 | 108.8 |

PPM defective = 109

These calculations can be sensitive to round-off error. Don't round off the mean and standard deviation when you enter them into the calculator.

Error DF from the Analysis of Variance

**Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Model | 1 | 8836.6440 | 8836.64 | 1093.146 |
| Error | 8 | 64.6695 | 8.08 | Prob > F |
| C. Total | 9 | 8901.3135 | | <.0001* |

Tested against reduced model: Y=0

## Notes

206

## Exercise 10.1



Open *LSSV2 data sets \ production vs capacity.jmp*.

a) Find the best fitting distribution for *Production qty*.

b) What is the predicted % of days on which production quantity will fall below 3000?

c) What is the % of *data values* that fall below 3000?

d) We found earlier that capacity utilization 52.8% gives a mean daily production quantity of 3500. The RMSE was 406.06, the error degrees of freedom was 35. Assuming 52.8% capacity utilization, what is the predicted % of days on which production quantity will be less than 3000?

e) Save your scripts, close and save the data table.

## Exercise 10.2



Open *LSSV2 data sets \ outgassing process* (in JMP). *Current* (the Y variable) is the current required to heat a filament to a target temperature. *Resist* (the X variable) is the electrical resistance of the filament. *Machine* is the processing unit. This example shows how to reduce % defective by separate optimization of each machine.

a) Find the % of *Current* data values that fall outside the interval (1.9, 2.1).

b) Fit a regression for *Current* as a function of *Resist*, using *Machine* as the *By variable*. For each machine, give the RMSE, the error degrees of freedom, and the resistance that predicts a mean current of 2.

| Machine | RMSE | DF | Resistance | % Outside |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |

c) Assuming we use the indicated resistance values, find for each machine the % of *Current* values predicted to fall outside the interval (1.9, 2.1).

d) Save your scripts, close and save the data table.

# 11 Multiple Regression



- Multiple regression model
- Examples
- Fitting regression models
- Interactive effects
- Predicted values and uncertainty
- Modeling and optimization

## Multiple regression model



$$Y = b_0 + b_1X_1 + b_2X_2 + \ldots + b_kX_k + \text{“error”}$$

| $Y$ | $X_1, X_2, \ldots, X_k$ | $b_0$ | $b_1, b_2, \ldots, b_k$ | “Error” |
|---|---|---|---|---|
| Dependent variable | Independent variables | Intercept | Regression coefficients | Mean = 0 |
| Response variable | Explanatory variables | Parameter | Parameters | Standard deviation = $\sigma$ (RMSE) |
| Output | Inputs | | | Distribution = Normal |
| | Predictors | | | |
| | Regressors | | | |
| | Factors (in DOE) | | | |

## Model and error components, one X

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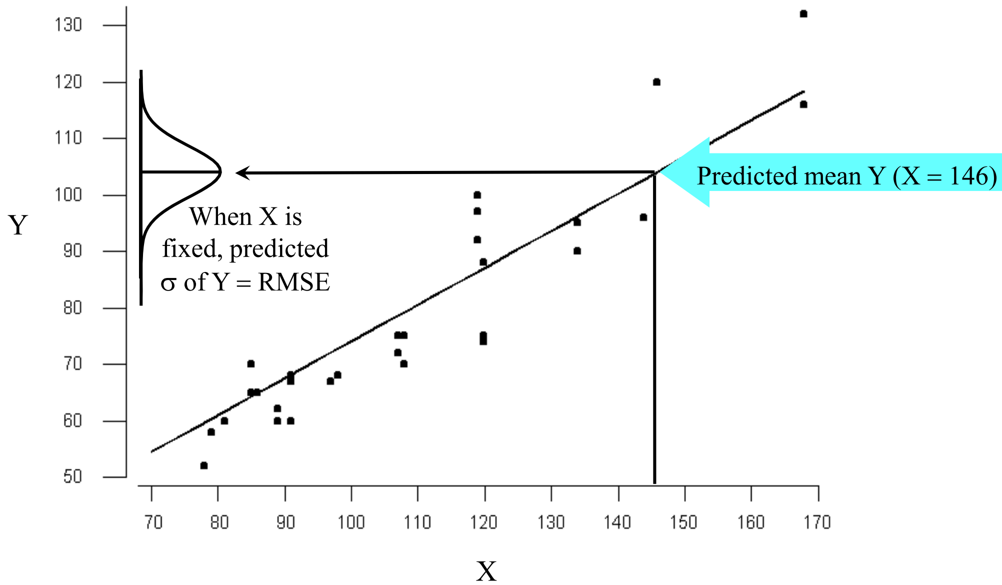
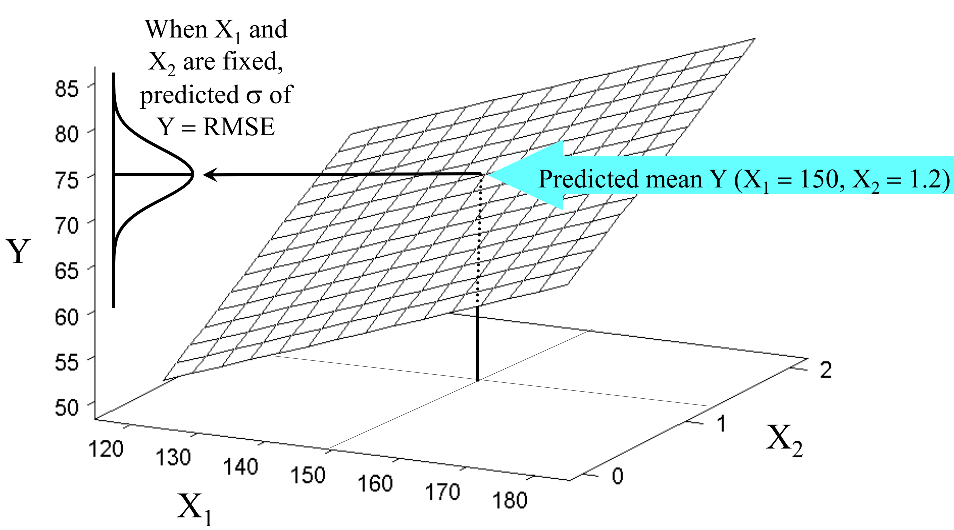
$$Y = b_0 + b_1X + \text{"error"}$$




## Model and error components, two Xs

212

$$Y = b_0 + b_1X_1 + b_2X_2 + \text{"error"}$$



## Multiple regression examples



| Y | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ |
|---|---|---|---|---|---|---|
| *Life of cutting tool* | RPM | Tool type | Material | Feed rate | | |
| *MPG* | Displace-ment | Horsepower | Weight | | | |
| *Salary* | Education | Experience | Performance | Seniority | Gender | |
| *Vending machine service time* | Amount of product stocked | Distance from truck to machine | | | | |
| | | | | | | |
| | | | | | | |

*Fill in examples of interest to you*

## Regression model equations



| Y | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
|---|---|---|---|---|---|
| *MPG* | Displacement (D) | Horsepower (H) | Weight (W) | | |

$$\text{MPG} = b_0 + b_1 D + b_2 H + b_3 W + \text{error}$$

| Y | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
|---|---|---|---|---|---|
| *Bond strength* | Temperature (T) | Dwell time (D) | $T \times D$ | $T^2$ | $D^2$ |

$$\text{Bond} = b_0 + b_1 T + b_2 D + b_3 TD + b_4 T^2 + b_5 D^2 + \text{error}$$

**Response surface model** (RSM) with two continuous Xs

## Linearizing nonlinear models



| Nonlinear model | Equivalent linear model |
| --- | --- |
| $Y = b_0 (X_1)^{b_1} (X_2)^{b_2}$ | $\log(Y) = \log(b_0) + b_1 \log(X_1) + b_2 \log(X_2)$ |
| $Y = b_0 (b_1)^{X_1} (b_2)^{X_2}$ | $\log(Y) = \log(b_0) + \log(b_1) X_1 + \log(b_2) X_2$ |

Notes

## Fitting regression models



Open *LSSV2 data sets \ teenage growth* (in JMP)

| Y | $X_1$ | $X_2$ |
|---|---|---|
| height | age | gender |
| weight | age | gender |

teenage growth

5/0 Cols, 40/0 Rows

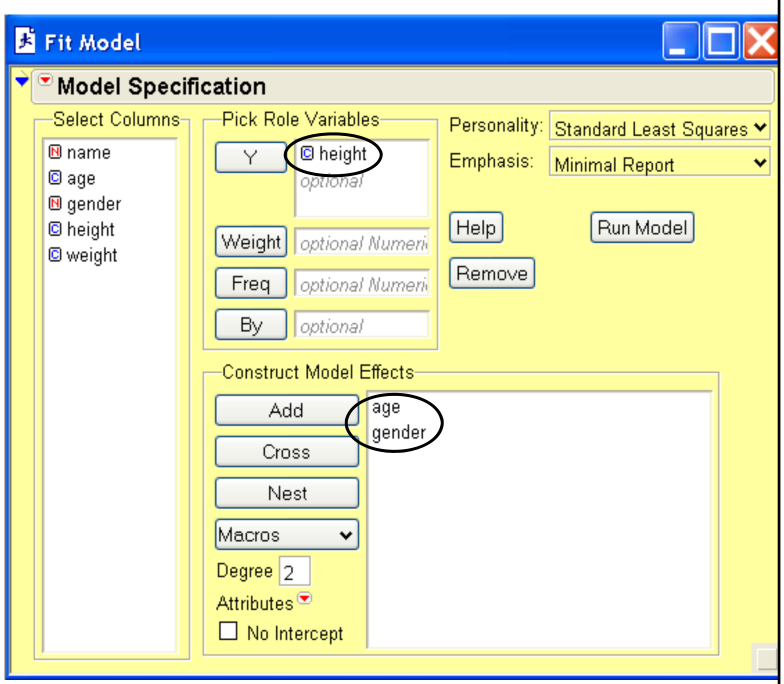
| | name | age | gender | height | weight |
|---|---|---|---|---|---|
| 1 | ALFRED | 14 | M | 64 | 99 |
| 2 | ALICE | 13 | F | 61 | 107 |
| 3 | AMY | 15 | F | 64 | 112 |
| 4 | BARBARA | 13 | F | 60 | 112 |
| 5 | CAROL | 14 | F | 63 | 84 |
| 6 | CHRIS | 14 | M | 64 | 99 |
| 7 | CLAY | 15 | M | 66 | 105 |
| 8 | DANNY | 15 | M | 66 | 106 |
| 9 | DAVID | 13 | M | 59 | 79 |
| 10 | EDWARD | 14 | M | 68 | 112 |
| 11 | ELIZABETH | 14 | F | 62 | 91 |
| 12 | FREDERICK | 14 | M | 63 | 93 |
| 13 | HENRY | 14 | M | 65 | 119 |
| 14 | JACLYN | 12 | F | 66 | 145 |
| 15 | JAMES | 12 | M | 61 | 128 |
| 16 | JANE | 12 | F | 55 | 74 |
| 17 | JEFFREY | 14 | M | 69 | 113 |
| 18 | JOE | 13 | M | 63 | 105 |
| 19 | JOHN | 13 | M | 65 | 98 |
| 20 | JUDY | 14 | F | 61 | 81 |
| 21 | KATIE | 12 | F | 59 | 95 |
| 22 | KIRK | 17 | M | 68 | 134 |
| 23 | LAWRENCE | 17 | M | 70 | 172 |
| 24 | LESLIE | 14 | F | 65 | 142 |
| 25 | LEWIS | 14 | M | 64 | 92 |
| 26 | LILLIE | 12 | F | 52 | 64 |
| 27 | LINDA | 17 | F | 62 | 116 |

## Fitting models (cont'd)



Say we want to model *height* as a function of *age* and *gender*

Analyze
↓
Fit Model

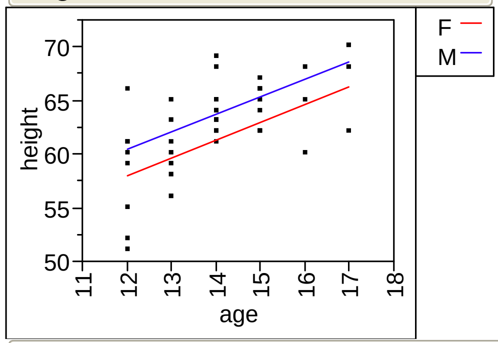


# Categorical X variables



Alt click red triangle next to *Response Height*
↓
Parameter Estimates

**Regression Plot**




**Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Model | 2 | 317.25956 | 158.630 | 15.2592 |
| Error | 37 | 384.64044 | 10.396 | Prob > F |
| C. Total | 39 | 701.90000 | | <.0001* |

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 39.416521 | 4.945343 | 7.97 | <.0001* |
| age | 1.6466542 | 0.352418 | 4.67 | <.0001* |
| gender[F] | -1.214868 | 0.516246 | -2.35 | 0.0240* |

Dummy variable representing the effect of *gender* in the equation

# Numeric coding for two-level categorical X



$$\text{gender}[F] \;=\; \begin{cases} +1 & \text{if gender is F} \\ -1 & \text{if gender is M} \end{cases}$$

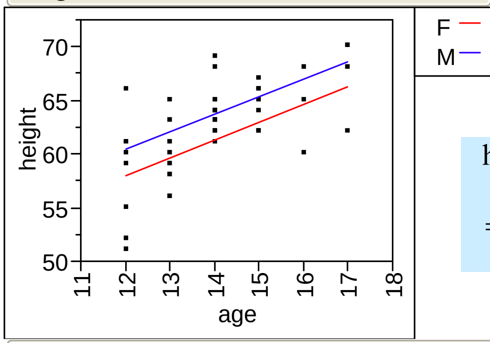
$$\text{height} \;=\; b_0 + b_1\,\text{age} + b_2\,\text{gender}[F]$$

$$\text{height} = \begin{cases} (b_0 + b_2) + b_1\text{age} & \text{if gender is F} \\ (b_0 - b_2) + b_1\text{age} & \text{if gender is M} \end{cases}$$

# Constructing the model equation



## Regression Plot




height

$$= \begin{cases} 38.21 + 1.65\ \text{age} & \text{if gender} = \text{F} \\ 40.63 + 1.65\ \text{age} & \text{if gender} = \text{M} \end{cases}$$

height

$$= 39.42 + 1.65\ \text{age} - 1.21\ \text{gender}[\text{F}]$$

## Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Ratio |
|---|---|---|---|---|
| Model | 2 | 317.25956 | 158.630 | 15.2592 |
| Error | 37 | 384.64044 | 10.396 | Prob > F |
| C. Total | 39 | 701.90000 | | <.0001* |

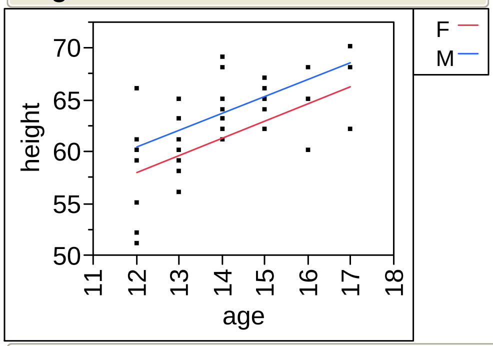
## Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 39.416521 | 4.945343 | 7.97 | <.0001* |
| age | 1.6466542 | 0.352418 | 4.67 | <.0001* |
| gender[F] | -1.214868 | 0.516246 | -2.35 | 0.0240* |

Notes

## The need for interaction effects

**Regression Plot**




**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 39.416521 | 4.945343 | 7.97 | <.0001* |
| age | 1.6466542 | 0.352418 | 4.67 | <.0001* |
| gender[F] | -1.214868 | 0.516246 | -2.35 | 0.0240* |

- With this model, the growth curves are parallel
- That is an *assumption* of the model, not a result of the analysis
- How do we *test* for parallel curves?



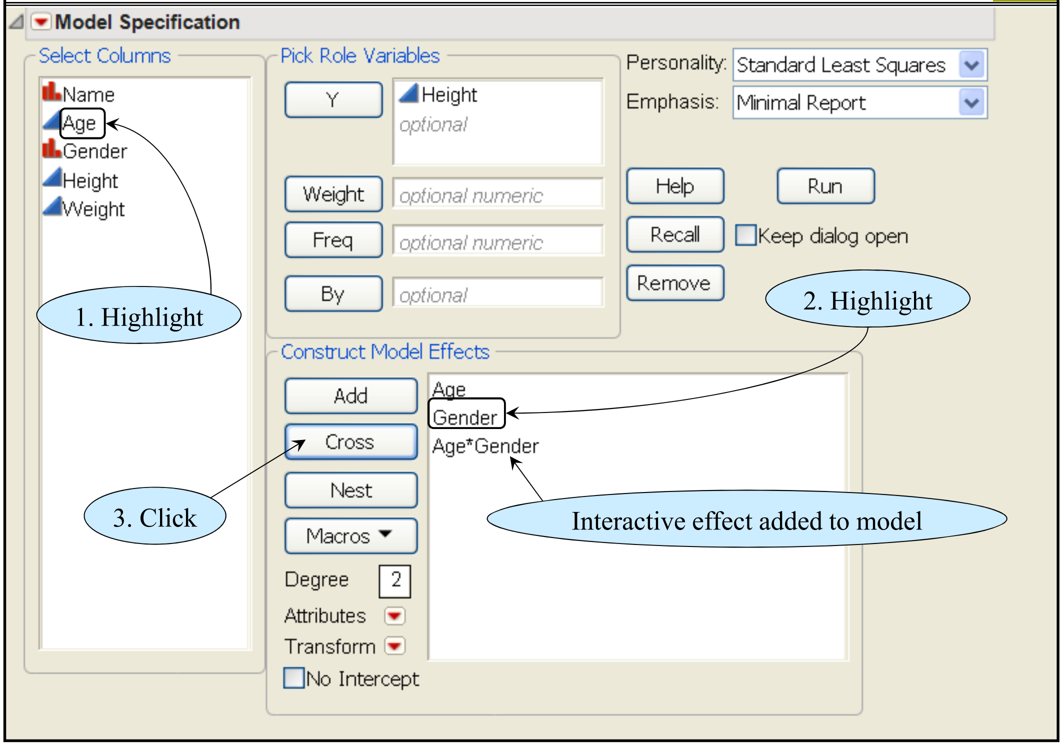
## Interaction effects (cont'd)

$$\text{height} = b_0 + b_1\,\text{age} + b_2\,\text{gender}[\text{F}] + b_3\,\text{age*gender}[\text{F}]$$

This product term allows different slopes for M and F

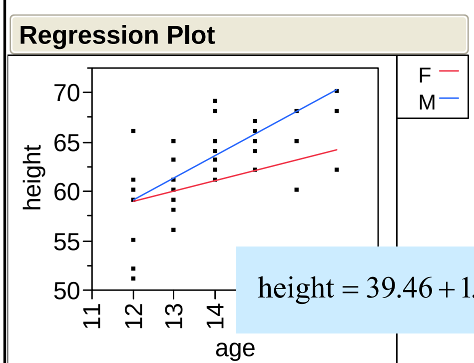
# Adding an interactive effect






# Non-parallel growth curves






$$\text{height} = \begin{cases} 46.62 + 1.04\,\text{age} & \text{if gender} = \text{F} \\ 32.30 + 2.24\,\text{age} & \text{if gender} = \text{M} \end{cases}$$

$$\text{height} = 39.46 + 1.64\,\text{age} + 7.16\,\text{gender[F]} - 0.60(\text{age})(\text{gender[F]})$$

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 39.457057 | 4.812681 | 8.20 | <.0001* |
| age | 1.6360307 | 0.343014 | 4.77 | <.0001* |
| gender[F] | -1.227546 | 0.502444 | -2.44 | 0.0196* |
| (age-13.975)*gender[F] | -0.600896 | 0.343014 | -1.75 | 0.0883 |

$$\text{height} = 39.46 + 1.64\,\text{age} - 1.23\,\text{gender[F]} - 0.60(\text{age} - 13.98)\,\text{gender[F]}$$

# Testing the interaction effect



## Response height

### Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 39.457057 | 4.812681 | 8.20 | <.0001 |
| age | 1.6360307 | 0.343014 | 4.77 | <.0001 |
| gender[F] | -1.227546 | 0.502444 | -2.44 | 0.0196 |
| gender[F]*(age-13.975) | -0.600896 | 0.343014 | -1.75 | 0.0883 |

Some evidence that growth curves for girls and boys have different slopes

### Effect Tests

| Source | Nparm | DF | Sum of Squares | F Ratio | Prob > F |
|---|---|---|---|---|---|
| age | 1 | 1 | 223.96652 | 22.7488 | <.0001 |
| gender | 1 | 1 | 58.76589 | 5.9690 | 0.0196 |
| gender*age | 1 | 1 | 30.21331 | 3.0688 | 0.0883 |

Same information in a more compact format

### Summary of Fit without Interaction

| | |
|---|---|
| RSquare | 0.452001 |
| RSquare Adj | 0.42238 |
| Root Mean Square Error | 3.224234 |
| Mean of Response | 62.55 |
| Observations (or Sum Wgts) | 40 |

✓ Adjusted $R^2$ went up
✓ RMSE went down

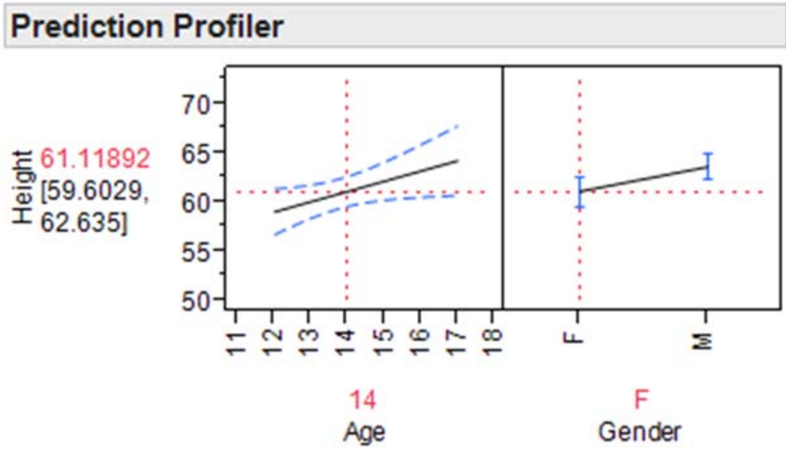
### Summary of Fit with Interaction

| | |
|---|---|
| RSquare | 0.495046 |
| RSquare Adj | 0.452967 |
| Root Mean Square Error | 3.137706 |
| Mean of Response | 62.55 |
| Observations (or Sum Wgts) | 40 |

Notes

## Predicted values and associated uncertainty






| | |
|---|---|
| Predicted avg. height in the population of 14 year old girls | 61.12 |
| 95% confidence interval for avg. height of 14 year old girls | [59.60, 62.64]<br>61.12 ± 1.52 |

## Notes



The model without interaction gave 61.25 ± 1.55 (slightly larger margin of error).

## Exercise 11.1



a) In the table below, record the Adjusted $R^2$ and RMSE from the analysis of *height* in this section. Also, record the P values from *Effects Tests*. Run the same analysis for *weight* and record the corresponding results.

| | | | P values | | |
|---|---|---|---|---|---|
| Response | Adj. $R^2$ | RMSE | Age | Gender | Age*Gender |
| Height | | | | | |
| Weight | | | | | |

b) Which variable (*height* or *weight*) has the greater proportion of variation explained by *age* and *gender*?

c) Explain why it wouldn't make sense to compare the two models in terms of RMSE.

## Exercise 11.1 (cont'd)



d) Both *age* and *gender* were statistically significant for predicting *height*. Is this true for *weight*?

e) For *height* we found evidence that the growth curves for girls and boys have different slopes. Is this true for *weight* as well? Give the P value that is relevant to this question, and explain what it means.

f) Give the predicted average *weight* in the population of 15-year-old boys. Give a 95% confidence interval for this average.

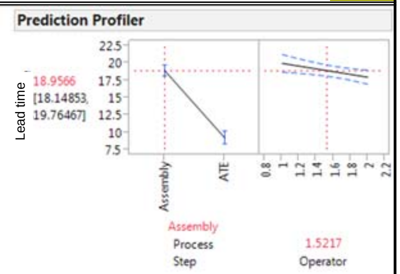
g) Save your scripts, close and save the data table.

## Exercise 11.2

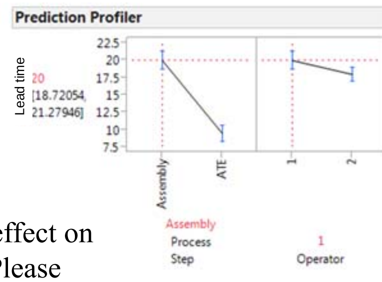


Open *LSSV2 data sets \ lead time 2* (in JMP).

a) Fit a model for *Lead time* including the terms *Process Step*, *Operator*, and their interactive effect.




b) If you got the upper right profiler, you did something wrong. (What is the correct modeling type for *Operator*?) The lower right profiler is the correct one.




c) Select *Model Dialog* on the *Response* red triangle menu. Remove terms under *Effects Tests* with P values exceeding 0.15 (use the *Remove* button). Run the model again. Which terms are left?

d) Based on the profiler, which factor has the larger effect on lead time? Does this correlate with the P values? Please explain.

e) Save your script, close and save the data table.

## Exercise 11.3



Open *LSSV2 data sets \ number and size of defects.jmp*.

a) Fit a model for *Max size* including the terms *Welder*, *# Defects*, their interactive effect, and the quadratic effect for *# Defects* (cross it with itself). This is the *response surface model* for one categorical factor and one continuous factor.

b) Select *Model Dialog* on the *Response* red triangle menu. Remove terms under *Effects Tests* with P value exceeding 0.15 (use the *Remove* button). Run the model again. Which terms are left in the model?

c) Based on the profiler, which factor has the larger effect on *Max size*? Does this correlate with the P values? Please explain.

d) Save your script, close and save the data table.

## Exercise 11.4



In this example you will analyze data from an optimization experiment concerning the removal of excess metal from castings by belt grinding.

The belt supplier had been recommending that belts be discarded when they are "50% used up." This rule was based on tests conducted by the supplier to define the usage point at which the total of labor and belt costs will be minimized. One of the grinders thought the supplier's rule caused grinders to discard belts too soon. Aside from being suspicious that the supplier just wanted to sell more belts, he argued that the supplier's tests did not take into account the time lost to belt changes.

This grinder developed a new standard under which belts would be discarded only after they were "75% used up." He wanted to do a comparative study to show that his method was cheaper overall. After he explains the study with his fellow grinders, 3 additional factors are added to the experiment.

Each casting in the experiment was weighed before and after the grinding operation. A technician kept track of how many belts were used and how long it took the grinder to complete each casting. From this information the total cost per unit of metal removed was calculated for each casting.

## Exercise 11.4 (cont'd)



- Y variable: *cost per unit of metal removed*
- X variables:

| | | | |
|---|---|---|---|
| ➢ Contact wheel land-groove ratio (LGR): | Low | or | High |
| ➢ Contact wheel material (MATL): | Steel | or | Rubber |
| ➢ Belt usage limit (USAGE): | "50%" | or | "75%" |
| ➢ Belt grit size (GRIT): | 30 | or | 50 |

- Open *LSSV2 data sets \ belt grinding*
- Run the *Fit Model* script provided in the left panel, run the model. This is the response surface model for 4 categorical Xs. Remove insignificant terms ($P > 0.15$) then re-run the model. Which terms are left?
- Use the *Prediction Profiler* to find the minimum cost factor settings.
- What do you expect the mean and standard deviation of *Cost* to be after implementing the optimal factor settings?
- Save your script, close and save the data table.

## Exercise 11.5



In this example you will analyze data from an optimization experiment concerning the bond strength of potato chip bags.

Chips 'R' Us was receiving customer complaints about stale chips, especially from customers on airplanes. They traced the problem to the bag sealing process. The current process involved a temperature of 150°C, a pressure of 100 psi and a dwell time of 1.1 secs. The current average bond strength was about 85 psi.

Process Engineer Chip Kettle ran an experiment to increase the bond strength. Production Manager Justin Thyme reminded Chip that he would very much like to avoid an increase in the dwell time.

Justin is able to free up a bag sealer for only so much time each shift. Chip realizes he will need two shifts to complete the experiment. He decides to include *Shift* as an additional variable in the analysis just in case there is an operator and/or equipment effect.

## Exercise 11.5 (cont'd)



- Y variable: *bond strength*
- X variables and feasible ranges:
  - Temperature (TEMP): 120 to 180
  - Pressure (PRESS): 50 to 150
  - Dwell time (DWELL): 0.2 to 2.0
  - Shift: 1 or 2
- *Open LSSV2 data sets \ heat sealing 1*
- Run the *Fit Model* script provided in the left panel, run the model. This is the response model for 3 continuous Xs. Remove insignificant terms from the model (P > 0.15), then re-run it. Which terms are left?
- Use the *Prediction Profiler* to maximize the average bond strength. If your solution requires a long dwell time, find another solution with a short dwell time.
- What do you expect the mean and standard deviation of *bond* to be after implementing the optimal factor settings?
- Save your script, close and save the data table.

## Exercise 11.6



Open *LSSV2 data sets \ outgassing process.jmp*. *Current* (the Y variable) is the electrical current required to heat a filament to a specified temperature. *Resist* (one of the X variables) is the electrical resistance of the filament. *Machine* (the other X variable) identifies which of three processing units was used. We want to develop a model for *Current* as a function of *Resist* and *Machine*.

a) Fit a response surface model for *Current*. (The terms will be *Resist*, *Machine*, the interaction term *Resist*Machine*, and the quadratic term *Resist*Resist*. To get the quadratic term, cross *Resist* with itself.)

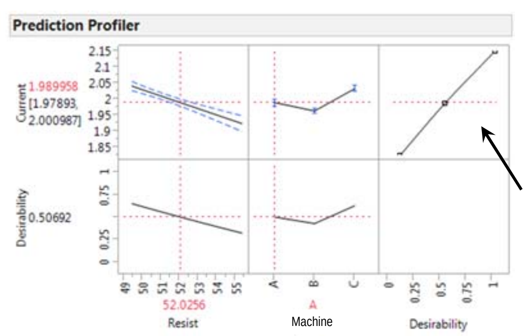
b) Select *Model Dialog* on the *Response* red triangle menu. Remove any terms under *Effects Tests* with P value exceeding 0.15. (Use the *Remove* button.) Run the model again. Record the RMSE.

c) Use the *Prediction Profiler* to find the predicted average *Current* for each machine if we always use filaments with resistance 52.

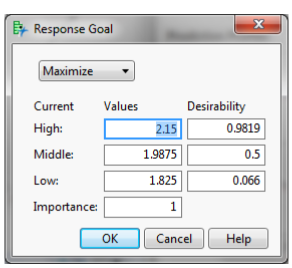
## Exercise 11.6 (cont'd)



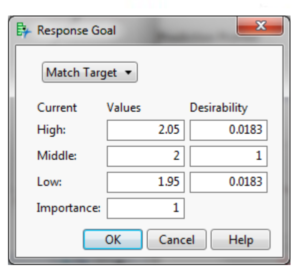
d) The target value for *Current* is 2. For each machine, we want to find the resistance for which the average current is 2. On the *Prediction Profiler* red triangle, select *Desirability Functions*. It should look like this:




e) Double click in the upper right hand panel of the profiler. (Try to avoid the plotted line.) You should get the dialog shown below.



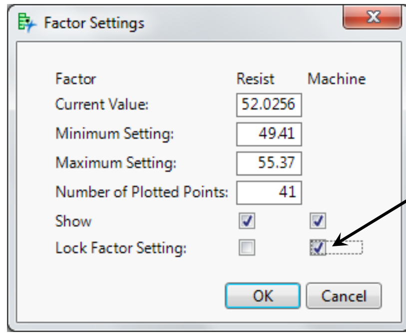

f) Modify the dialog as shown to the right, then select OK. Proceed to the next slide.



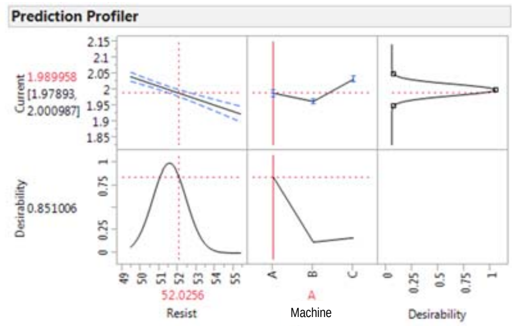
## Exercise 11.6 (cont'd)



g) On the *Prediction Profiler* red triangle, select *Reset Factor Grid*. We want to lock the factor setting for *Machine*, so check the *Lock Factor Setting* box as shown here.



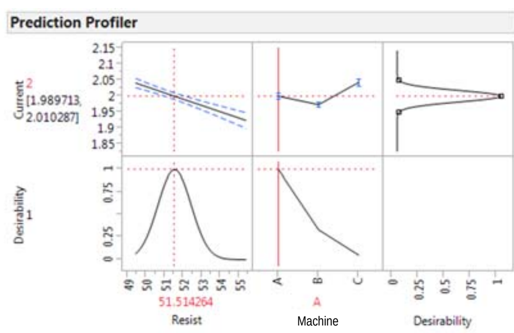

h) The vertical line for *Machine* should now be solid instead of dotted. This will allow you to optimize *Resist* separately for each machine. On the *Prediction Profiler* red triangle, select *Maximize Desirability*. Proceed to the next slide.




## Exercise 11.6 (cont'd)



i) The optimal resistance value for machine A is 51.5. Move the solid vertical line to machine B, find the optimal resistance value. Do the same for machine C.



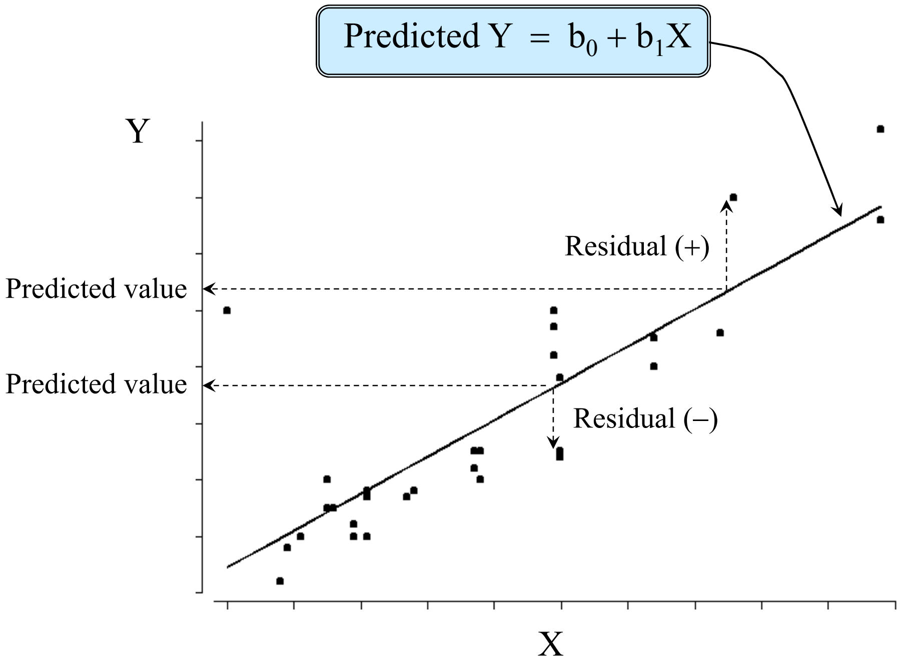

j) What will the average current be if we always use the optimal resistance values?

k) What will the standard deviation of current be if we always use the optimal resistance values?

l) Save your scripts, close and save the data table.

# 12 Transforming the Y variable






## Notes



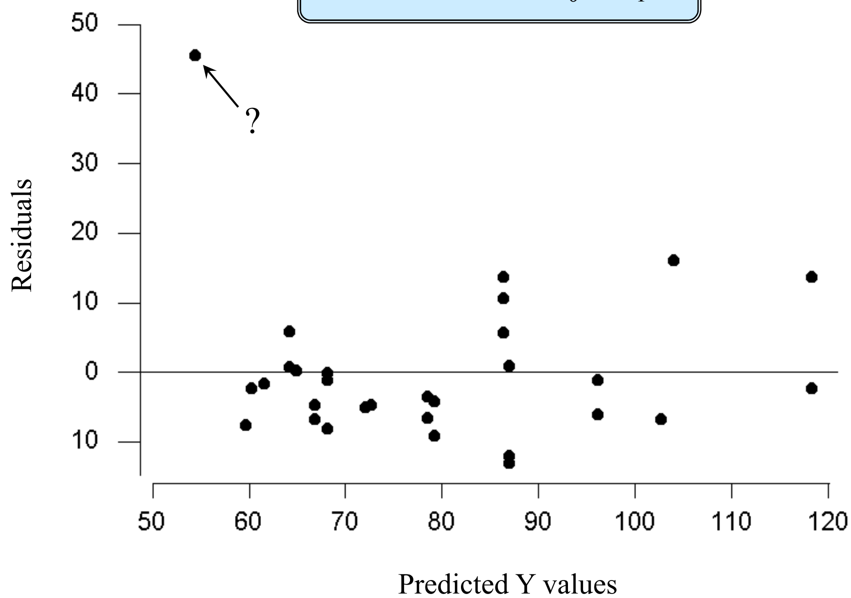
A fitted model gives the predicted mean value of the response variable as a function of the predictor variables. These predicted mean values are also called *predicted values*, or just *predicted* for short. The *residual value* is the data value minus the predicted value. Residual values are called *residuals* for short.

These terms are easiest to visualize in the simple linear model shown above. A predicted value is the fitted line evaluated at some X value. A residual is the difference between a measured Y value and the predicted value at the corresponding X.

Residuals contain information about the magnitude and direction of variability in the data relative to the fitted model. An unusually large residual might signal a measurement error, data entry error or some other type of outlier. A systematic trend or pattern in the residuals might signal an inadequacy in the fitted model.

## Plot of residuals by predicted values

$$\text{Predicted Y} = b_0 + b_1X$$






## Notes

Here the residuals from the preceding slide are plotted against the predicted values. This is a good all-around diagnostic plot. “Healthy” residuals look like random scatter around 0—like a control chart with no assignable causes. Here, it looks like there might be a suspicious data point. If it turns out to be just a data entry error, we simply enter the correct value, then all is well. Most of the time it’s not that simple. When you have an outlier of unknown origin, it helps to run the analysis with and without the questionable data point. If you’re lucky, the results will be pretty much the same both ways, hence no worries.

If excluding the outlier does make a significant difference in the results, then you have a hard decision to make. The official rule is: leave the data point in unless you can identify the cause. The idea is to throw it out only if you can demonstrate to an impartial jury that it does not come from the population you want to study. This is the “pure” approach. This should be tempered with the following practical consideration: you don’t want your results to be determined completely by one extreme outlier, even if you can’t explain it.

## Same thing for any number of X's

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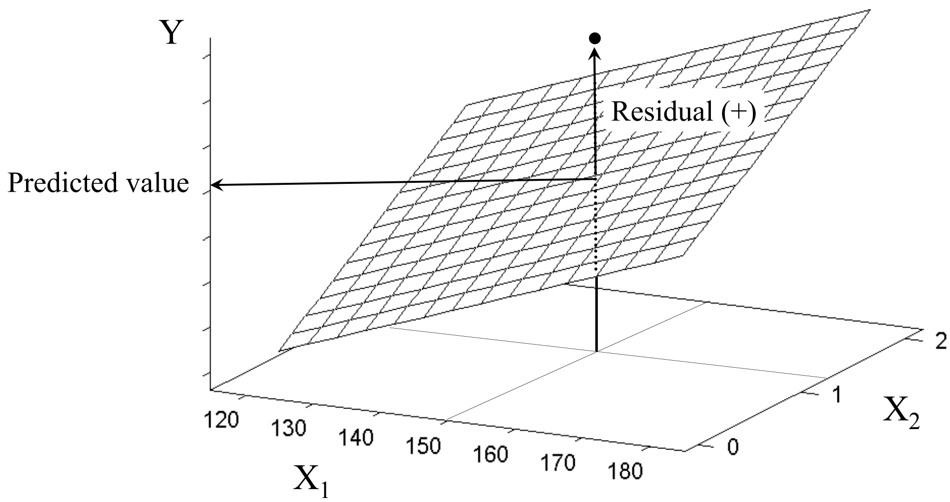
$$\text{Predicted Y} = b_0 + b_1 X_1 + b_2 X_2$$




## Same thing (cont'd)

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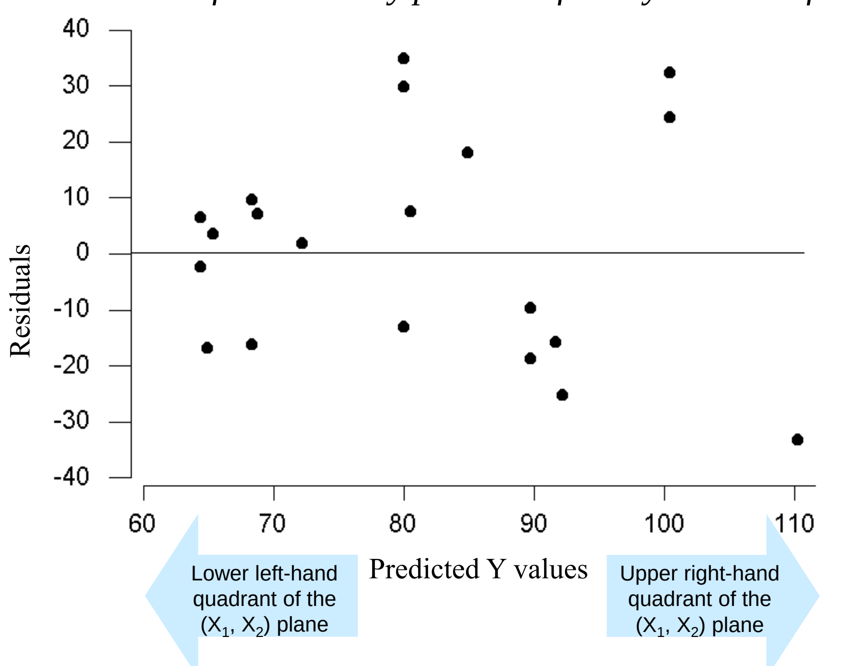
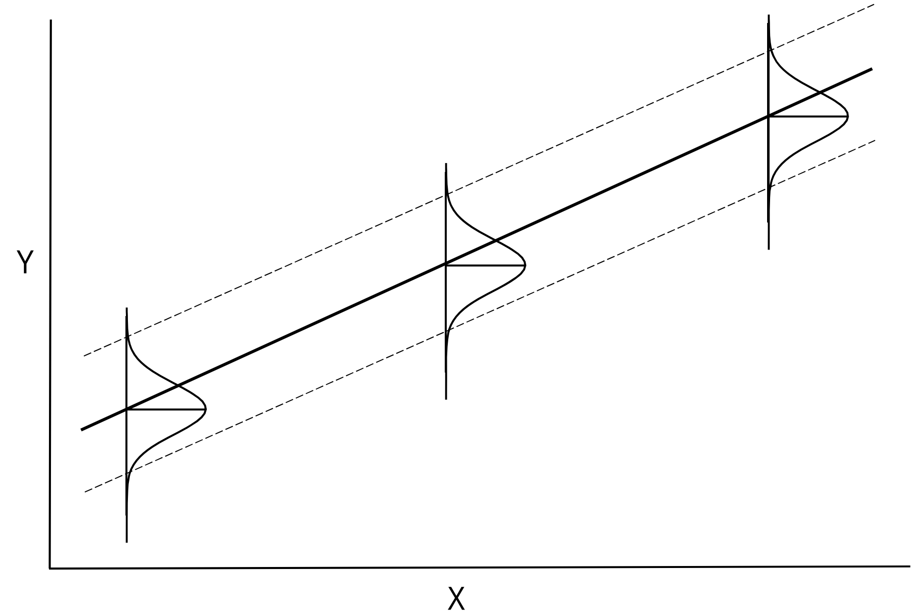
*Plot of residuals by predicted for any number of Xs*



## Basic model assumption



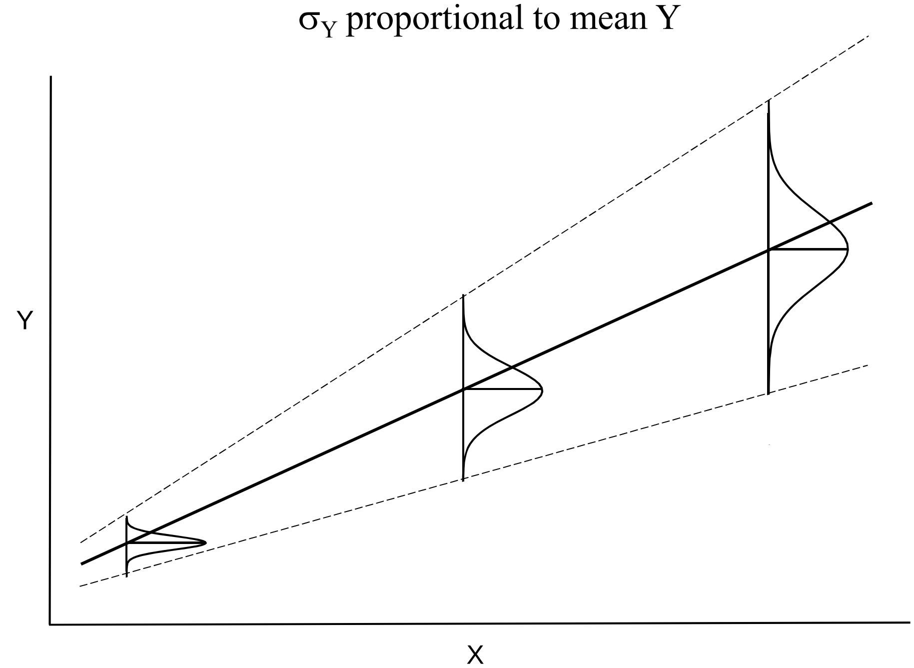
$\sigma_Y$ constant (does not depend on the Xs)




## Most common violation of the basic assumption



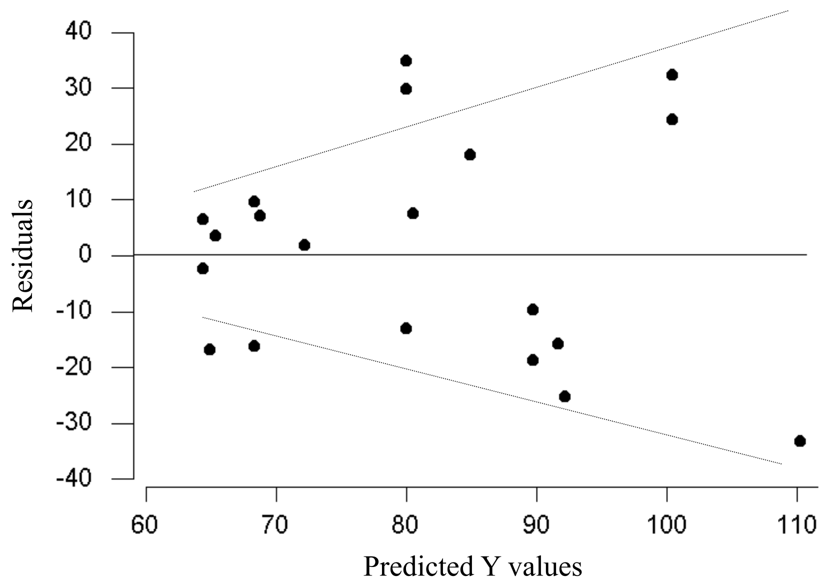
$\sigma_Y$ proportional to mean Y



## Plot of residuals by predicted values



$\sigma_Y$ proportional to mean Y → "sideways V"




## Notes



The standard assumption in all comparison and correlation analyses involving a quantitative Y variable is that the noise (unexplained/error/residual) variation follows a Normal distribution with mean 0 and a standard deviation that does not depend on the X variables.

This simple model has served us well. However, when Normality or constant $\sigma$ is grossly violated, something must be done. The most common remedy is to use log(Y) as the dependent variable instead of Y. This "trick of the trade" is simple and, in most cases, effective.

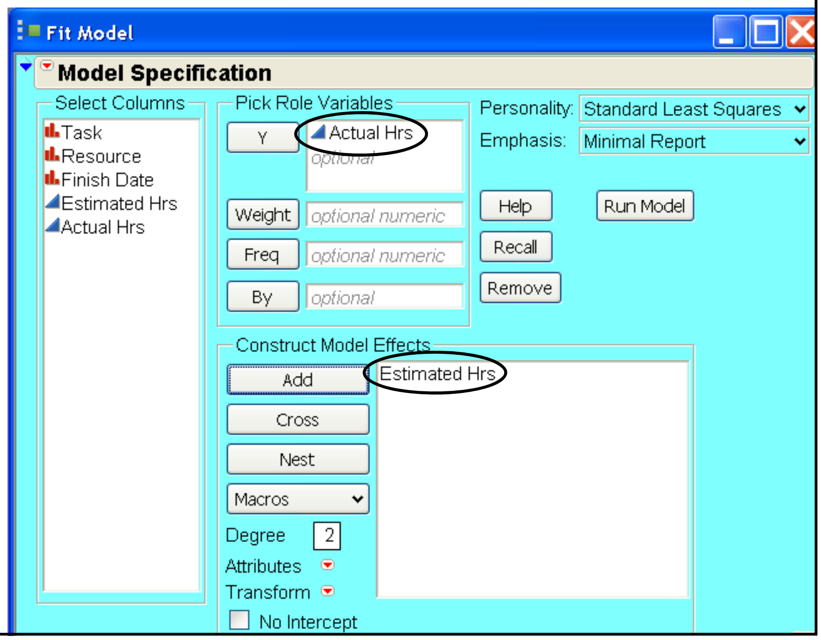
## Transforming the Y variable



Open *LSSV2 data sets \ actual vs estimated* (in JMP)

We want to see how accurately we can estimate the time it takes to do certain tasks

Analyze
↓
Fit Model

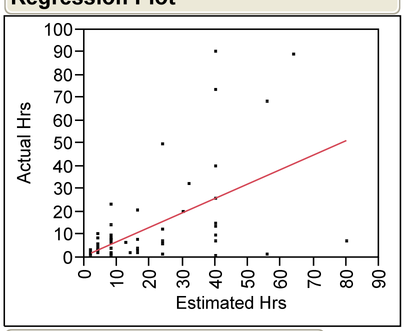



## Transforming Y (cont'd)



**Response Actual Hrs**

**Regression Plot**




**Summary of Fit**

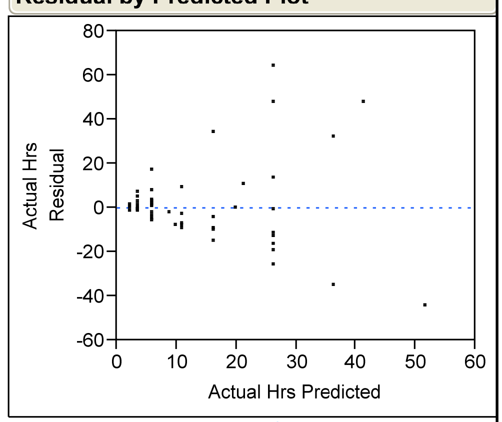
| | |
|---|---|
| RSquare | 0.307347 |
| RSquare Adj | 0.296176 |
| Root Mean Square Error | 16.95281 |
| Mean of Response | 12.23828 |
| Observations (or Sum Wgts) | 64 |

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 0.8352064 | 3.035964 | 0.28 | 0.7842 |
| Estimated Hrs | 0.6321871 | 0.120529 | 5.25 | <.0001* |

$Y = 0.835 + 0.632\ X$

**Residual by Predicted Plot**

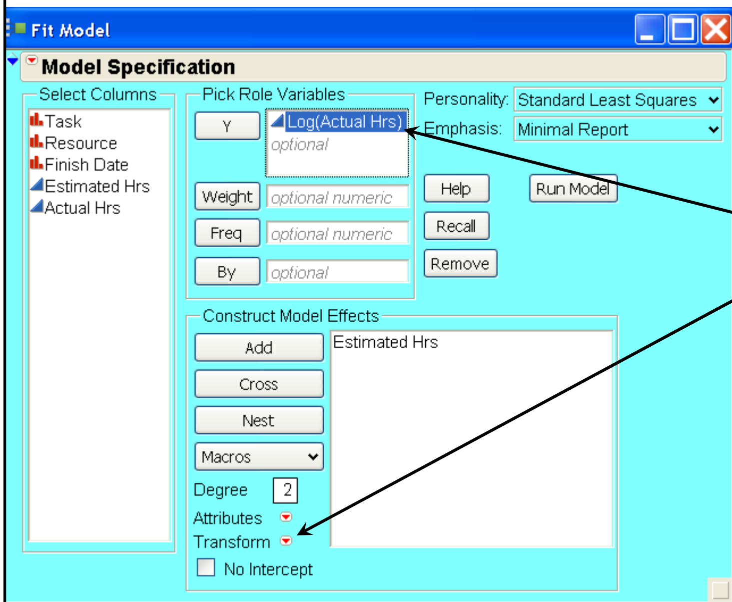



Variation increases as average *Actual Hrs* increases

# Transforming Y (cont'd)



$\sigma_Y$ proportional to mean Y $\Leftrightarrow$ $\sigma_{Log(Y)}$ constant

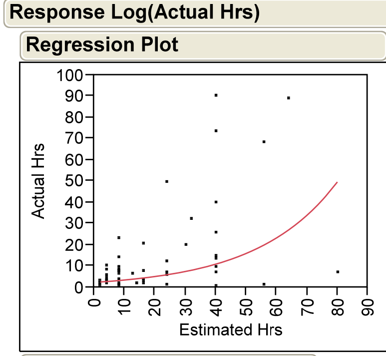



- Click on *Actual Hrs*
- Click on *Transform* red triangle
- Select *Log*
- Run the model

# Effects of log transformation



**Response Log(Actual Hrs)**

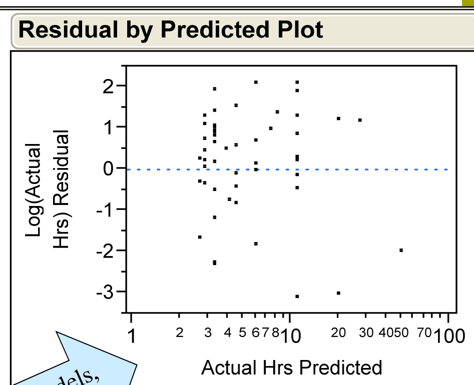



**Summary of Fit**

| | |
|---|---|
| RSquare | 0.233276 |
| RSquare Adj | 0.22091 |
| Root Mean Square Error | 1.217933 |
| Mean of Response | 1.576584 |
| Observations (or Sum Wgts) | 64 |

**Parameter Estimates**

| Term | Estimate | Std Error | t Ratio | Prob>\|t\| |
|---|---|---|---|---|
| Intercept | 0.8982207 | 0.218111 | 4.12 | 0.0001* |
| Estimated Hrs | 0.0376085 | 0.008659 | 4.34 | <.0001* |




For Log(Y) models, use Log scale here

Nonlinear model for Y

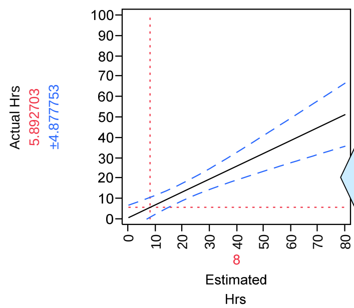
$$\text{Log}(Y) = 0.898 + 0.038X \longrightarrow Y = \exp(0.898 + 0.038X) = e^{0.898}\left(e^{0.038}\right)^X = 2.45(1.04)^X$$

# Effect of not using Log(Y) when you should



**Response Actual Hrs**

**Prediction Profiler**

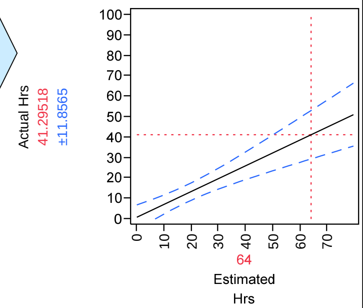



Uncertainty is *overstated* at the low end

Uncertainty is *understated* at the high end

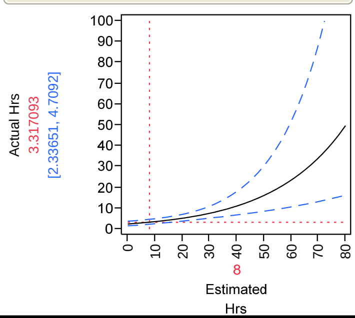
**Response Actual Hrs**

**Prediction Profiler**




**Response Log(Actual Hrs)**

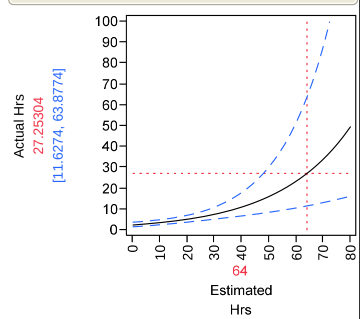
**Prediction Profiler**




- Y data shows σ proportional to mean
- Log(Y) model incorporates this behavior
- Uncertainty is *correctly stated* everywhere

**Response Log(Actual Hrs)**

**Prediction Profiler**




# Summary



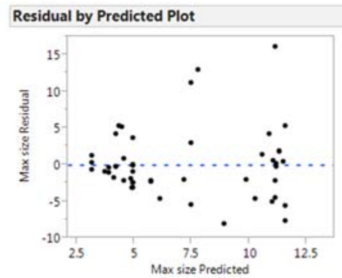
- In this example, the model for Y was questionable because it assumed constant σ.
- Using Log(Y) as the response is better because it models an important aspect of the data: σ proportional to mean.
- Answers to frequently asked questions:
  - ✓ Satisfying model assumptions always takes precedence over Adjusted $R^2$.
  - ✓ P values always take precedence over Adjusted $R^2$.
  - ✓ Adjusted $R^2$ should be used only when there are no other ways to distinguish one model from another.

# Exercise 12.1

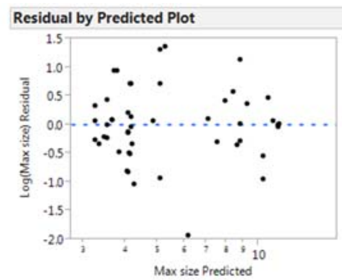


Open *LSSV2 data sets \ number and size of defects.jmp*

a) Fit a model for *Max size* including the terms *Welder*, *# Defects*, their interactive effect, and the quadratic effect for *# Defects* (*response surface model* for one continuous factor and one categorical factor). You should see a distinct sideways V.




b) Select *Model Dialog* on the *Response* red triangle menu, apply a Log transformation to *Max size*, re-run the model. The sideways V isn't completely gone, but close enough.




c) Select *Model Dialog* on the *Response* red triangle menu, remove terms with P > 0.15, run the model again (and again, if necessary).

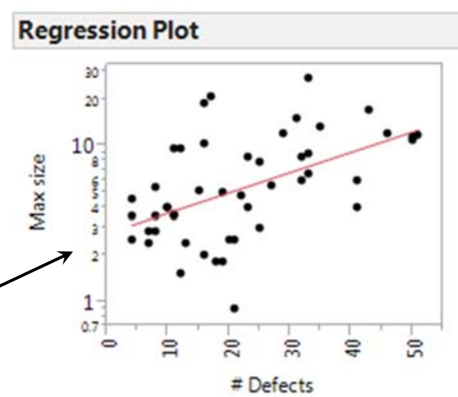
d) Which terms are left in the model equation?

# Exercise 12.1 (cont'd)



e) Now we have a log-linear simple regression.

When you use a Log or square root transformation on Y, it is helpful to use log scale for the Y axes of the plots




f) Save your script, close and save the data table.

## Exercise 12.2

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An aerospace manufacturer uses integral castings as structural components of jet engines. Integral castings give design engineers more flexibility and simplify the assembly process. Defect-free castings are known to have long cycle fatigue life, but defects often arise in the casting process and must be weld repaired. The engine manufacturer's metallurgical team has proposed a finishing process of the following type to ensure adequate cycle fatigue life of weld-repaired castings:

Heat Treat → Polish → Peen

The team wants to optimize the first two steps in this process to achieve maximum cycle fatigue life. Also, though other applications of similar processes have included peening, they would like to see if it can be omitted to reduce processing time and cost.

Due to project time constraints and limited availability of test fixtures, the team can perform at most 12 cycle fatigue tests for their experiment.

## Exercise 12.2 (cont'd)

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- Y variable: *Cycles* (to failure)
- X variables:

| | |
|---|---|
| ➢ Heat treat: | Anneal or Solution/age |
| ➢ Polish: | Chemical or Mechanical |
| ➢ Peen: | Yes or No |

- Open *LSSV2 data sets \ weldment fatigue.jmp*
- Run the *Model* script provided in the left panel, run the model.
- Notice the extreme sideways V on the *Residual by Predicted Plot*.
- Rerun the model using a Log transformation on *Cycles*. Remove insignificant terms from the model (P > 0.15), then run the model again.
- Use the *Prediction Profiler* to maximize the cycle fatigue life.

## Exercise 12.3



A Black Belt wants to minimize the *leak rate* in plastic containers ultrasonically welded together. The X variables and ranges are:

- Force: 70 to 150
- Energy: 275 to 325
- Amplitude: 70 to 90

- Open *LSSV2 data sets \ ultrasonic welding 1.jmp*
- Run the *Model* script provided in the left panel, run the model.
- Use the *Prediction Profiler* to minimize the leak rate.
- Notice anything odd about your minimized leak rate?

## Exercise 12.3 (cont'd)



- Confirm that the *Residual by Predicted Plot* shows a sideways V.
- Re-run the script provided in the data table, apply a Log transformation to *leak rate*, run the model again.
- Remove insignificant terms from the model ($P > 0.15$), then run the model again.
- Use the *Prediction Profiler* to minimize the leak rate.

# Exercise 12.4

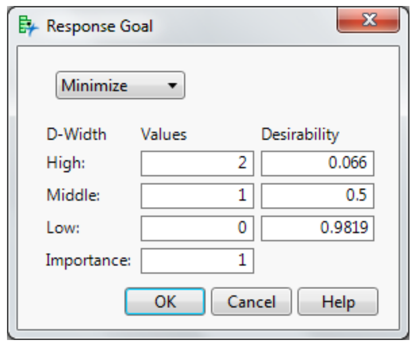
Open *LSSV2 data sets \ electron microscope.jmp*

a) Run the *Fit Model* script using *D-Width* (a 3σ measure of non-repeatability) as the response. You should see a distinct sideways V.

b) Select *Model Dialog* on the *Response* red triangle menu, apply a Log transformation to *D-Width*, re-run the model. The sideways V isn't completely gone, but close enough.

c) Select *Model Dialog* on the *Response* red triangle menu, remove terms with P > 0.15. (Have *Effects Tests* and model dialog open at the same time to avoid errors.) You will have to repeat this cycle, then run the model a third time.

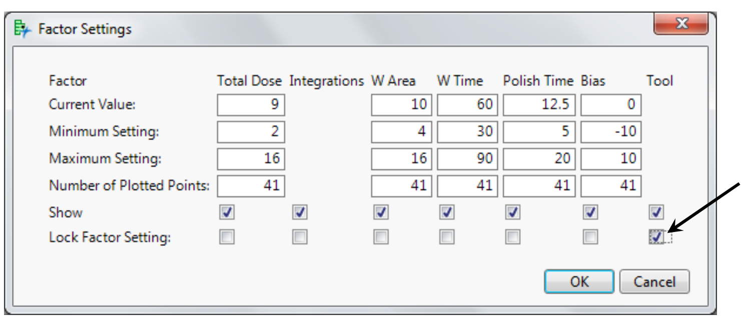
d) We want to minimize *D-Width*. Set up the response goal as shown here.






# Exercise 12.4 (cont'd)

e) It is quite likely that the optimal factor settings will be different for each tool. On the *Prediction Profiler* red triangle, select *Reset Factor Grid*. We want to lock the factor setting for *Tool*, so check the *Lock Factor Setting* box as shown here.




f) Maximize the desirability for each tool. Verify the table below.

| Tool | Total Dose | W Area | W Time | Polish Time | Bias | D-Width |
|---|---|---|---|---|---|---|
| A | 11.8 | 4 | 30 | 5 | -10 | 0.74 |
| B | 9.6 | 4 | 90 | 5 | -10 | 0.72 |
| C | 9.6 | 4 | 90 | 5 | 10 | 1.13 |

g) Save your script, close and save the data table.

# 13 DOE vs "File Cabinet" Data

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*All experiments are experiences, but not all experiences are experiments.*—R A Fisher

| | File cabinet data | DOE |
|---|---|---|
| Data sets | Larger, "messy" | Smaller, "clean" |
| Data collection | Routine operation | Controlled conditions |
| Information provided | Correlations | Cause and effect |
| Interactive effects? | Maybe | Definitely |
| Time period covered | Longer | Shorter |

Notes

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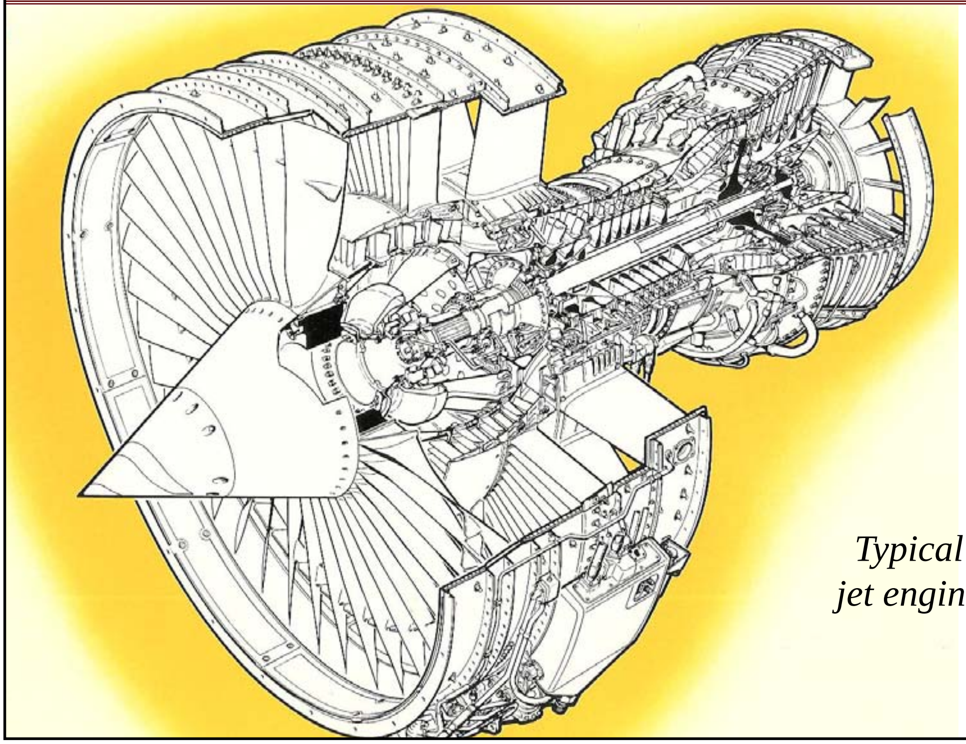
Ronald Fisher was an English geneticist and mathematician trying to increase crop yields in the 1920s. There were limited numbers of plots available for field trials, gradients in the soil, variable proximity to water sources, differing amounts of sunlight, and long lead times. To solve these problems, Fisher developed a body of statistical methods known as Design of Experiments (DOE).

During World War II, Fisher's techniques were extended and applied to military optimization problems. After the war, they were further extended and applied to industrial problems like improving the quality and reliability of manufactured products. For his lifelong contributions to science and statistics, Dr Ronald Fisher eventually became Sir Ronald Fisher.

The quote above was Fisher's way of emphasizing the difference between observational studies (analysis of "file cabinet" data) and designed experiments. This distinction is as important today in Six Sigma as it was a century ago in agriculture. After all, both are concerning with increasing yields!

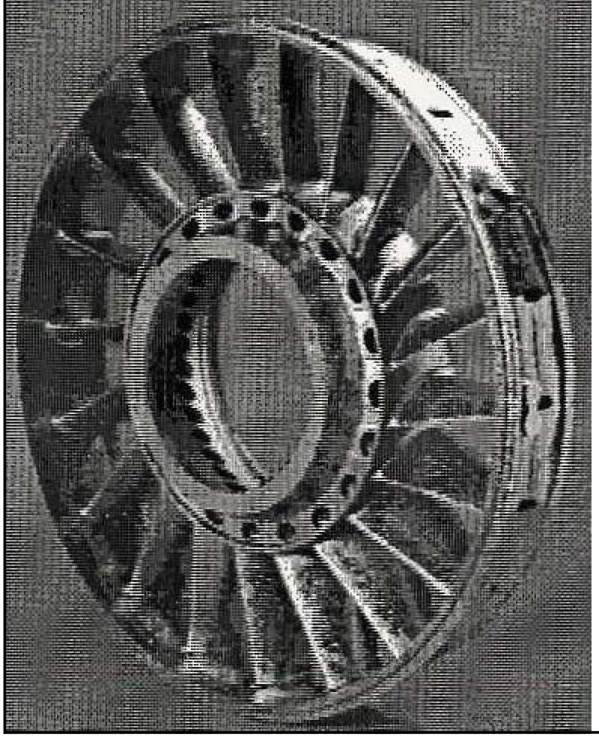
## Case study: structural jet engine components






## Typical structural component of jet engine





- Back in the day: many small pieces welded together
- Now: one piece casting
- 3 to 6 feet in diameter
- Stainless steel, nickel alloys, titanium alloys

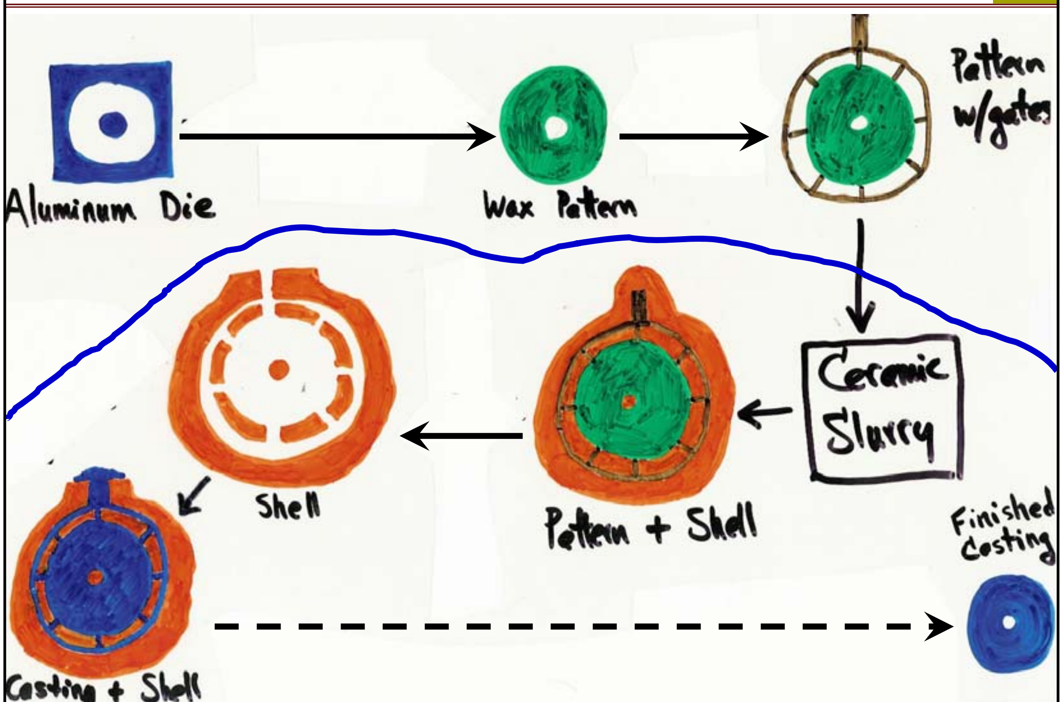
## Case study (cont'd)



- Value stream: investment casting of nickel alloy structural components
- Process boundaries: shell making through backend processing
- Experiencing "orange peel" surface condition violating customer smoothness requirements
- 12% scrap rate (big parts → big $$)
- Y = $f$(X): analyze existing production data

## Investment casting process





# A big signal

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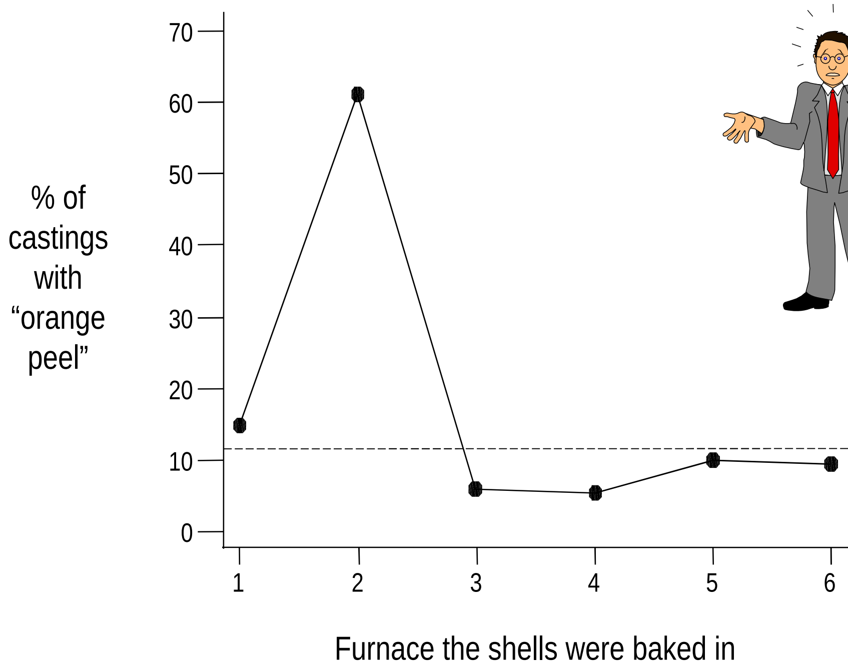


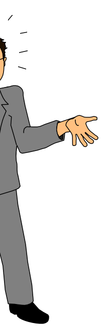

## Notes

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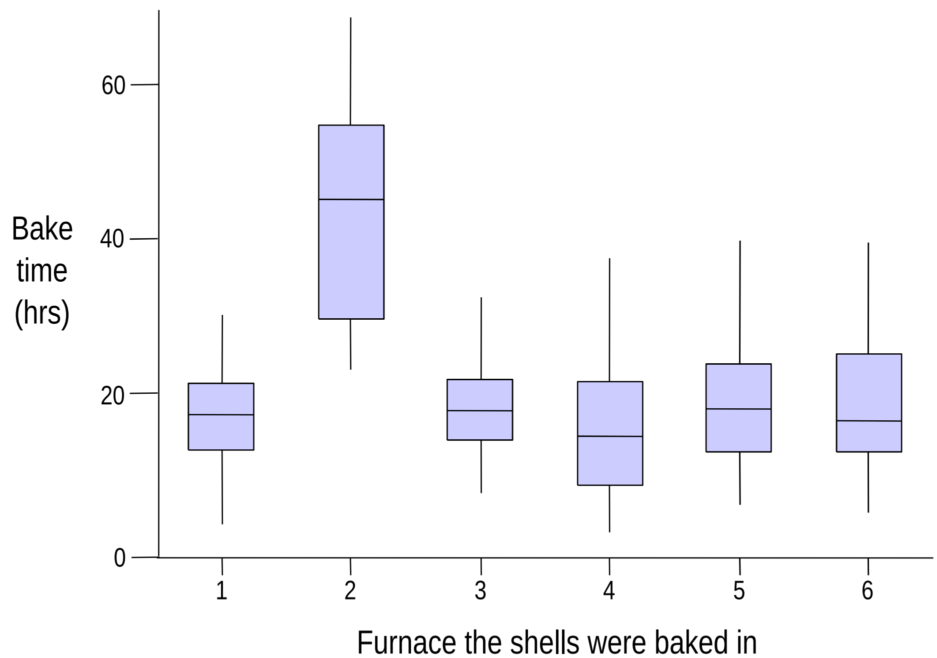
The strongest correlation in the database involved one of the pre-heat furnaces used to bake the ceramic shells before transfer to the casting furnace. Furnace 2 was new and had come on line just about the same time orange peel started occurring. Almost everyone agreed the new furnace was the problem.

The casting area manager refused to take Furnace #2 off line. He needed all six pre-heats to keep the casting furnace running nonstop so he could meet his production quotas.

Process Engineer Dave (shown above) was skeptical that Furnace 2 was causing the problem. For one thing, the other pre-heats were also producing scrap castings. Also, he had spent the better part of the past three months evaluating and qualifying the new furnace.

## Another big signal






## Notes



Dave pointed out that the shell bake times were much longer for Furnace 2 than for the other furnaces. There was a minimum required bake time, but no upper limit. Dave's theory was that orange peel was caused by long bake times.

Why did shells stay longer in Furnace 2?

It turned out there wasn't room to put the new furnace next to the original five, so it had to be located further away from the casting furnace. The fork-lift operators wouldn't drive over there unless they had no shells ready from the closer furnaces, so shells tended to sit in Furnace 2 for a long time.

## Autopsy

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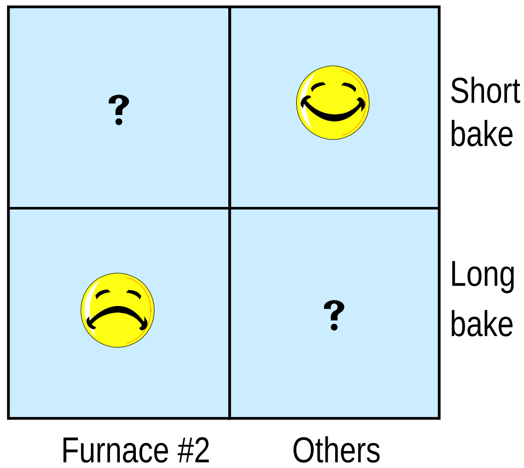
- The file cabinet data suggested some plausible hypotheses
- It could not establish the cause of the defect
- The *quantity* of data was not the problem
- The data lacked the *structure* required to determine cause and effect




## Notes

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There was lots of data in the upper right-hand and lower left-hand cells in the table above, but virtually nothing in the other two cells. Making sure that data tables like the one above are completely filled out is one of the basic principles of experimental design.

Subsequently, engineers ran enough parts in the upper left-hand corner of the table to determine that long bakes were indeed causing the problem. An upper limit on the bake time was developed and put in place. Shells that exceeded this limit were scrapped. This cost the company much less than scrapping the resulting castings.

The new procedure made the fork-lift operators’ job harder, but it made the orange peel problem go away.

# 14 The Role of DOE in LSS Projects

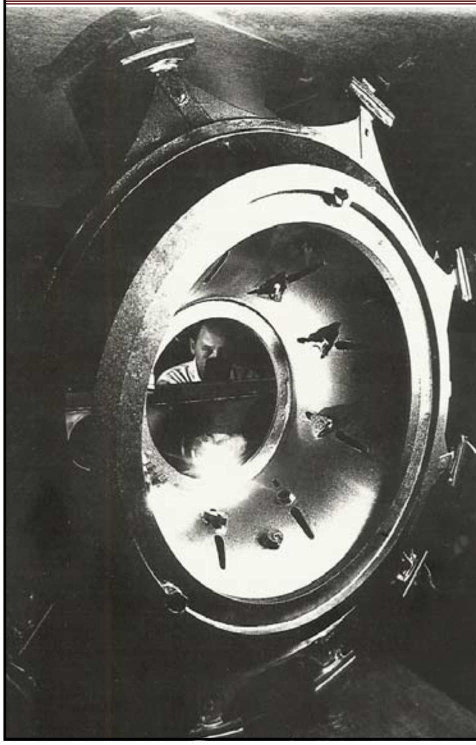


| | |
|---|---|
| Y = ***f***(X) analysis | • Data collection in the Measure phase may have produced little or no useful information<br>• DOE is an effective way to collect useful current state data in a relatively short period of time |
| Developing the future state | • May have multiple potential improvement ideas on the table<br>• DOE is an effective way to evaluate these ideas prior to defining the future state |

Notes

## Example





- Titanium castings → strong & light
- Ti develops surface oxidation during the cooling phase
- Large Ti castings were failing the customer $O_2$ requirement
- Analysis of file cabinet data yielded no significant correlations
- Engineers developed a list of factors for a DOE

## Example (cont'd)



| Factor | Levels | Current state X variable | Possible future state solution |
|---|---|---|---|
| Slurry for shell | Batch 1 vs Batch 2 | ✓ | |
| Shell thickness | 14 dips vs 18 dips | | ✓ |
| Shell bake time | 6 hrs vs 48 hrs | ✓ | |
| Shell bake temp | 1950° vs 2050° | | ✓ |
| Alloy grade | Low $ vs High $ | | ✓ |
| Alloy status | New vs Revert | ✓ | |
| Heat shield steel | Mild vs SS | | ✓ |
| Cooling fan speed | 2400 vs 3200 | | ✓ |

## 15 One Factor at a Time?

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- In this approach, each factor is varied with all others held constant. This way, it is felt, we can see the "pure effect" of each factor.
- This is one way to apply the scientific method, but it is not the only way.
- For any proposed one at a time experiment, there is usually a multifactor experiment providing:
  - ✓ More information
  - ✓ Better results
  - ✓ Same (or possibly smaller) total sample size
- One at a time trials *are* useful for determining feasible ranges for factor in a DOE

## Example: potato chip bags

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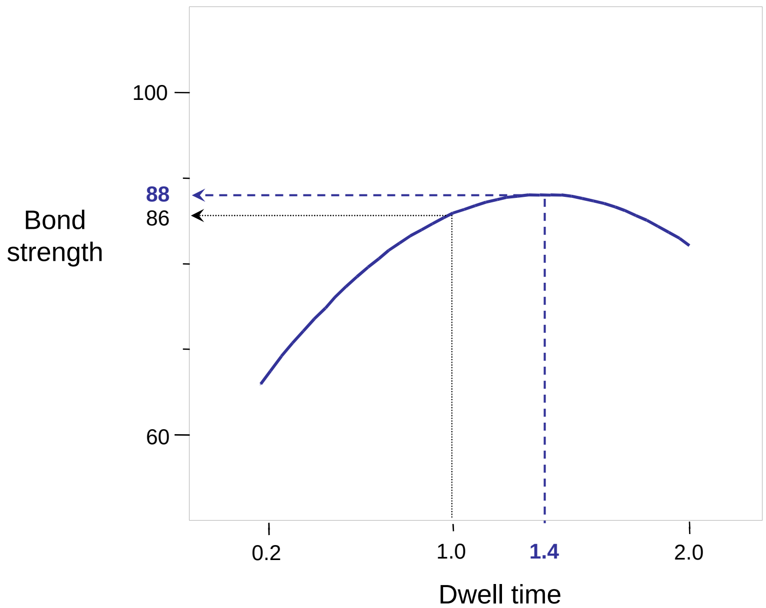
- The current average bond strength of our potato chip bags is 86 psi
- Based on customer complaints, we need to increase the bond strength
- The most important control factors in the bag sealing operation are *temperature* and *dwell time* (see below)
- Secondary objective: decrease the *dwell time* if possible

| Factor | Current level | Feasible range |
|---|---|---|
| Temperature | 150° | 120 to 180 |
| Dwell time | 1.0 secs | 0.2 to 2.0 |

# One-at-a-time experiment #1



Vary *dwell time* over its feasible range while holding *temperature* at 150




N = 9

## Notes



Our process engineer Chip Kettle first studies the effect of dwell time while holding temperature constant. He seals and tests 9 bags using dwell times ranging from 0.2 to 2.0. Chip finds he can increase the bond strength by 2 psi by increasing the dwell time to 1.4.

Our production manager Justin Thyme is not pleased with the prospect of a 40% increase in dwell time.

# One-at-a-time experiment #2

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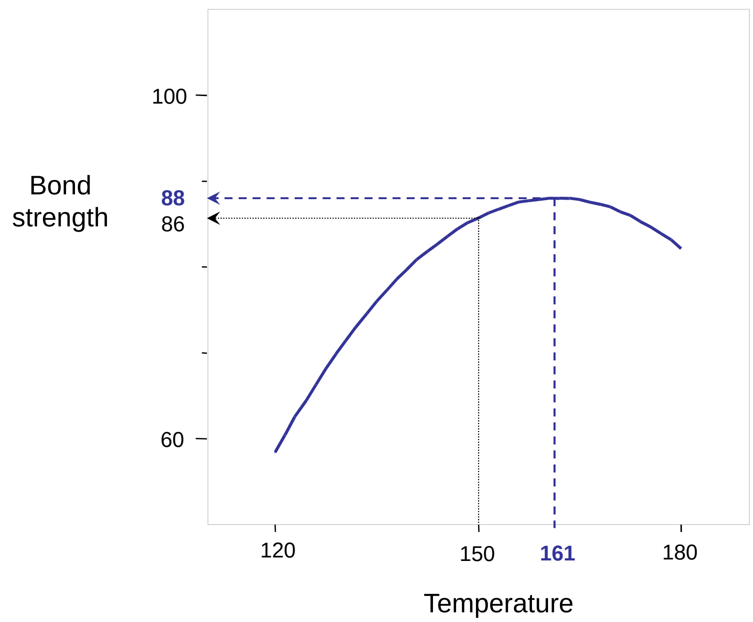
Vary *temperature* over its feasible range while holding *dwell time* at 1.0




N = 9

## Notes

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Chip now studies the effect of temperature while holding dwell time constant. He seals and tests 9 bags using temperatures ranging from 120 to 180. Chip finds he can increase the bond strength by 2 psi by increasing the temperature to 161.

Chip predicts that changing the dwell time to 1.4 and the temperature to 161 will increase the average bond strength by 4 psi (2 + 2). However, it is highly likely that Justin will oppose the increase in dwell time, in which case the increase in average bond strength will be only 2 psi.

## The multi-factor approach

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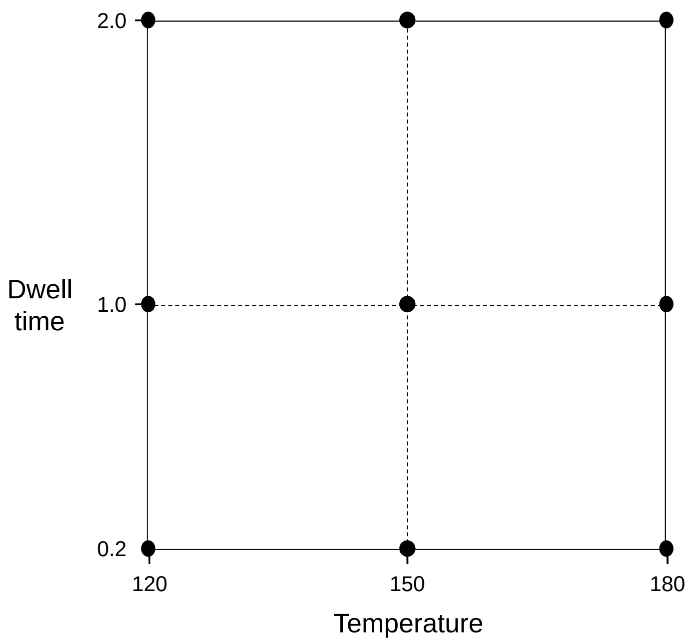


- ✓ 9 design points (●)
- ✓ 2 bags sealed at each point
- ✓ Total N = 18

## Contour plot of predicted average bond strength

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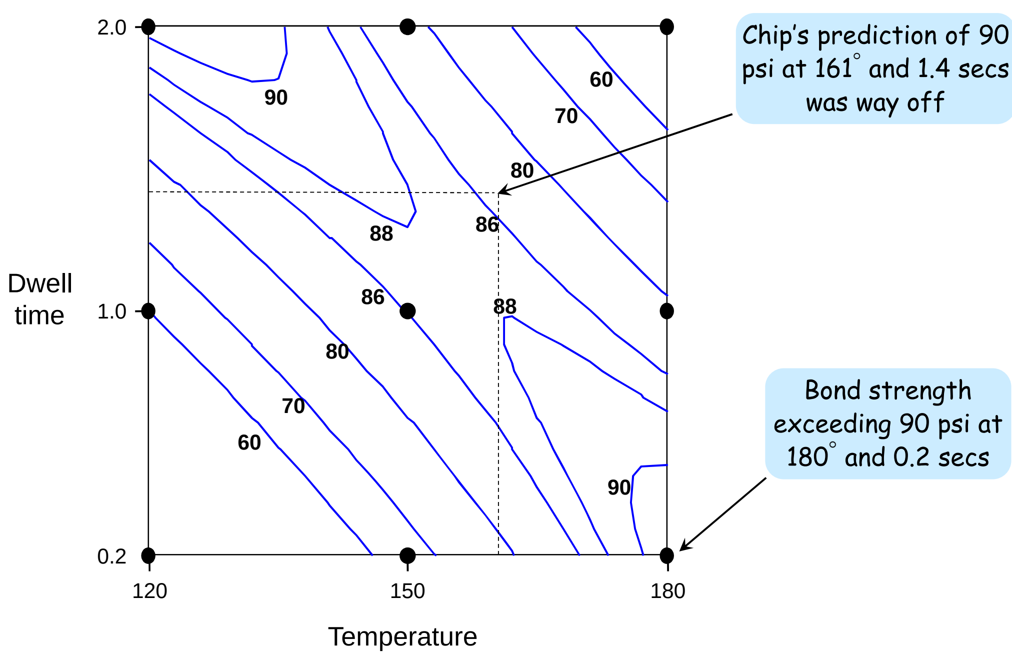

# Why one-at-a-time doesn't work

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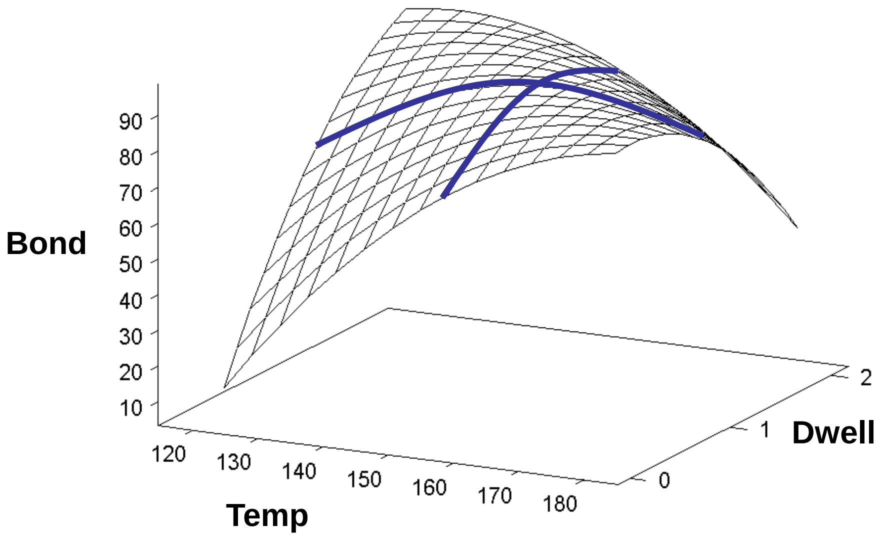
*The 3D perspective*




## Notes

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When we experiment with all factors but one held constant, we optimize sequentially over one-dimensional profiles. The sequence of solutions generated by this process is highly dependent on the starting point. It has very little chance of finding a global optimum, and often fails to move a significant distance from the starting point.

# 16 DOE Terminology



*Experimental unit*

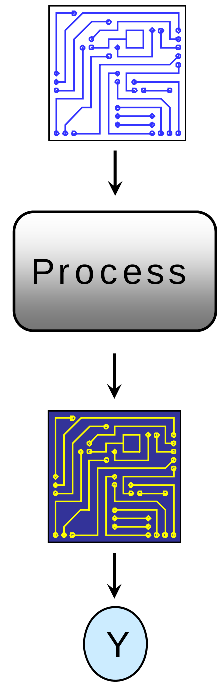
The outcome of a single application of the process being studied

*Sample size*

The total number of experimental units (“number of runs”)

*Response variable*

A Y variable measured or inspected on each experimental unit




## Notes



The experimental unit is often a part, lot, batch or single transaction of some kind. It may also be a test specimen or sample of material. It is important to identify the experimental unit—it provides the basis for counting sample size, and sample size is critical in determining the statistical significance of the results.

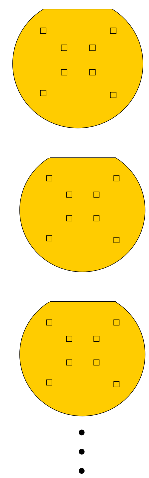
The experimental unit is determined by the process on which we are experimenting, not the measurement plan used to evaluate the results. For example, suppose we test 100 devices for product life. Suppose we measure a degradation parameter on each device every 10 hours until the end of the test at 100 hours. The sample size for the study is the number of units (100), not the number of measurements (1000).

# Example



- 11 silicon wafers were subjected to vapor deposition at various temperatures, pressures, and Argon flow rates
- The thickness of the resulting layer was measured at 8 locations on each wafer
- What is the sample size?

| Temp | Press | Flow | Thickness |
|---|---|---|---|
| 180 | 0.3 | 30 | |
| 180 | 0.3 | 30 | |
| 180 | 0.3 | 30 | |
| 160 | 0.4 | 10 | |
| 160 | 0.4 | 50 | |
| 160 | 0.2 | 50 | |
| 160 | 0.2 | 10 | |
| 200 | 0.4 | 10 | |
| 200 | 0.2 | 10 | |
| 200 | 0.2 | 50 | |
| 200 | 0.4 | 50 | |

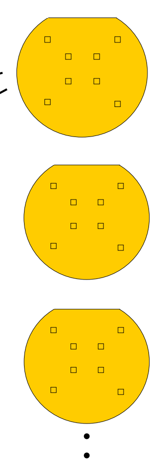


# Example (cont'd)



- The sample size is the number of experimental units, not the total number of measurements taken
- The response variables of interest may be statistical summaries of multiple measurements on each unit

| Temp | Press | Flow | Avg. | Std. dev. |
|---|---|---|---|---|
| 180 | 0.3 | 30 | | |
| 180 | 0.3 | 30 | | |
| 180 | 0.3 | 30 | | |
| 160 | 0.4 | 10 | | |
| 160 | 0.4 | 50 | | |
| 160 | 0.2 | 50 | | |
| 160 | 0.2 | 10 | | |
| 200 | 0.4 | 10 | | |
| 200 | 0.2 | 10 | | |
| 200 | 0.2 | 50 | | |
| 200 | 0.4 | 50 | | |



# DOE terminology (cont'd)

*Factor*

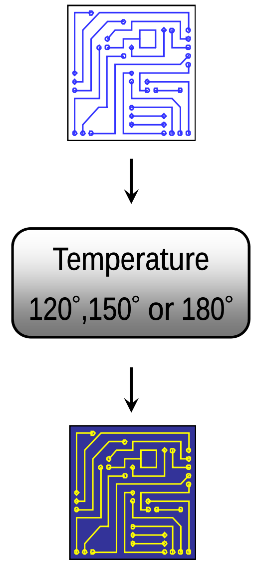
An X variable controlled in an experiment, varied on purpose to determine its effect on the responses

*Level*

A particular value or setting of a factor to be used in the experiment

*Requirements*

All levels of each factor must be logically and physically compatible with all levels of the other factors






## Notes

Variables used as factors in a designed experiment may or may not be controlled in the routine process. What matters is that they can be controlled for the purpose of experimentation.

## DOE terminology



*Examples of continuous factors*

| | |
|---|---|
| Time | Volume |
| Temperature | Weight |
| Pressure | Length |
| Energy | Width |
| Voltage | Density |
| Resistance | Rate |
| Concentration | RPM |
| Flow | Intensity . . . |

Notes



- A factor is *continuous* if it can be varied within some range on a scale of measurement
- It is generally preferable to use 3 equally-spaced levels (low, medium, and high) for continuous factors

## DOE terminology

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*Examples of categorical factors*

| | |
|---|---|
| Method | Old or New |
| Tool set | 1, 2 or 3 |
| Material | A, B, C or D |
| Supplier | X, Y or Z |
| Operator | Bob, Carol, Ted or Alice |
| Color | Cyan, Magenta or Yellow |
| Size | Small, Medium or Large |

## Notes

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- A factor is *categorical* if it represents a set of discrete choices
- It is easier to design and analyze experiments with categorical factors because the levels are given, the models are simpler, and we don't need to interpolate.
- Treating a factor as quantitative implies that any value in the range can be used in the process
- If the levels used in the experiment are the only values that can be used in the process, the factor should be treated as categorical
- For example, one of the controls parameters for certain electron microscopes has to be a power of 2.
- If you treat this as a continuous factor, the optimal value from the DOE will most likely not be a power of 2

## Distinctions relative to design and analysis



| *Categorical factors* | *Continuous factors* |
|---|---|
| Any number of levels | Usually 3 levels |
| Discrete set of design points | Region in factor space |
| Test for significant differences | Response surface modeling |
| Select best design point | Interpolate between design points |

## DOE terminology (cont'd)



*Control factors*

Can be controlled in the routine process

↓

Type of material
Temperature
Pressure
Method
Time
⋮

*Noise factors*

Cannot be controlled in the routine process

↓

Ambient conditions
Raw materials
Operators
Suppliers
Batches
Setups
Shifts
Lots
⋮

When possible, it is good practice to include selected noise factors in experiments. Why?

## DOE terminology



### *Design point*

A particular combination of levels of the factors.

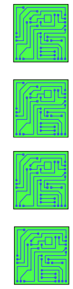
### *Design matrix*

The set and sequence of design points to be used in the experiment.

### *Full factorial*

The set of all possible design points for a given set of factors and levels.

| Temp | Press |
|---|---|
| 120 | 50 |
| 120 | 150 |
| 180 | 50 |
| 180 | 150 |



Experimental units

- ✓ Full factorial
- ✓ 4 design points
- ✓ No repeats
- ✓ Sample size = 4

## DOE terminology



### *Repeat run*

An experimental unit created independently of other units at the same design point

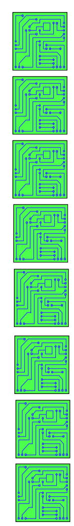
### *False repeat*

- Repeated measurement of one unit
- Units in the same batch, when optimizing a batch process for which there is very little variation within batches

### *Replicate*

A set of repeat runs, one for each unit in a given set

| Temp | Press |
|---|---|
| 120 | 50 |
| 120 | 150 |
| 180 | 50 |
| 180 | 150 |
| 120 | 50 |
| 120 | 150 |
| 180 | 50 |
| 180 | 150 |



Experimental units

- ✓ Full factorial
- ✓ 4 design points
- ✓ 1 replicate
- ✓ Sample size = 8

## Exercise 16.1



A bank wants to increase the yield of its credit card offers. It plans to collect VOC data by means of a DOE involving the factors in the table below. The bank plans to send out 1000 offers for each combination of the factor levels. Based on the data, they will determine the combination with the greatest % yield.

(a) How many design points are in the full factorial?

(b) What is the experimental unit?

(c) What is the sample size?

(d) What is the Y variable?

(e) For each factor, decide whether you would treat it as quantitative or categorical (give your answers and reasons in the table below).

## Exercise 16.1 (cont'd)



| Factor | Levels | Continuous or categorical? |
|---|---|---|
| Introductory APR | 0, 2.5, or 5% | |
| Introductory time period | 3, 6, or 9 months | |
| Gift | None, iPhone, iPad, or espresso machine | |

# 17 Creating a Full Factorial Design

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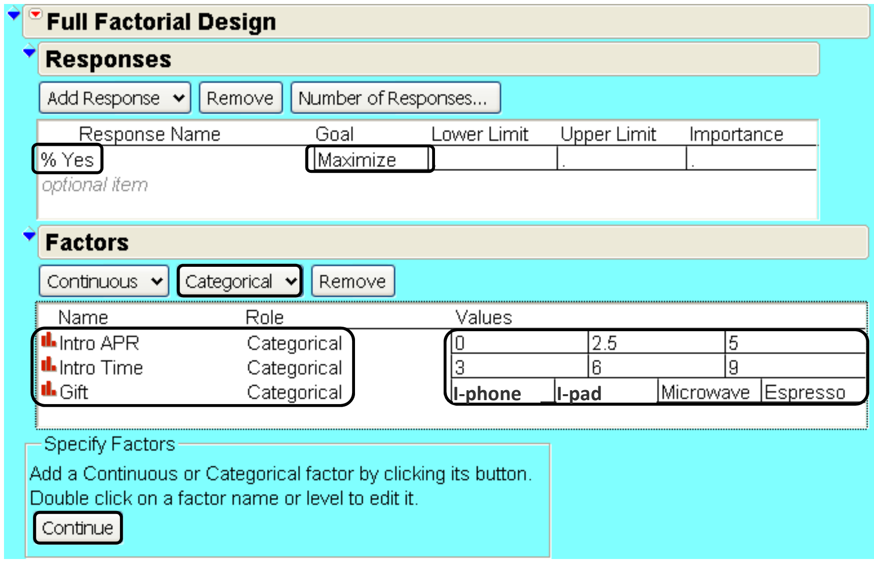
*DOE → Full Factorial Design*

1. Define responses, factors, numerical ranges for quantitative factors, and levels for categorical factors.




# Creating a full factorial (cont'd)

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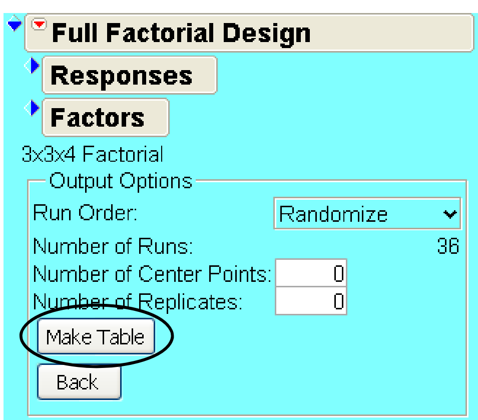
*DOE → Full Factorial Design*

2. Add extra "center" points, request one or more replicates, and/or pre-sort the matrix if desired. Use the *Back* button to modify Step 1.




| 5/0 Cols 36/0 | Pattern | Intro APR | Intro Time | Gift | % Yes |
|---|---|---|---|---|---|
| 1 | 324 | 5 | 6 | Espresso | • |
| 2 | 114 | 0 | 3 | Espresso | • |
| 3 | 212 | 2.5 | 3 | Toaster | • |
| 4 | 132 | 0 | 9 | Toaster | • |
| 5 | 211 | 2.5 | 3 | None | • |
| 6 | 314 | 5 | 3 | Espresso | • |
| 7 | 223 | 2.5 | 6 | Microwave | • |
| 8 | 134 | 0 | 9 | Espresso | • |
| 9 | 111 | 0 | 3 | None | • |
| 10 | 131 | 0 | 9 | None | • |
| 11 | 112 | 0 | 3 | Toaster | • |
| 12 | 323 | 5 | 6 | Microwave | • |
| 13 | 334 | 5 | 9 | Espresso | • |
| 14 | 313 | 5 | 3 | Microwave | • |
| 15 | 224 | 2.5 | 6 | Espresso | • |
| 16 | 222 | 2.5 | 6 | Toaster | • |
| 17 | 232 | 2.5 | 9 | Toaster | • |
| 18 | 332 | 5 | 9 | Toaster | • |
| 19 | 122 | 0 | 6 | Toaster | • |
| 20 | 322 | 5 | 6 | Toaster | • |
| 21 | 213 | 2.5 | 3 | Microwave | • |
| 22 | 234 | 2.5 | 9 | Espresso | • |
| 23 | 123 | 0 | 6 | Microwave | • |
| 24 | 331 | 5 | 9 | None | • |
| 25 | 233 | 2.5 | 9 | Microwave | • |
| 26 | 333 | 5 | 9 | Microwave | • |
| 27 | 312 | 5 | 3 | Toaster | • |
| 28 | 311 | 5 | 3 | None | • |
| 29 | 121 | 0 | 6 | None | • |
| 30 | 321 | 5 | 6 | None | • |
| 31 | 214 | 2.5 | 3 | Espresso | • |
| 32 | 231 | 2.5 | 9 | None | • |
| 33 | 133 | 0 | 9 | Microwave | • |
| 34 | 113 | 0 | 3 | Microwave | • |
| 35 | 124 | 0 | 6 | Espresso | • |
| 36 | 221 | 2.5 | 6 | None | • |

- Each "center" point = one additional row in the matrix
- Each "replicate" = one additional set of 36 rows

## Simulating response data



3. Create two new columns as shown here.

4. Define *Sent* with a formula consisting of the constant value 1000.

5. The *Returned* column is where we would enter the number of offers accepted and returned for each of the 36 offer types.

6. For this exercise, define *Returned* with the formula defined on the next slide.

7/0 Cols
36/0

| | Pattern | Intro APR | Intro Time | Gift | % Yes | Sent | Returned |
|---|---|---|---|---|---|---|---|
| 1 | 324 | 5 | 6 | Espresso | • | 1000 | • |
| 2 | 114 | 0 | 3 | Espresso | • | 1000 | • |
| 3 | 212 | 2.5 | 3 | Toaster | • | 1000 | • |
| 4 | 132 | 0 | 9 | Toaster | • | 1000 | • |
| 5 | 211 | 2.5 | 3 | None | • | 1000 | • |
| 6 | 314 | 5 | 3 | Espresso | • | 1000 | • |
| 7 | 223 | 2.5 | 6 | Microwave | • | 1000 | • |
| 8 | 134 | 0 | 9 | Espresso | • | 1000 | • |
| 9 | 111 | 0 | 3 | None | • | 1000 | • |
| 10 | 131 | 0 | 9 | None | • | 1000 | • |
| 11 | 112 | 0 | 3 | Toaster | • | 1000 | • |
| 12 | 323 | 5 | 6 | Microwave | • | 1000 | • |
| 13 | 334 | 5 | 9 | Espresso | • | 1000 | • |
| 14 | 313 | 5 | 3 | Microwave | • | 1000 | • |
| 15 | 224 | 2.5 | 6 | Espresso | • | 1000 | • |
| 16 | 222 | 2.5 | 6 | Toaster | • | 1000 | • |
| 17 | 232 | 2.5 | 9 | Toaster | • | 1000 | • |
| 18 | 332 | 5 | 9 | Toaster | • | 1000 | • |
| 19 | 122 | 0 | 6 | Toaster | • | 1000 | • |
| 20 | 322 | 5 | 6 | Toaster | • | 1000 | • |
| 21 | 213 | 2.5 | 3 | Microwave | • | 1000 | • |
| 22 | 234 | 2.5 | 9 | Espresso | • | 1000 | • |
| 23 | 123 | 0 | 6 | Microwave | • | 1000 | • |
| 24 | 331 | 5 | 9 | None | • | 1000 | • |
| 25 | 233 | 2.5 | 9 | Microwave | • | 1000 | • |
| 26 | 333 | 5 | 9 | Microwave | • | 1000 | • |
| 27 | 312 | 5 | 3 | Toaster | • | 1000 | • |
| 28 | 311 | 5 | 3 | None | • | 1000 | • |
| 29 | 121 | 0 | 6 | None | • | 1000 | • |
| 30 | 321 | 5 | 6 | None | • | 1000 | • |
| 31 | 214 | 2.5 | 3 | Espresso | • | 1000 | • |
| 32 | 231 | 2.5 | 9 | None | • | 1000 | • |
| 33 | 133 | 0 | 9 | Microwave | • | 1000 | • |
| 34 | 113 | 0 | 3 | Microwave | • | 1000 | • |
| 35 | 124 | 0 | 6 | Espresso | • | 1000 | • |
| 36 | 221 | 2.5 | 6 | None | • | 1000 | • |

## Simulating response data (cont'd)



Functions (grouped)
↓
Random
↓
Random Integer[n1]
↓
Random Integer[50]

7. Define *% Yes* with the formula

$$\left[\frac{\text{Returned}}{\text{Sent}}\right] * 100$$

8. Run the *Model* script provided in the left panel.

3x3x4 Factorial
Design 3x3x4 Factoria
Model

Columns (6/0)
Intro APR
Intro Time
Gift
% Yes
Sent
Returned

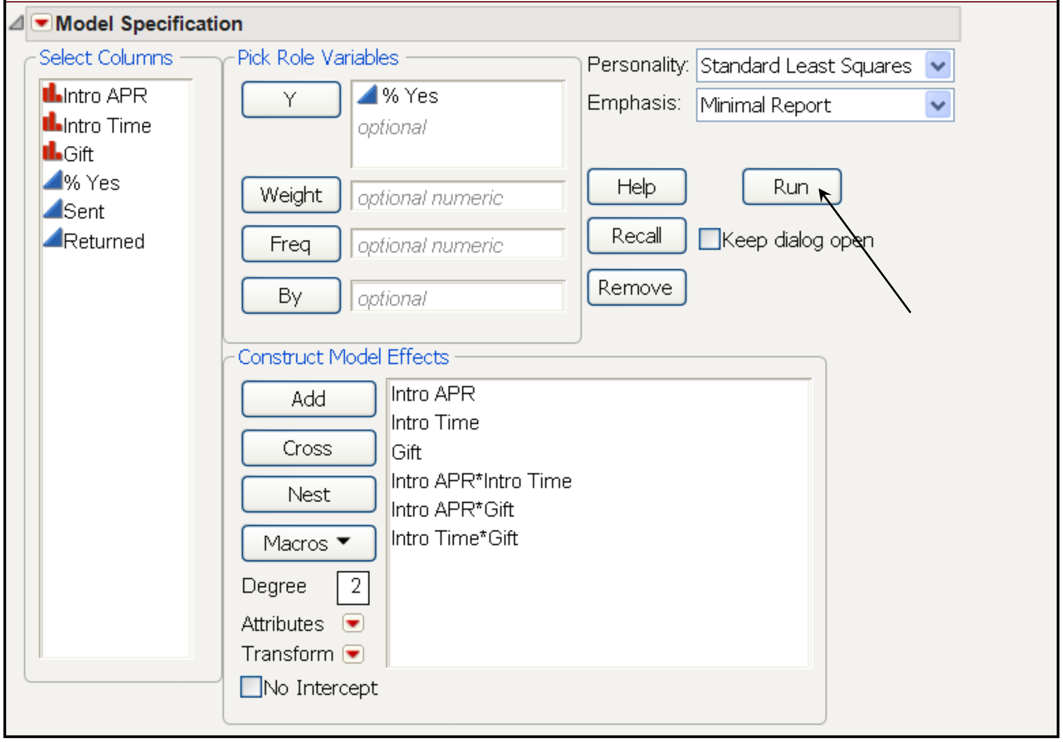
Rows

| | |
|---|---|
| All rows | 36 |
| Selected | 0 |
| Excluded | 0 |
| Hidden | 0 |
| Labelled | 0 |

| | Intro APR | Intro Time | Gift | % Yes | Sent | Returned |
|---|---|---|---|---|---|---|
| 1 | 0 | 3 | Microwave | 3.9 | 1000 | 39 |
| 2 | 0 | 9 | None | 3.7 | 1000 | 37 |
| 3 | 0 | 9 | Toaster | 4.9 | 1000 | 49 |
| 4 | 0 | 3 | Espresso | 0.1 | 1000 | 1 |
| 5 | 5 | 9 | Microwave | 4.4 | 1000 | 44 |
| 6 | 0 | 6 | Microwave | 2.6 | 1000 | 26 |
| 7 | 2.5 | 9 | None | 0.6 | 1000 | 6 |
| 8 | 5 | 6 | Espresso | 4.2 | 1000 | 42 |
| 9 | 2.5 | 9 | Microwave | 1.6 | 1000 | 16 |
| 10 | 0 | 3 | None | 0.9 | 1000 | 9 |
| 11 | 0 | 6 | Espresso | 3 | 1000 | 30 |
| 12 | 2.5 | 6 | Microwave | 1.4 | 1000 | 14 |
| 13 | 5 | 6 | Toaster | 3.8 | 1000 | 38 |
| 14 | 0 | 9 | Espresso | 2.4 | 1000 | 24 |
| 15 | 0 | 9 | Microwave | 0.3 | 1000 | 3 |
| 16 | 2.5 | 6 | None | 4.1 | 1000 | 41 |
| 17 | 2.5 | 6 | Toaster | 2.5 | 1000 | 25 |
| 18 | 5 | 3 | Espresso | 1.1 | 1000 | 11 |
| 19 | 0 | 3 | Toaster | 3.1 | 1000 | 31 |
| 20 | 5 | 3 | Microwave | 4.6 | 1000 | 46 |
| 21 | 5 | 3 | None | 3.5 | 1000 | 35 |
| 22 | 5 | 3 | Toaster | 4.8 | 1000 | 48 |
| 23 | 2.5 | 3 | Toaster | 0.6 | 1000 | 6 |
| 24 | 2.5 | 3 | Microwave | 3.6 | 1000 | 36 |
| 25 | 0 | 6 | Toaster | 3.5 | 1000 | 35 |
| 26 | 2.5 | 9 | Toaster | 0.4 | 1000 | 4 |
| 27 | 2.5 | 6 | Espresso | 1.5 | 1000 | 15 |
| 28 | 5 | 9 | Toaster | 1.6 | 1000 | 16 |
| 29 | 2.5 | 9 | Espresso | 0.1 | 1000 | 1 |
| 30 | 5 | 9 | Espresso | 3.8 | 1000 | 38 |

# Analyzing the data

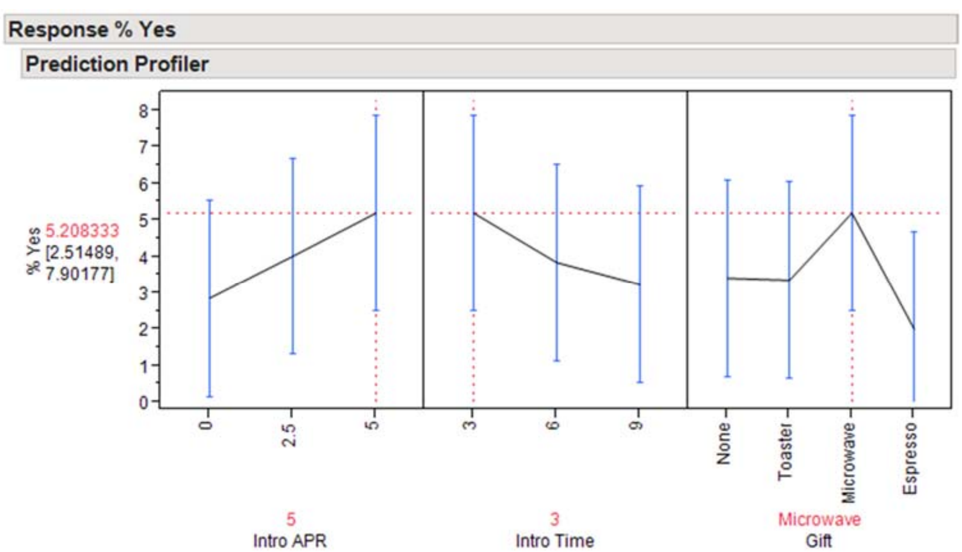





# Maximizing % Yes



- Point and click to find the combination with the highest % Yes
- Your profiler won't look like this one
- Your best combination may not make sense



# 18 Statistical Assumptions

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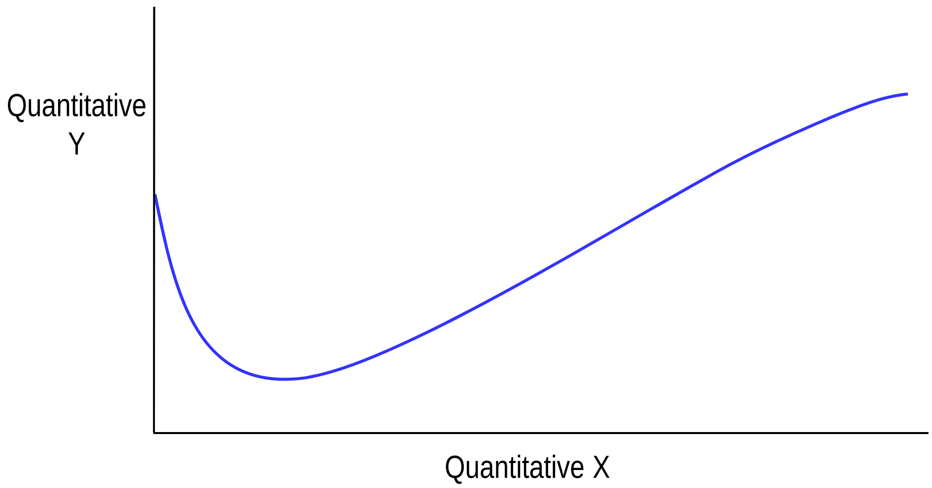
Average Y as a function of X has no jumps or corners
(assumption of *smoothness*)




Notes

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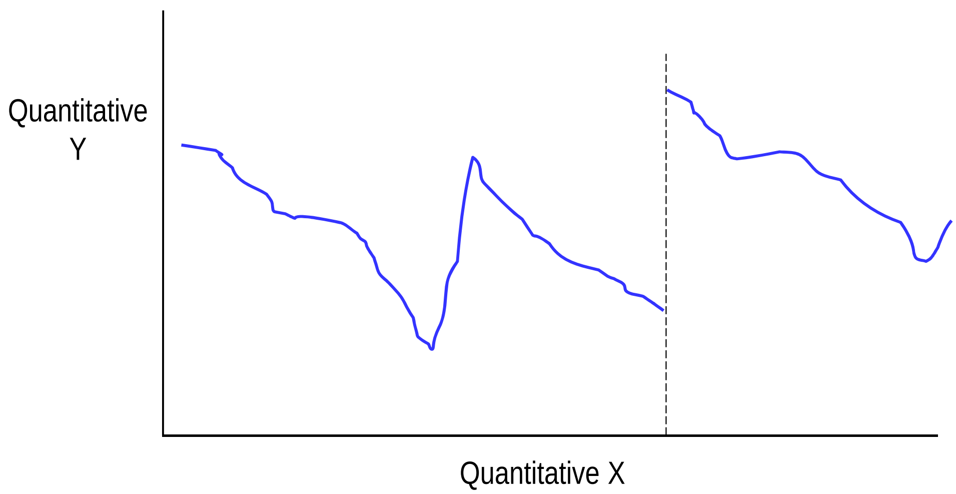
A hypothetical smooth response function.

We never know the true response function, but often we have information about its general properties. For quantitative X and Y, *smoothness* of the $Y = f(X)$ relationship is one such property. It means the function can be well approximated over sufficiently short intervals by a polynomial, usually linear or quadratic. This is necessary in optimization experiments where we want to *interpolate* between the experimental design points.

## Non-smooth response function



*Average Y as a function of X has jumps and/or corners*




## Notes



A hypothetical non-smooth response function.

A function with jumps or sharp corners will not be well approximated by low-order polynomials in neighborhoods of the associated X values. This is a problem in optimization experiments because we want to interpolate.

It is not a problem in screening experiments, because there we are merely trying to identify factors with large first-order effects. Accurate approximation throughout the X range is not required.

Jumps and sharp corners often occur outside the feasible operating range of the process. In fact, such discontinuities often *define* the feasible operating range. A smooth response function is usually a safe assumption as long as we are not operating too close to a "cliff."

## Occam’s razor

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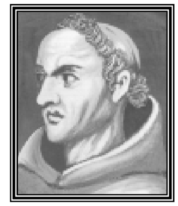
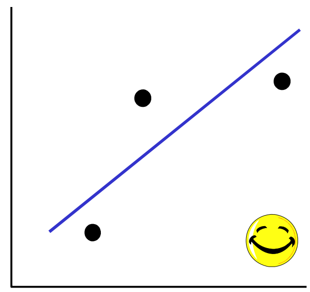
*“One should not increase, beyond what is necessary, the number of entities required to explain something.”*

—William of Occam, medieval philosopher



Exact “French curve”



Linear plus noise

## Notes

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Occam’s razor represents a preference for simple explanations over complex ones. This reflects a belief that simple hypotheses are more likely to be true than complex ones. This belief is not always justified, but it is efficient in that it leads to models with just enough complexity to explain a given set of observations.

We can always find a sufficiently complex curve passing exactly through any given set of data points. The predictive ability of this “over-fitting” method is notoriously poor. The more successful “Occam” strategy is illustrated by random variation superimposed on a simple linear model.

## Standard assumptions on the response function



- ✓ $Y = f(X_1, X_2, X_3, \ldots) + noise$
- ✓ Can't assume $f$(X) explains everything (hence noise term)
- ✓ Can't assume $f$(X) is linear, but quadratic is sufficient
- ✓ Don't need cubic or higher order models
- ✓ $f$(X) must include all second order interactive effects
- ✓ Don't need higher order interactive effects

Notes

## Standard assumption on the noise

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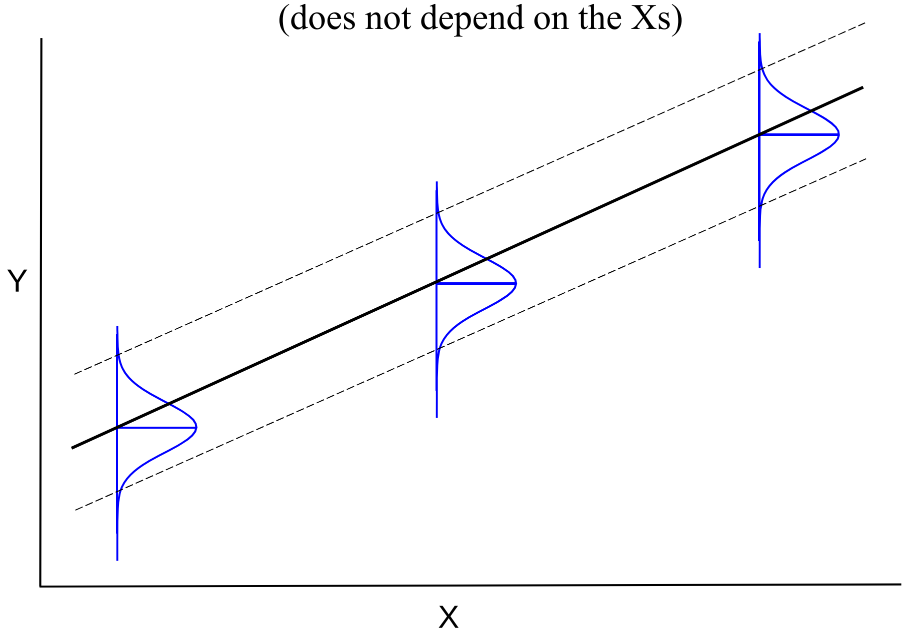
Normal distribution with $\sigma_Y$ constant
(does not depend on the Xs)




## Most common violation of the noise assumption

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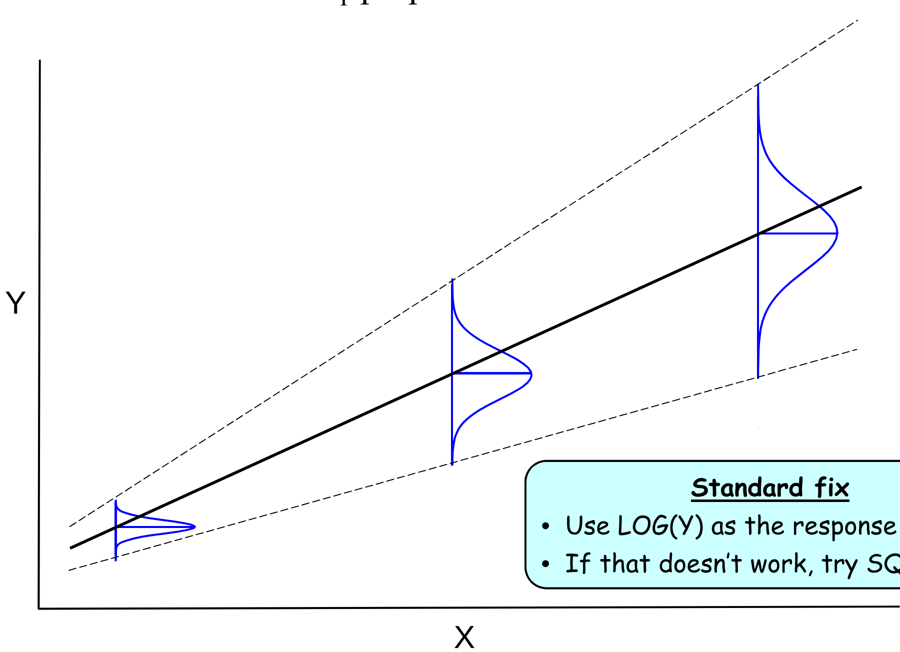
$\sigma_Y$ proportional to mean Y




**Standard fix**

- Use LOG(Y) as the response variable
- If that doesn't work, try SQRT(Y)

## Least squares model fitting



For each of 18 potato chip bags, we have data on

T = bonding temperature
D = bonding time (duration)
Y = bond strength

The best fitting *response surface model* (RSM) is the one whose parameters

$$b_0, b_1, b_2, b_3, b_4, b_5$$

minimize the sum of squared residuals:

$$\sum_{\{18\text{ bags}\}} \left[Y - \left(b_0 + b_1 T + b_2 D + b_3 TD + b_4 T^2 + b_5 D^2\right)\right]^2$$

## Least squares RSM model



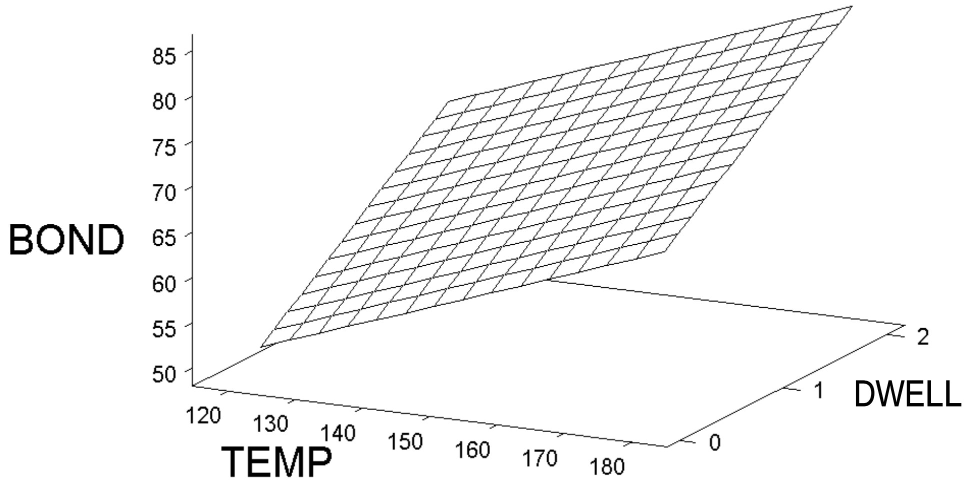
$$\text{Avg. } Y = 87.2 + 8.3(T) + 7.7(D) - 31.8(TD) - 16.1\left(T^2\right) - 13.2\left(D^2\right)$$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **TEMP** | **DWELL** | **BOND** | | **Prediction** | | **Noise** |
| 2 | -1 | -1 | 11.0 | | 10.08 | | 0.92 |
| 3 | -1 | -1 | 8.9 | | 10.08 | | -1.18 |
| 4 | -1 | 0 | 63.9 | | 62.80 | | 1.10 |
| 5 | -1 | 0 | 60.4 | | 62.80 | | -2.40 |
| 6 | -1 | 1 | 93.2 | | 89.07 | | 4.13 |
| 7 | -1 | 1 | 86.5 | | 89.07 | | -2.57 |
| 8 | 0 | -1 | 65.7 | | 66.30 | | -0.60 |
| 9 | 0 | -1 | 67.7 | | 66.30 | | 1.40 |
| 10 | 0 | 0 | 88.4 | | 87.20 | | 1.20 |
| 11 | 0 | 0 | 88.0 | | 87.20 | | 0.80 |
| 12 | 0 | 1 | 82.0 | | 81.65 | | 0.35 |
| 13 | 0 | 1 | 78.5 | | 81.65 | | -3.15 |
| 14 | 1 | -1 | 88.1 | | 90.37 | | -2.27 |
| 15 | 1 | -1 | 92.1 | | 90.37 | | 1.73 |
| 16 | 1 | 0 | 77.2 | | 79.45 | | -2.25 |
| 17 | 1 | 0 | 81.0 | | 79.45 | | 1.55 |
| 18 | 1 | 1 | 39.5 | | 42.08 | | -2.58 |
| 19 | 1 | 1 | 45.9 | | 42.08 | | 3.82 |
| 20 | Sum of squares (SS) | | 93876.58 | = | 93792.35 | + | 84.18 |
| 21 | Degrees of freedom (DF) | | 18 | = | 6 | + | 12 |
| 22 | | RMSE | *Square root of noise (SS/DF)* | | | | 2.65 |
| 23 | | | | | | | |

*least squares modeling.xls*

*Linear in the Xs*

Average Bond = 67.2 + 8.3(TEMP) + 8.3(DWELL)




Notes



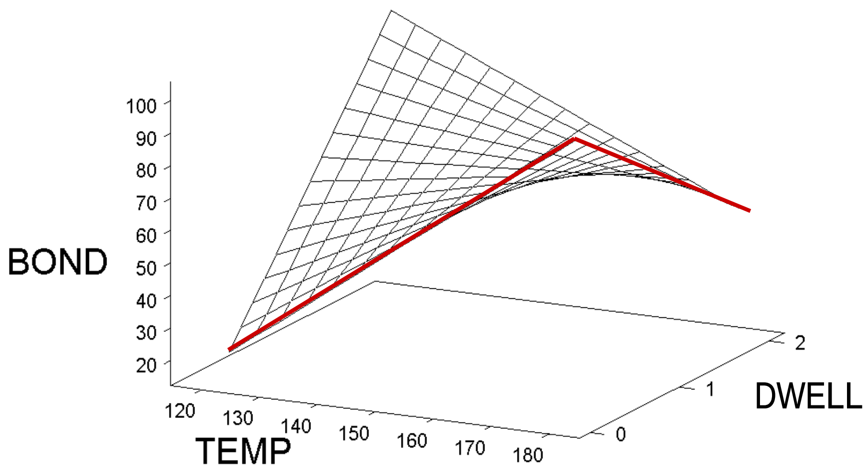
Response surface: tilted plane.

Simple linear models like the one shown above are used in screening designs. In many cases, simple linear models fit the data poorly, and do not give accurate predictions. They should not be used for optimization experiments.

# Linear interaction model



Avg. BOND = 67.2 + 8.3(TEMP) + 8.3(DWELL) - 31.5(TEMP$\times$DWELL)




## Notes



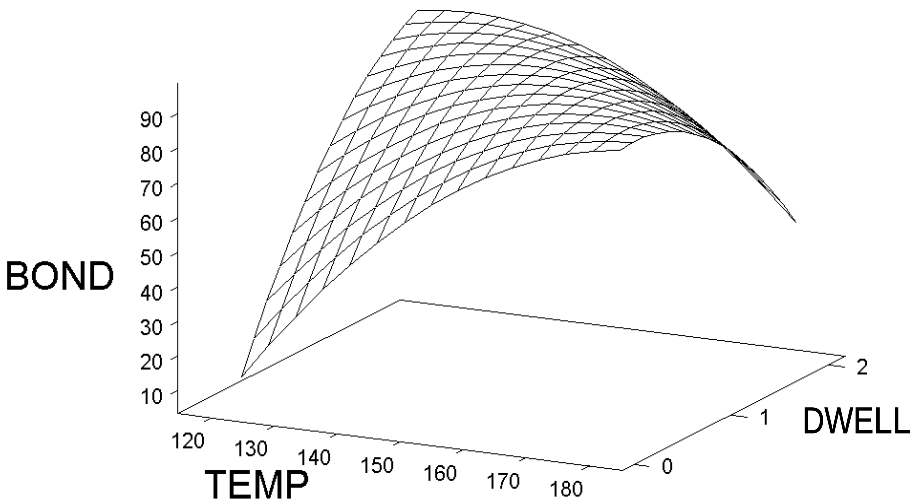
Response surface: saddle.

Linear interaction models like the one shown above usually fit the data much better than simple linear models. They are good for optimization experiments where all factors are categorical, but they should not be used for optimization experiments involving quantitative factors.

## Response surface model (RSM)



$$\text{Avg. BOND} = 86.8 + 8.3(\text{TEMP}) + 8.1(\text{DWELL}) - 32.4(\text{TEMP}\times\text{DWELL}) - 15.5(\text{TEMP}\times\text{TEMP}) - 12.9(\text{DWELL}\times\text{DWELL})$$




## Notes



Response surface: ridge.

The response surface model (RSM) shown above is the standard model for optimization experiments. It differs from the linear interaction model in that it includes quadratic (squared) terms for all quantitative factors. In most experiments involving quantitative factors, the RSM fits the data much better than the linear interaction model.

## RSM for a different data set

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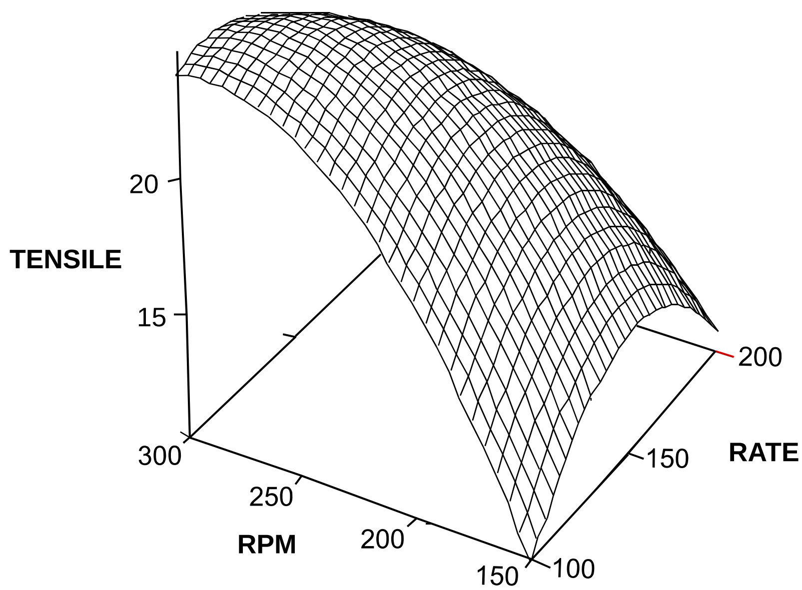
$$\text{Avg. TENSILE} = 22.5 - 3.3(\text{RATE}) + 3.4(\text{RPM}) - 3.6(\text{RATE}\times\text{RPM}) - 4.8(\text{RATE}\times\text{RATE}) - 5.6(\text{RPM}\times\text{RPM})$$




## Notes

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Response surface: hilltop.

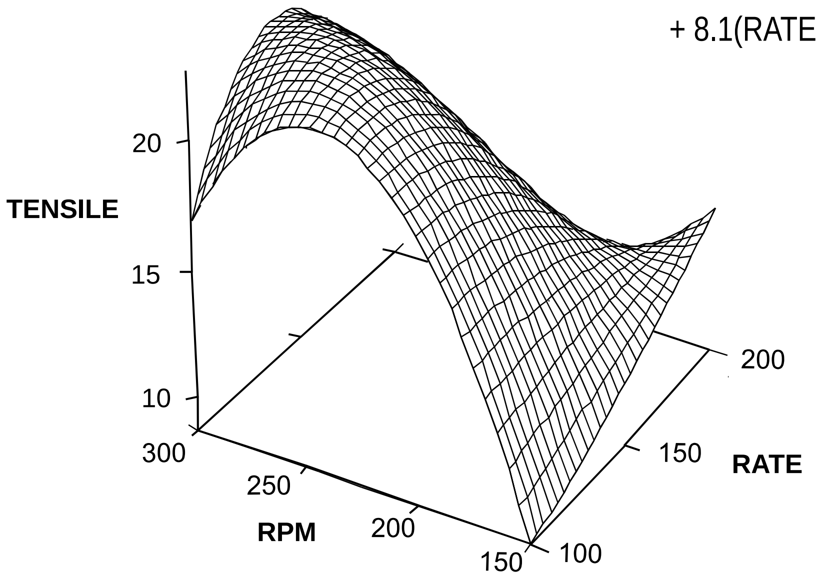
Other RSM shapes include inverted saddles, inverted ridges, and bowls.

You can't tell from the plot, but in this example the RSM model does not fit the data very well.

## RSM plus quadratic interactions



$$\text{Avg. TENSILE} = 22.4 - 8.5(\text{RATE}) + 8.6(\text{RPM}) - 3.2(\text{RATE} \times \text{RPM}) - 6.1(\text{RATE}^2) - 4.8(\text{RPM}^2) - 7.0(\text{RATE}^2 \times \text{RPM}) + 8.1(\text{RATE} \times \text{RPM}^2)$$




## Notes



The shows a more complicated quadratic model fit to the same data as on the previous page. This model turns out to fit the data well.

Model terms like

$$\text{RATE} \times \text{RATE} \times \text{RPM}$$

$$\text{RATE} \times \text{RPM} \times \text{RPM}$$

$$\text{RATE} \times \text{RATE} \times \text{RPM} \times \text{RPM}$$

are called *quadratic interactions*. Adding one or more quadratic interactions is a good thing to try when an RSM model does not fit.

## Higher-order polynomial models?

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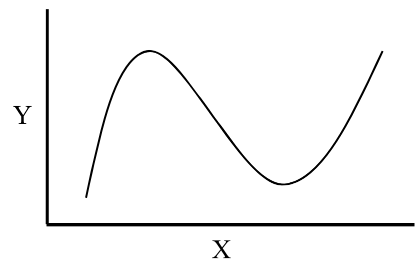
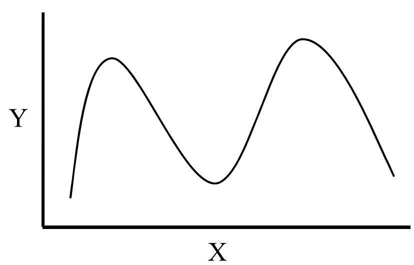
<u>3rd order polynomial (cubic)</u>

$$\text{Avg. } Y = b_0 + b_1X + b_2X^2 + b_3X^3$$




<u>4th order polynomial (quartic)</u>

$$\text{Avg. } Y = b_0 + b_1X + b_2X^2 + b_3X^3 + b_4X^4$$




## Notes

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Even though third- or higher-order models may fit the data better than quadratic (second-order) models, they are rarely used in DOE. Why? They require much larger samples sizes for any given set of factors.

It is much more common to use quadratic models in an iterative fashion. A quadratic model may not fit the data well over a large initial factor space, but it almost always tells us which subset of the initial factor space is most likely to give the results we are looking for. The next step is to run another quadratic experiment in the smaller region. The smaller the factor space, the better the quadratic model will fit the data.

This concept is illustrated on the next page.

## Iterated quadratic experiments

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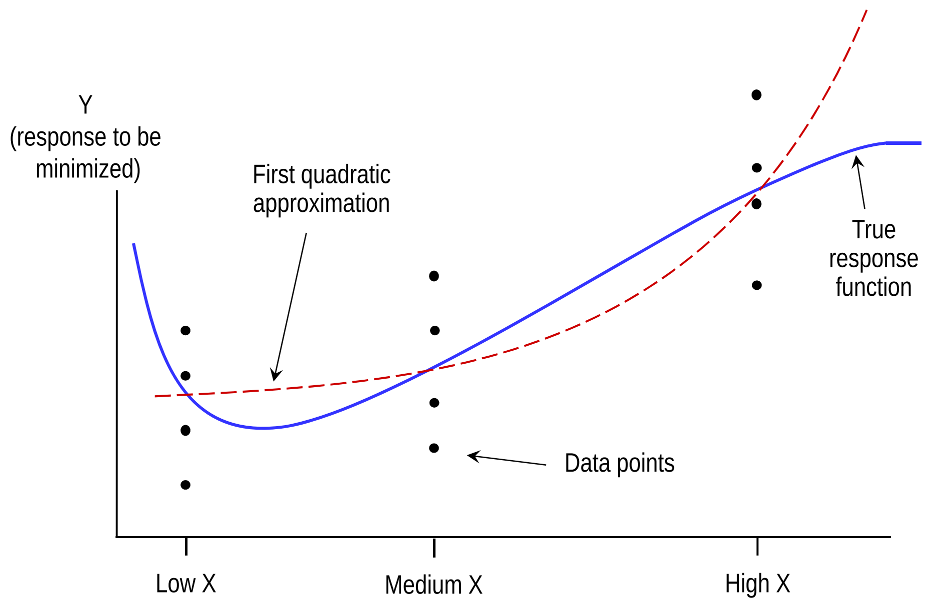
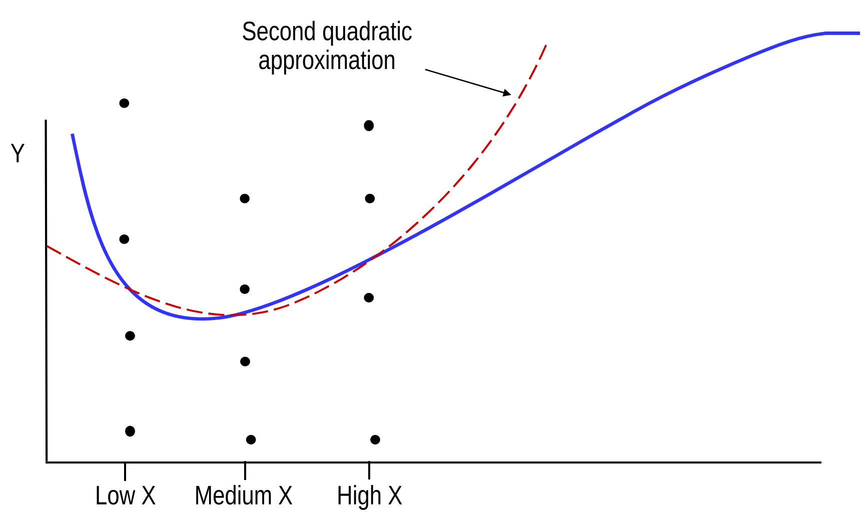
*First experiment, wide ranges → "big picture"*




## Iterated quadratic experiments (cont'd)

340

*Second experiment, narrow ranges → accurate modeling*



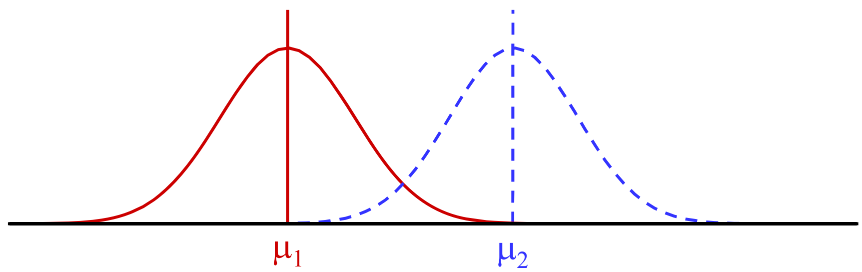
## Models for categorical factors



*Two-level categorical factor*

MATL = Steel or Rubber

$$\text{Average COST} = \begin{cases} \mu_1 & \text{if MATL} = \text{Steel} \\ \mu_2 & \text{if MATL} = \text{Rubber} \end{cases}$$




## Equation form of model



Categorical factors are represented by *indicator* variables
(also known as *dummy* variables)

$$\text{Average COST} = b_0 + b_1 \,\text{MATL[Steel]}$$

$$\text{MATL[Steel]} = \begin{cases} 1 & \text{if MATL} = \text{Steel} \\ -1 & \text{if MATL} = \text{Rubber} \end{cases}$$

## Simple linear model with all factors categorical

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$$\text{Avg. COST} = b_0 + b_1\,\text{LGR[Low]} + b_2\,\text{MATL[Steel]} + b_3\,\text{USAGE[50\%]} + b_4\,\text{GRIT[30]}$$

4.868125
+ Match(LGR): "High" ⇒ -0.616875; "Low" ⇒ 0.616875; else ⇒ .
+ Match(MATL): "Rubber" ⇒ 1.145625; "Steel" ⇒ -1.145625; else ⇒ .
+ Match(USAGE): "50%" ⇒ 1.054375; "75%" ⇒ -1.054375; else ⇒ .
+ Match(GRIT): "30" ⇒ -0.048125; "50" ⇒ 0.048125; else ⇒ .

- Analogy: blue book pricing of used cars
- Base price + extra for power windows
  + extra for air conditioning
  + extra for cruise control
  etc.

## Categorical interaction model

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$$\begin{aligned}\text{Avg. COST} = {} & b_0 \\ & + b_1\,\text{LGR[Low]} \\ & + b_2\,\text{MATL[Steel]} \\ & + b_3\,\text{USAGE[50\%]} \\ & + b_4\,\text{GRIT[30]} \\ & + b_5\,\text{LGR[Low]} \times \text{MATL[Steel]} \\ & + b_6\,\text{LGR[Low]} \times \text{USAGE[50\%]} \\ & + b_7\,\text{LGR[Low]} \times \text{GRIT[30]} \\ & + b_8\,\text{MATL[Steel]} \times \text{USAGE[50\%]} \\ & + b_9\,\text{MATL[Steel]} \times \text{GRIT[30]} \\ & + b_{10}\,\text{USAGE[50\%]} \times \text{GRIT[30]}\end{aligned}$$

| # Factors | 4 | 5 | 6 |
|---|---|---|---|
| Full factorial (FF) | 16 | 32 | 64 |
| Min. sample size | 11 | 16 | 22 |
| % of FF | 69 | 50 | 34 |

# 20 Design Principles



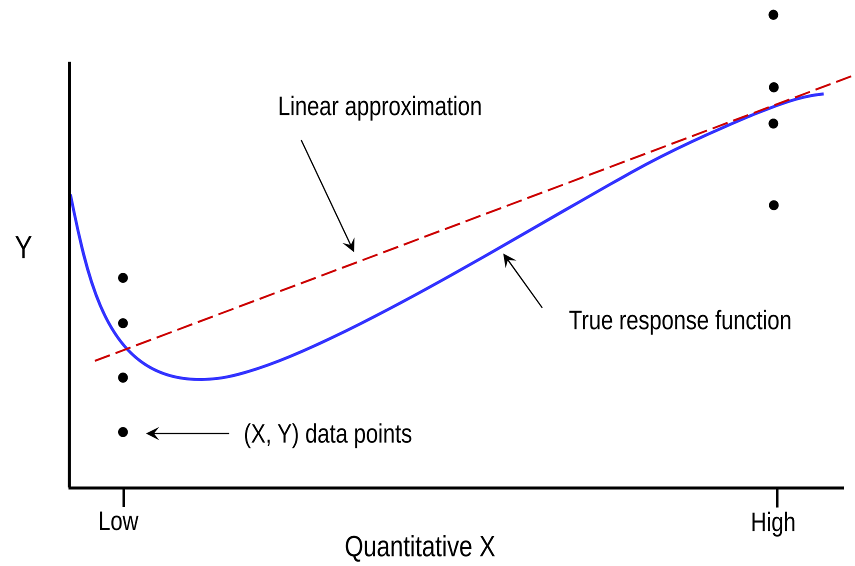
- Bold strategy
- Control group
- Replication
- Randomization
- "Blocking"

Notes

## Bold strategy



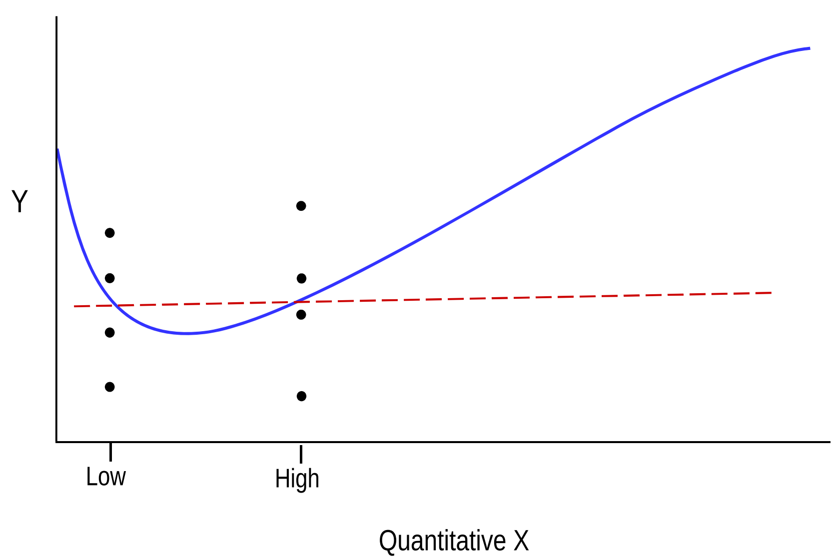
*Use the entire feasible operating range in a first experiment*




## Not bold enough



- Low and high levels of X are too close together
- We mistakenly conclude that X has no effect on Y



# Control group



*For each factor, one of the levels should match the current value*

| Temp | Press | Dwell | Mat'l |
|---|---|---|---|
| 120 | 50 | 0.2 | A |
| 120 | 100 | 1.1 | B |
| 120 | 150 | 2.0 | C |
| 150 | 50 | 1.1 | C |
| 150 | 100 | 2.0 | A |
| 150 | 150 | 0.2 | B |
| 180 | 50 | 2.0 | B |
| 180 | 100 | 0.2 | C |
| 180 | 150 | 1.1 | A |

## Notes



The units involved in a DOE may turn out to be uniformly different from those in current production — either better or worse. This can be due to the effects of noise variables on production units, or to special circumstances surrounding the creation and handling of experimental units.

For each factor, one of the DOE levels should match the current state value of that factor. This allows valid comparisons between current state and experimental process settings. This is especially important when non-routine measurements, tests or inspections are applied to experimental units.

# Replication



*Use repeat runs to quantify the noise in the experiment and increase the signal-to-noise ratios*

| Temp | Press | Experimental units |
|---|---|---|
| 120 | 50 | 1 |
| 120 | 50 | 2 |
| 120 | 150 | 3 |
| 120 | 150 | 4 |
| 180 | 50 | 5 |
| 180 | 50 | 6 |
| 180 | 150 | 7 |
| 180 | 150 | 8 |

## Notes



Replication forces redundancy into the experiment. This is necessary for two reasons:

- To quantify the magnitude of noise in the experimental data — differences between units at the same design point are, by definition, due to noise variables.
- To reduce the influence of noise variables on the experimental results by averaging multiple units at the same design point. In other words, to increase the signal-to-noise ratios.

Assume that you are the person responsible for running the experiment and for the validity of the results. Is there anything about the run order shown above that makes you nervous? Please explain.

## Randomization

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*Use a random number generator determine the sequence in which experimental units are created and/or evaluated*

| Temp | Press | Experimental units |
|---|---|---|
| 120 | 150 | 1 |
| 180 | 50 | 2 |
| 180 | 50 | 3 |
| 120 | 150 | 4 |
| 180 | 50 | 5 |
| 120 | 150 | 6 |
| 180 | 150 | 7 |
| 120 | 50 | 8 |

## Randomization

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*Benefits*

- Reduces the chance of biased results due to noise variables
- Results are more convincing to skeptics
- Doesn't require control of noise variables

*Drawbacks*

- Impractical when some of the factors are hard to change
- Does not quantify the effects of noise variables

*What happens if you don't randomize?*

- Same thing as driving without car insurance
- Nothing happens, unless you have bad luck
- Warning:  strong temptation to do false repeats

## "Blocking"

*Include noise variables as additional factors in the experiment*

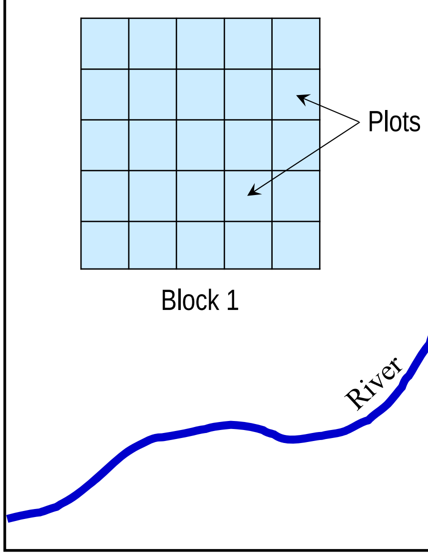
- Noise variables are used to divide the experiment into homogeneous "blocks"
- Effects of control factors are determined within each block

| Temp | Press | Run | Block |
|---|---|---|---|
| 120 | 50 | 1 | Block 1<br>Operator: Bob<br>Raw material: Lot 1 |
| 120 | 150 | 2 | |
| 180 | 150 | 3 | |
| 180 | 50 | 4 | |
| 180 | 150 | 5 | Block 2<br>Operator: Carol<br>Raw material: Lot 2 |
| 180 | 50 | 6 | |
| 120 | 50 | 7 | |
| 120 | 150 | 8 | |

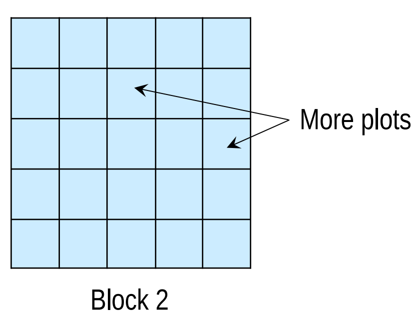


## Agricultural origin of "blocking"

- Want to increase crop yields
- Experimental units are plots of land in a field
- Compare varieties, fertilizers, etc.




- Need 50 plots, not 25
- Have to use a second field
- Differences in the soil will cause differences in yields



## Benefits of blocking



- Makes the experiment more representative of the real process
- Makes predictions more reliable
- Temporarily converts noise variables into signal variables
- Quantifies the effects of noise variables
- Increases signal to noise ratios without increasing sample size
- Usually protects against general time trends

## Notes

# 21 The Custom Design Process

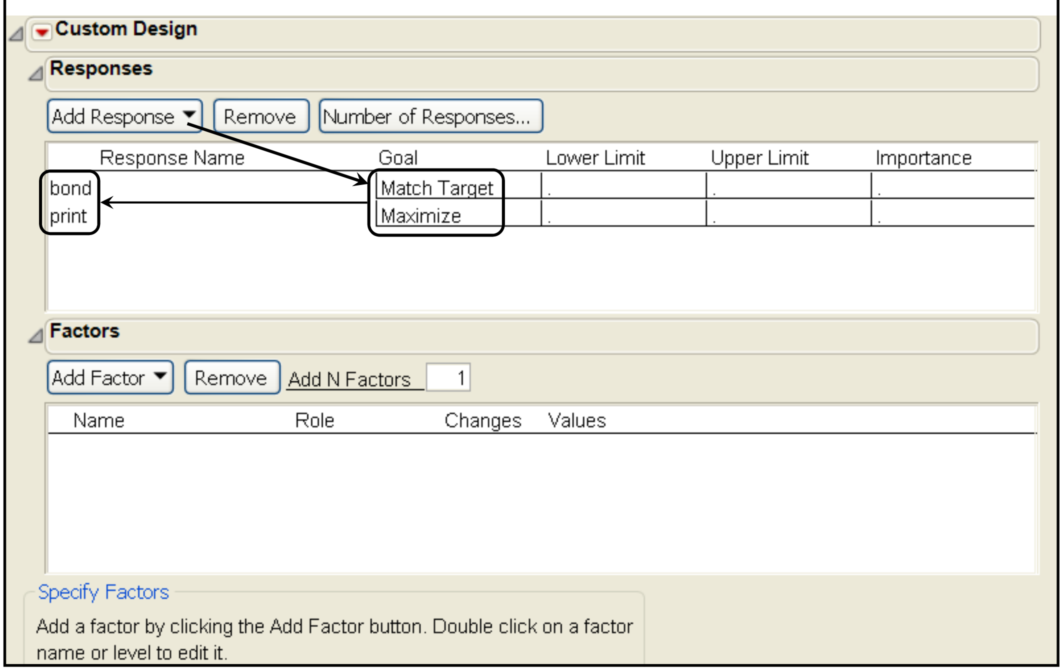


1. Specify the response variables and general goals (maximize, minimize, or match target).
2. Specify the factors. For continuous factors, give the desired numerical ranges (high and low). For categorical factors, give the levels.
3. Calculate and enter the required sample size (see section 22).
4. Specify the blocking variable(s).
5. Specify the statistical model (usually RSM).
6. Create the design matrix.
7. Back up to make changes, or create the data table.
8. Sort the data table if necessary, save for later.

# 1. Responses and general goals



*DOE → Custom Design*



## 2. Factors, ranges for continuous, levels for categorical

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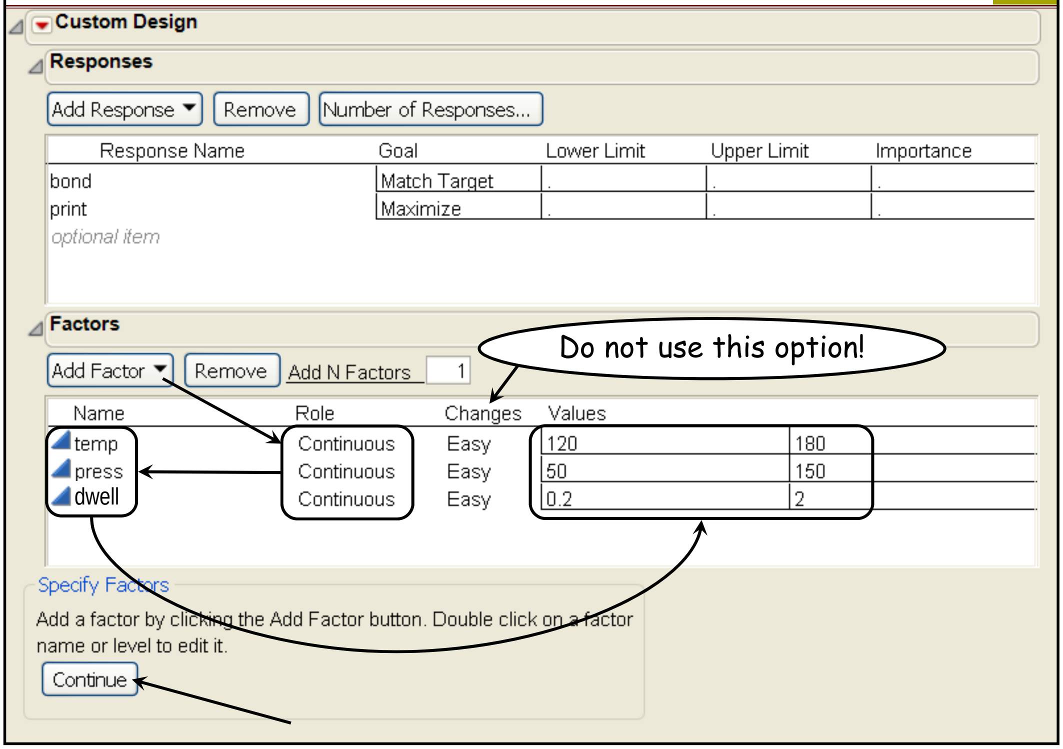


## 3. Calculate and enter the sample size

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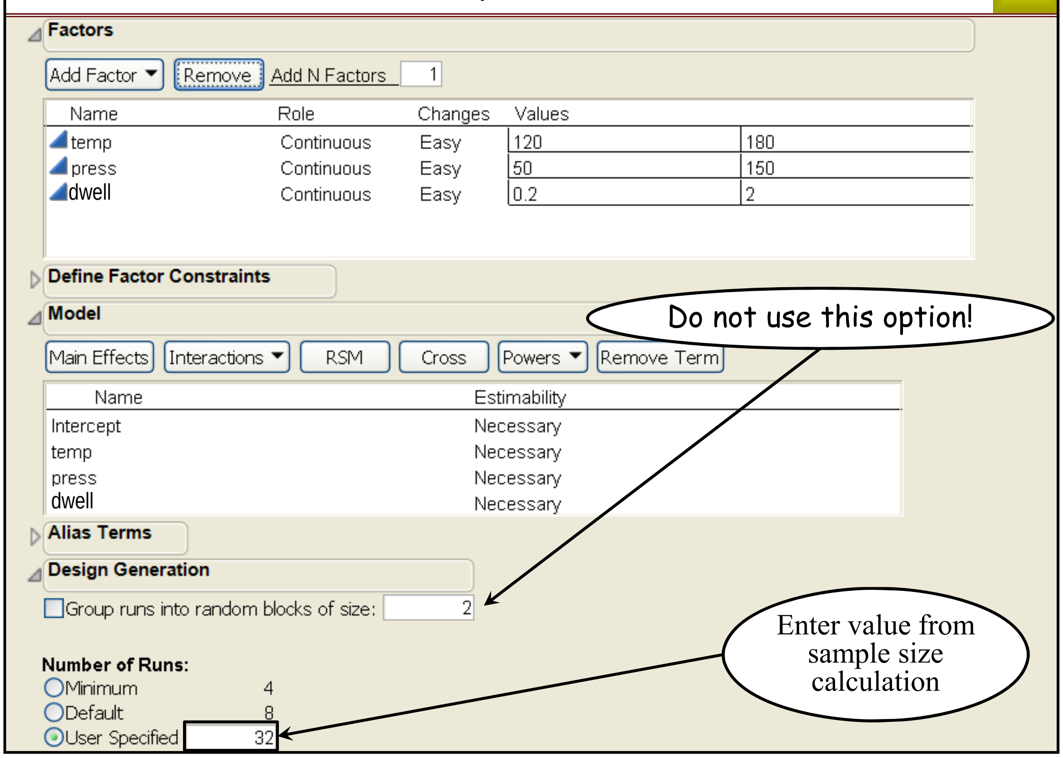
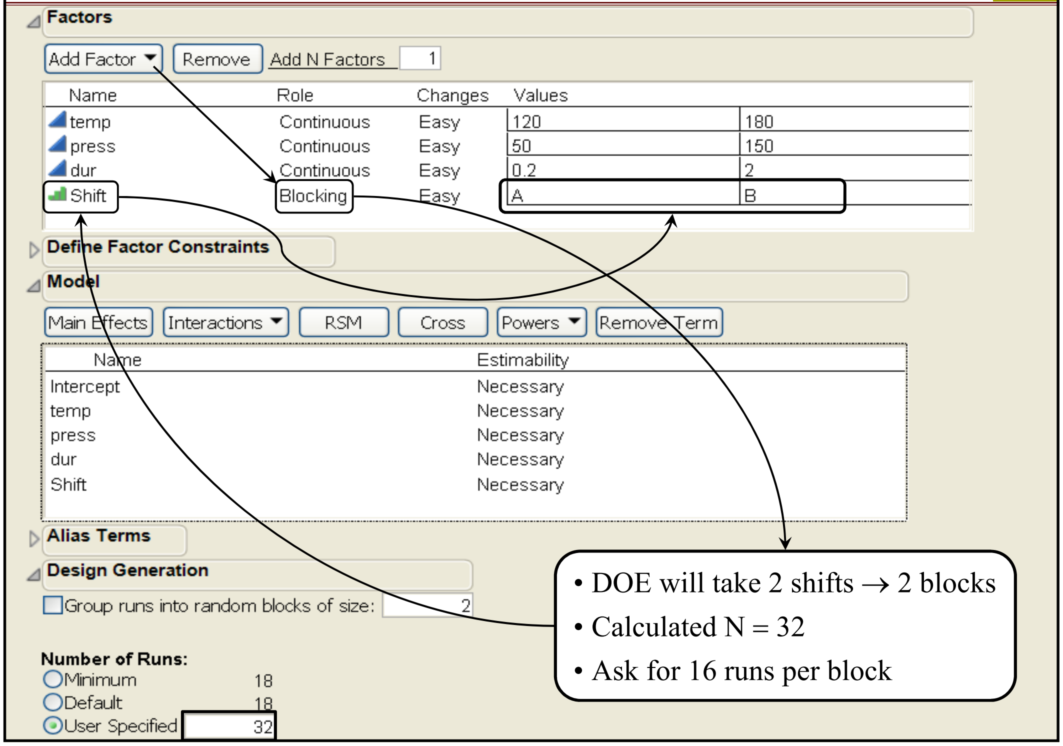


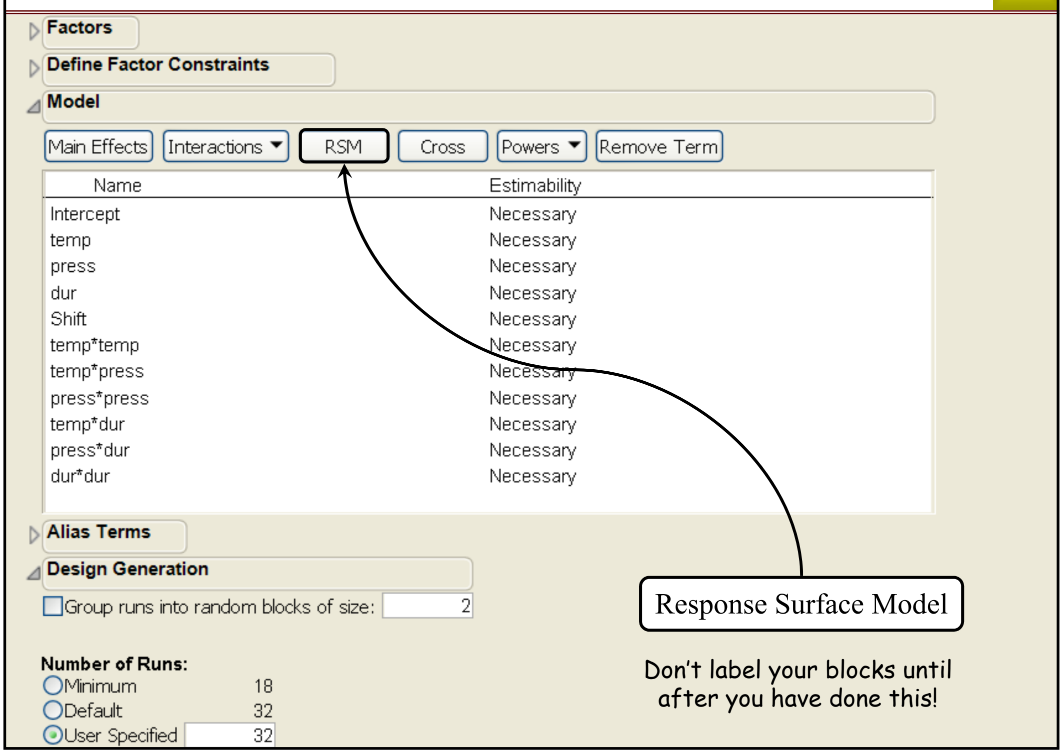
# 4. Specify the blocking variable(s)






# 5. Specify the statistical model





# 6. Create the design matrix



**Model**

Main Effects | Interactions | RSM | Cross

| Name | Estimability |
|---|---|
| Intercept | Necessary |
| temp | Necessary |
| press | Necessary |
| dur | Necessary |
| Shift | Necessary |
| temp*temp | Necessary |
| temp*press | Necessary |
| press*press | Necessary |
| temp*dur | Necessary |
| press*dur | Necessary |
| dur*dur | Necessary |

**Alias Terms**

**Design Generation**

Group runs into random blocks of size: 2

**Number of Runs:**

Minimum 18

Default 32

User Specified 32

Make Design

**Design**

| Run | temp | press | dur | Shift |
|---|---|---|---|---|
| 1 | 150 | 100 | 1.1 | B |
| 2 | 120 | 150 | 2 | A |
| 3 | 180 | 100 | 2 | A |
| 4 | 150 | 50 | 2 | B |
| 5 | 180 | 50 | 1.1 | B |
| 6 | 120 | 50 | 2 | A |
| 7 | 150 | 150 | 2 | A |
| 8 | 120 | 150 | 0.2 | A |
| 9 | 150 | 100 | 1.1 | A |
| 10 | 150 | 100 | 1.1 | A |
| 11 | 180 | 50 | 2 | B |
| 12 | 150 | 100 | 0.2 | B |
| 13 | 180 | 100 | 1.1 | A |
| 14 | 150 | 100 | 1.1 | A |
| 15 | 180 | 150 | 2 | B |
| 16 | 150 | 100 | 1.1 | A |
| 17 | 120 | 150 | 1.1 | B |
| 18 | 150 | 100 | 2 | B |
| 19 | 120 | 100 | 0.2 | B |
| 20 | 150 | 100 | 1.1 | B |
| 21 | 120 | 50 | 0.2 | A |
| 22 | 150 | 50 | 0.2 | B |
| 23 | 150 | 150 | 0.2 | A |
| 24 | 120 | 50 | 1.1 | B |
| 25 | 180 | 100 | 0.2 | B |
| 26 | 180 | 150 | 1.1 | A |
| 27 | 150 | 150 | 1.1 | B |
| 28 | 180 | 150 | 0.2 | B |
| 29 | 150 | 50 | 1.1 | A |
| 30 | 120 | 100 | 2 | A |
| 31 | 120 | 100 | 1.1 | B |
| 32 | 180 | 50 | 0.2 | A |

# 7. Back up to make changes, or create data table



**Design**

| Run | temp | press | dur | Shift |
|---|---|---|---|---|
| 1 | 150 | 100 | 1.1 | B |
| 2 | 120 | 150 | 2 | A |
| 3 | 180 | 100 | 2 | A |
| 4 | 150 | 50 | 2 | B |
| 5 | 180 | 50 | 1.1 | B |
| 6 | 120 | 50 | 2 | A |
| 7 | 150 | 150 | 2 | A |
| 8 | 120 | 150 | 0.2 | A |
| 9 | 150 | 100 | 1.1 | A |
| 10 | 150 | 100 | 1.1 | A |
| 11 | 180 | 50 | 2 | B |
| 12 | 150 | 100 | 0.2 | B |
| 13 | 180 | 100 | 1.1 | A |
| 14 | 150 | 100 | 1.1 | A |
| 15 | 180 | 150 | 2 | B |
| 16 | 150 | 100 | 1.1 | A |
| 17 | 120 | 150 | 1.1 | B |
| 18 | 150 | 100 | 2 | B |
| 19 | 120 | 100 | 0.2 | B |
| 20 | 150 | 100 | 1.1 | B |
| 21 | 120 | 50 | 0.2 | A |
| 22 | 150 | 50 | 0.2 | B |
| 23 | 150 | 150 | 0.2 | A |
| 24 | 120 | 50 | 1.1 | B |
| 25 | 180 | 100 | 0.2 | B |
| 26 | 180 | 150 | 1.1 | A |
| 27 | 150 | 150 | 1.1 | B |
| 28 | 180 | 150 | 0.2 | B |
| 29 | 150 | 50 | 1.1 | A |
| 30 | 120 | 100 | 2 | A |
| 31 | 120 | 100 | 1.1 | B |
| 32 | 180 | 50 | 0.2 | A |

**Custom Design**

**Responses**

**Factors**

**Define Factor Constraints**

**Model**

**Alias Terms**

**Design**

**Design Evaluation**

Rarely need to change this

Output Options

Run Order: Randomize within Blocks

Make Table

Back

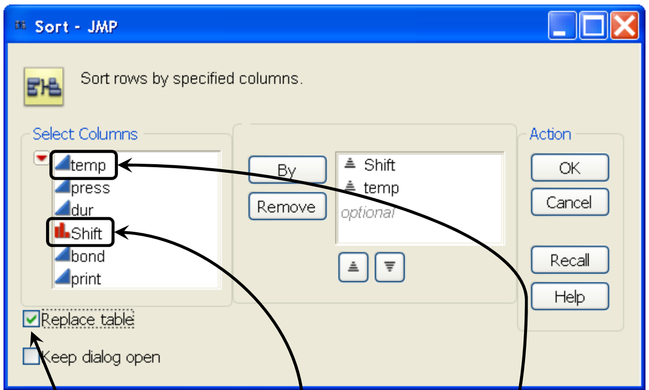
Use this if you want to make changes

Use this to create an editable data table

## 8. Sort “hard-to-change” factors within blocks, save



Tables → Sort




Double-click here . . . then here

Click here . . . then OK

| 32/0 Rows | temp | press | dur | Shift |
|---|---|---|---|---|
| 1 | 120 | 150 | 1.1 | A |
| 2 | 120 | 50 | 2 | A |
| 3 | 120 | 100 | 2 | A |
| 4 | 120 | 100 | 0.2 | A |
| 5 | 120 | 150 | 1.1 | A |
| 6 | 150 | 50 | 2 | A |
| 7 | 150 | 100 | 1.1 | A |
| 8 | 150 | 50 | 1.1 | A |
| 9 | 150 | 150 | 0.2 | A |
| 10 | 150 | 100 | 0.2 | A |
| 11 | 150 | 100 | 0.2 | A |
| 12 | 180 | 150 | 2 | A |
| 13 | 180 | 100 | 2 | A |
| 14 | 180 | 150 | 1.1 | A |
| 15 | 180 | 50 | 0.2 | A |
| 16 | 180 | 100 | 1.1 | A |
| 17 | 120 | 150 | 0.2 | B |
| 18 | 120 | 100 | 2 | B |
| 19 | 120 | 150 | 2 | B |
| 20 | 120 | 50 | 0.2 | B |
| 21 | 120 | 50 | 1.1 | B |
| 22 | 150 | 100 | 1.1 | B |
| 23 | 150 | 100 | 1.1 | B |
| 24 | 150 | 100 | 1.1 | B |
| 25 | 150 | 150 | 2 | B |
| 26 | 150 | 150 | 2 | B |
| 27 | 150 | 50 | 1.1 | B |
| 28 | 150 | 100 | 1.1 | B |
| 29 | 180 | 50 | 2 | B |
| 30 | 180 | 100 | 1.1 | B |
| 31 | 180 | 50 | 0.2 | B |
| 32 | 180 | 150 | 0.2 | B |

## Notes

# Exercises



Use the Custom Design process to create RSM design matrices for the exercises on the following pages. In addition to special instructions given in each case, follow these general instructions:

- If a numerical range is given, the factor is continuous.
- If levels are given, the factor is categorical.
- Use the given sample size when creating the design.
- Always sort the data table by the blocking factor.
- Some factors are "hard to change" (temperature, for example). Sort by hard-to-change factors secondary to the blocking factor. Leave other factors randomized.
- For each exercise, have the instructor review your matrix when you are finished.

For these exercises, you don't have to specify the Y variables

# Exercise 21.1



| Control factors | Levels | |
|---|---|---|
| Heat treat | Anneal | Solution/age |
| Polish | Chemical | Mechanical |
| Peen | Yes | No |

- Hard to change factors: *none*
- Blocking factor: *none*
- Experimental unit: *one small test piece*
- Sample size: 12 (constraint on available test fixtures, not from a calculation)

## Exercise 21.2



| Control factors | Levels | |
|---|---|---|
| Contact wheel land-groove ratio (LGR) | Low | High |
| Contact wheel material (MATL) | Steel | Rubber |
| Belt usage limit (USAGE) | “50%” | “75%” |
| Belt grit size (GRIT) | 30 | 50 |

- Hard to change factors: *LGR* and *MATL* (have to replace one wheel with another)
- Blocking factor: *Time of day* (morning vs. afternoon) (Why?)
- Experimental unit: *one large casting*
- Calculated sample size: 18

## Exercise 21.3



| Control factors | Ranges |
|---|---|
| Force | 70 to 150 |
| Energy | 275 to 325 |
| Amplitude | 70 to 90 |

- Hard to change factors: *none*
- Blocking factor: *Cavity* (parts are molded from 4 tool cavities)
- Experimental unit: *one welded plastic container*
- Calculated sample size: 68

## 22 Sample Size Calculation



- Sample size (N) is the total number of experimental units
- Also known as "number of runs"

| User inputs (Other than factor info) | Abbreviations |
|---|---|
| Expected standard deviation of noise/error/residual/unexplained variation in Y | RMSE, $\sigma_{noise}$ |
| Acceptable margin of error for predicting average Y in the population | MOE |
| Smallest change in average Y worth detecting (difference to detect) | DTD |

Notes



RMSE ($\sigma_{noise}$) can sometimes be estimated from historical data or trial runs before the experiment. It could also be the RMSE from a previous experiment with the same Y variable. As a last resort, data on a similar process or product could be used.

MOE and DTD are expressed in the units of the Y variable, not as percentages or proportions. The values assigned to MOE and DTD are judgments that must be made by the project team.

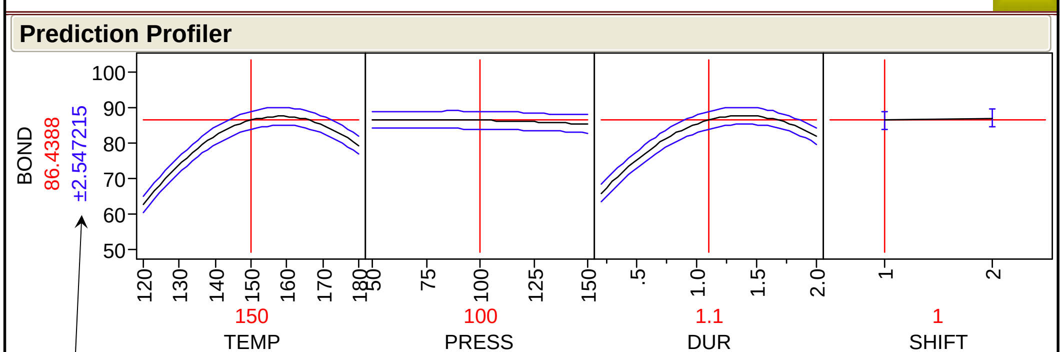
## Interpreting MOE and DTD



| Y variable | Population parameter to which MOE and DTD apply |
|---|---|
| Dimension | Average dimension |
| Strength | Average strength |
| Cycle time | Average cycle time |
| Variability in Y | Standard deviation |
| Pass/fail | Fraction or % defective |
| Time to failure | Failure probability at a given mission time, or Time with given reliability |

## MOE and DTD will vary over the factor space






- Profiler for an RSM model with 4 factors, N = 32
- MOE is 2.55 at the settings shown above, but it varies from 2.08 to 2.86
- The most we can get from a sample size calculation is an *average* MOE or DTD

## Average variance of prediction (AVP)



- AVP is a function of the design matrix, including N
- Average MOE is related to AVP

$$\text{Average}\left(\text{MOE}\right) \approx 2\left(\text{RMSE}\right)\sqrt{\text{AVP}}$$

- Solve this to find the required AVP for your expected RMSE and desired MOE

$$\text{Required AVP} \approx \frac{1}{4}\left(\frac{\text{MOE}}{\text{RMSE}}\right)^2$$

## AVP (cont'd)



- It is most common to specify DTD rather than MOE
- Often DTD is expressed in terms of a project metric:

(Current state value) minus (Goal value)

- There is a relationship: $\text{MOE} \approx \text{DTD}/\sqrt{2}$
- From this we get

$$\boxed{\text{Required AVP} \approx \frac{1}{8}\left(\frac{\text{DTD}}{\text{RMSE}}\right)^2}$$

## Steps for DOE sample size calculation



1. Determine your expected RMSE and desired DTD.
2. Calculate the required AVP.
3. In JMP, select DOE $\rightarrow$ *Custom Design* $\rightarrow$ red triangle $\rightarrow$ *Optimality Criterion* $\rightarrow$ *Make I–Optimal Design.*
4. Enter your factors into *Custom Design*. (For categorical factors, you must give the number of levels. Except for this, everything can be left generic.)
5. Select the statistical model (usually RSM).
6. Use *Make Design* to calculate AVP for each N you want to try.
7. Use *Back* to increase N until AVP is just below the value in step 2.

## Example



- For the bond strength experiment, the design team chose:

$$\text{DTD} = 3 \text{ psi}$$

- Based on the standard deviation of bond strength under stable process conditions in the past, they chose:

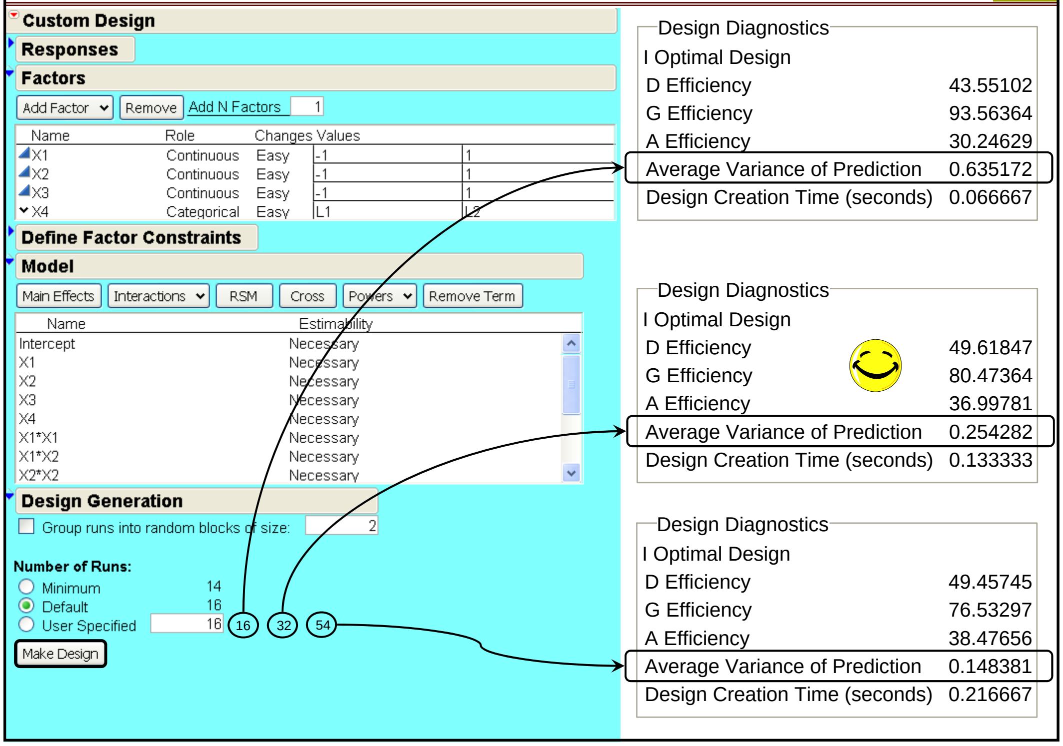
$$\text{RMSE} = 2.1 \text{ psi}$$

- Plug into the formula:

$$\text{AVP} = \frac{1}{8}\left(\frac{\text{DTD}}{\text{RMSE}}\right)^2 = \frac{1}{8}\left(\frac{3}{2.1}\right)^2 = 0.255102$$

# Example (cont'd)






## Notes



Shown on the left is the *Custom Design* "mock up" for an RSM experiment with 3 quantitative factors and 1 categorical factor with 2 levels. Shown on the right are the calculated AVPs for sample sizes 16, 32, and 54. For each sample size, click on *Make Design*. The AVP is given in the *Design Diagnostics* element. To do another calculation, click on *Back* and repeat this process.

The AVP for $N = 32$ is just under our require AVP (0.255). This is the sample size that was chosen for this experiment.

A disturbing feature of the AVP calculation is that it may vary when you hit *Make Design* multiple times with the same sample size. This happens because JMP may not select the same design every time. Fortunately, the variation in AVP isn't large enough to make a significant difference the sample size.

## Exercise 22.1



We are planning an experiment to optimize a monofilament extrusion process with 4 continuous factors. A key response variable is *tensile strength*. We want to do a sample size calculation based on the following information:

- The standard deviation of *tensile* in recent production during stable periods is 2314.4 psi.
- The difference to detect (DTD) in mean *tensile* is 3000 psi.

Open *LSSV2 other stuff \ sample size calculator*, go the sheet *AVP for DOE (cont Y)*. Enter the information given above to get the required AVP. Use this to determine the required sample size.

## Exercise 22.2



We are planning an experiment to optimize an ultrasonic welding process with 3 continuous factors and a 4-level categorical factor. A key response variable is the *weld depth.* We want to do a sample size calculation based on the following information:

- The standard deviation of *weld depth* in recent production during stable periods is 0.0236 mm.
- The difference to detect (DTD) in mean *weld depth* is 0.03 mm.

Open *LSSV2 other stuff \ sample size calculator*, go the sheet *AVP for DOE (cont Y)*. Enter the information given above to get the required AVP. Use this to determine the required sample size.

## Exercise 22.3



Calculate the sample size for this automotive paint experiment. The experimental units are 8½ × 11″ panels. The Y variable is % gloss. The standard deviation of Y is about 2.5. In the company's experience, customers cannot perceive a difference in gloss of less than 5 percentage points. Based on this, 5 was chosen as the DTD.

| *Factor* | *Range* | *Level 1* | *Level 2* | *Level 3* | *Level 4* |
|---|---|---|---|---|---|
| Resin amount | 25 - 75 | | | | |
| Catalyst amount | 0 - 1 | | | | |
| Pigment amount | 1 - 5 | | | | |
| Resin type | | Acrylic | Polymer | Urethane | |
| Catalyst type | | Tin | Zinc | | |
| Pigment type | | Cb | $FeO_2$ | $TiO_2$ | |

## Exercise 22.4



Calculate the sample size for this second automotive paint experiment. The experimental units are 8½ × 11″ panels. The Y variable is % gloss. The standard deviation of Y is about 2.5. In the company's experience, customers cannot perceive a difference in gloss of less than 5 percentage points. Based on this, 5 was chosen as the DTD.

| *Factor* | *Range* | *Level 1* | *Level 2* | *Level 3* | *Level 4* |
|---|---|---|---|---|---|
| Addition amount | 1 - 5 | | | | |
| Addition type | | Acrylic | Oil | Polymer | Urethane |
| Addition point | | Grind | Let-down | Post | |
| Addition method | | Cow | Lift | Shaker | |

# 23 Workshop: Paper Helicopters



Notes

# Helicopter DOE

We want to maximize the flight time of paper helicopters dropped from a fixed height. Here are the factors and levels:

| Factor | Type | Level 1 | Level 2 | Level 3 |
| --- | --- | --- | --- | --- |
| Paper | Categorical | Light | Heavy | |
| Blade length | Continuous | 3.25" | 4.5" | 5.75" |
| Stem length | Continuous | 3.5" | 4.0" | 4.5" |
| Midriff taper | Categorical | No | Yes | |
| Paper clip | Categorical | Small | Large | |

For continuous factors you enter only the low and high levels

1. Do a sample size calculation.

2. The instructor will tell you how many blocks will be needed. Create a design matrix with the calculated sample size and the indicated number of blocks.

3. Paper will be treated as a hard-to-change factor. Sort the matrix accordingly.

4. Save the data table as *Helicopter DOE*.

5. We will be dividing into teams to run this experiment. The instructor will show each team how to build the helicopters.

6. Build the helicopters called for in Block 1 of the design matrix in the data table saved by *one* of the team members. (The matrix produced by Custom Design will be different for each person.)

7. *Double-check the helicopters against the matrix.*

8. Fly the helicopters, enter the flight times into the data table.

9. For Block 2, everyone changes manufacturing jobs. Block 1 people will provide cross-training as needed before starting Block 2. The new teams will then build the helicopters called for in Block 2.

10. *Double-check the helicopters against the matrix.*

11. Fly the helicopters. The metrology teams should be the same as in Block 1. Enter the flight times into the data table.

12. When the data entry is complete, the data table should be saved and shared with all team members. Each team member is to independently run the analysis. Find the best factor settings, the resulting predicted average flight time, and the RMSE.

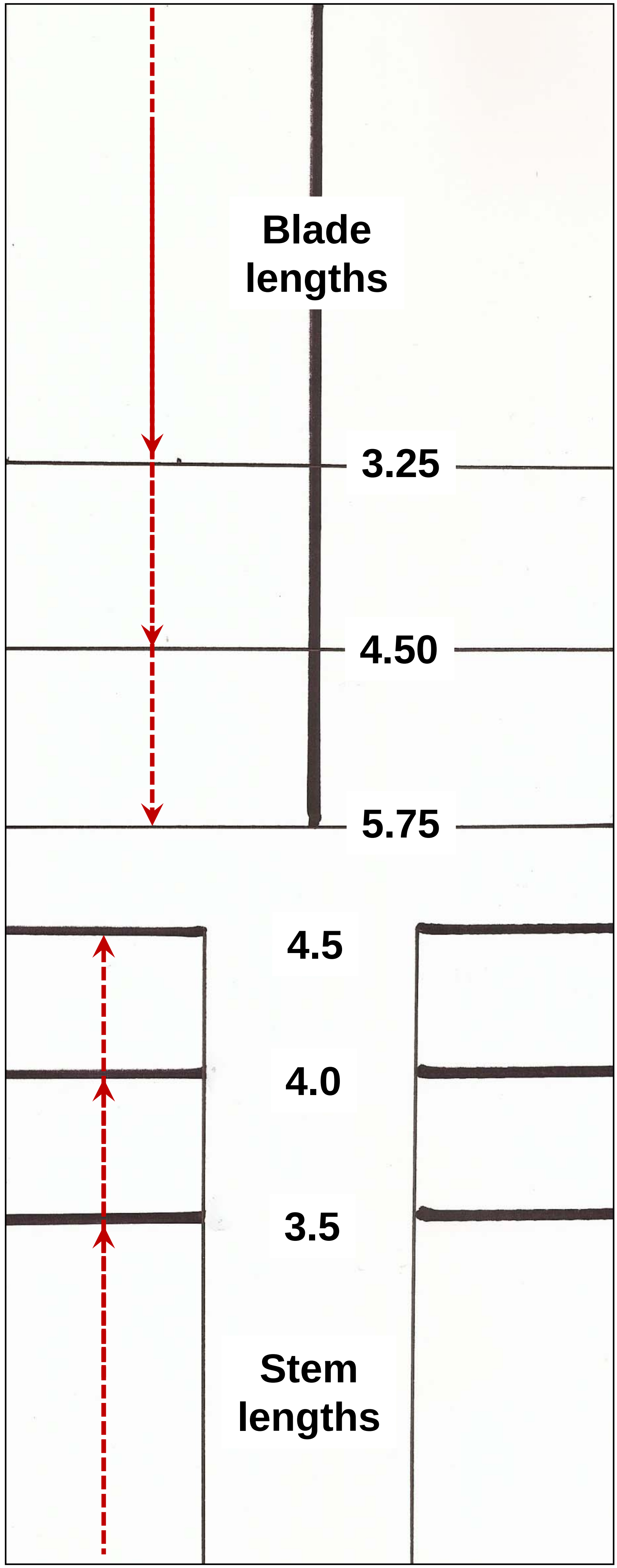
13. If different team members get different results, find the reasons for these differences and make the necessary corrections.

14. Build a confirmation helicopter using the best factor settings.

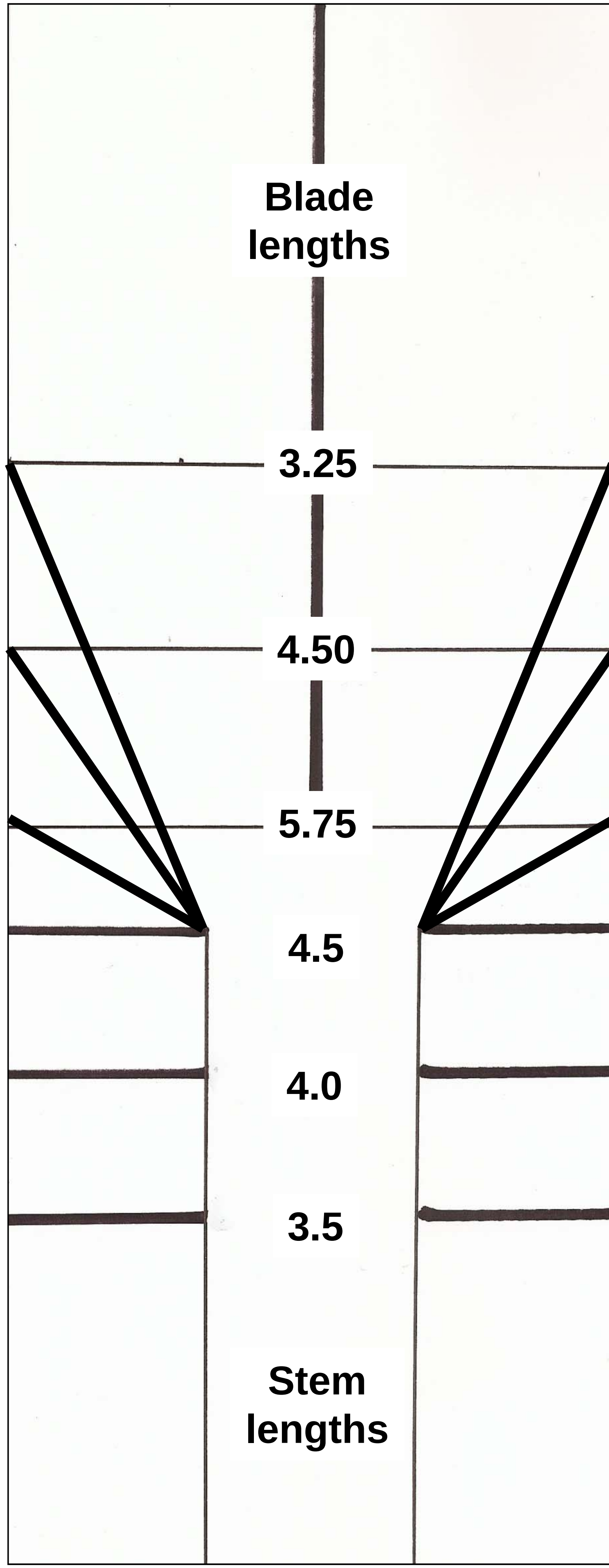
15. Fly and time the confirmation helicopter. The confirmation flight time should fall within 2 RMSEs of your predicted average flight time.

16. Save your script and data table.

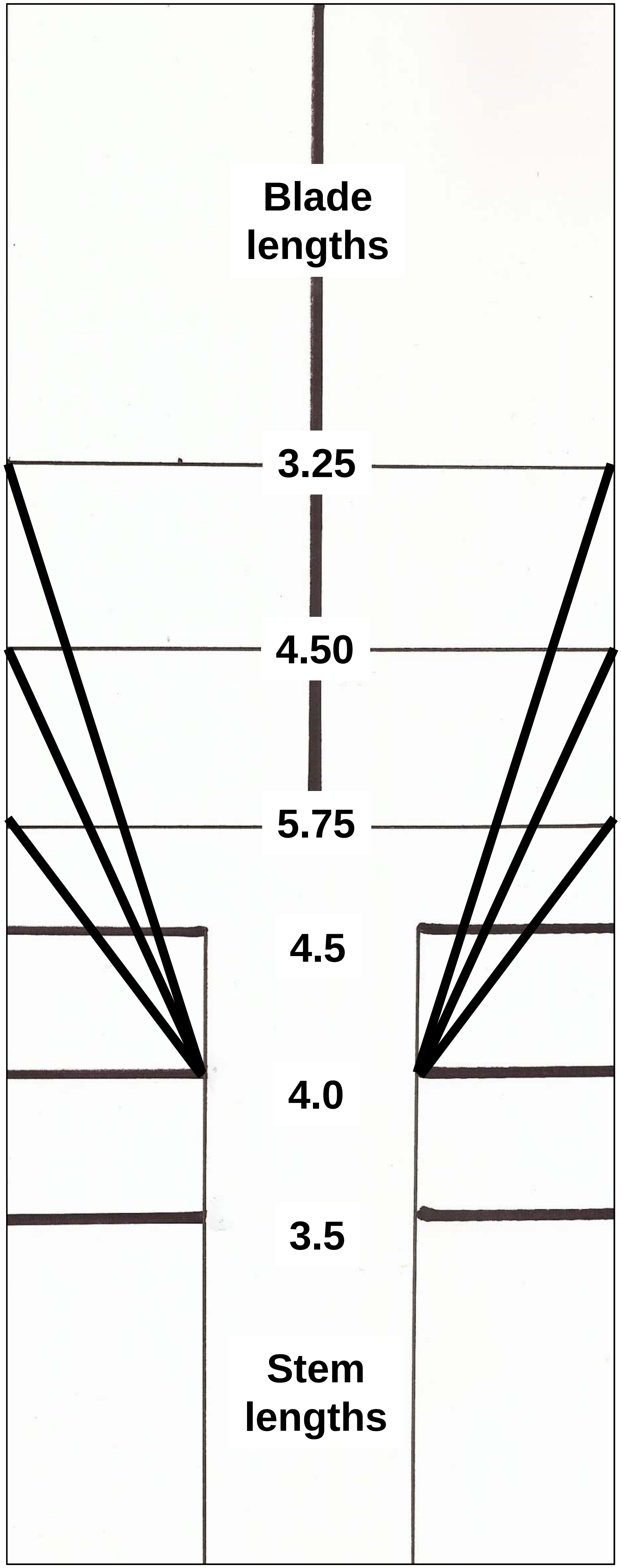
No taper



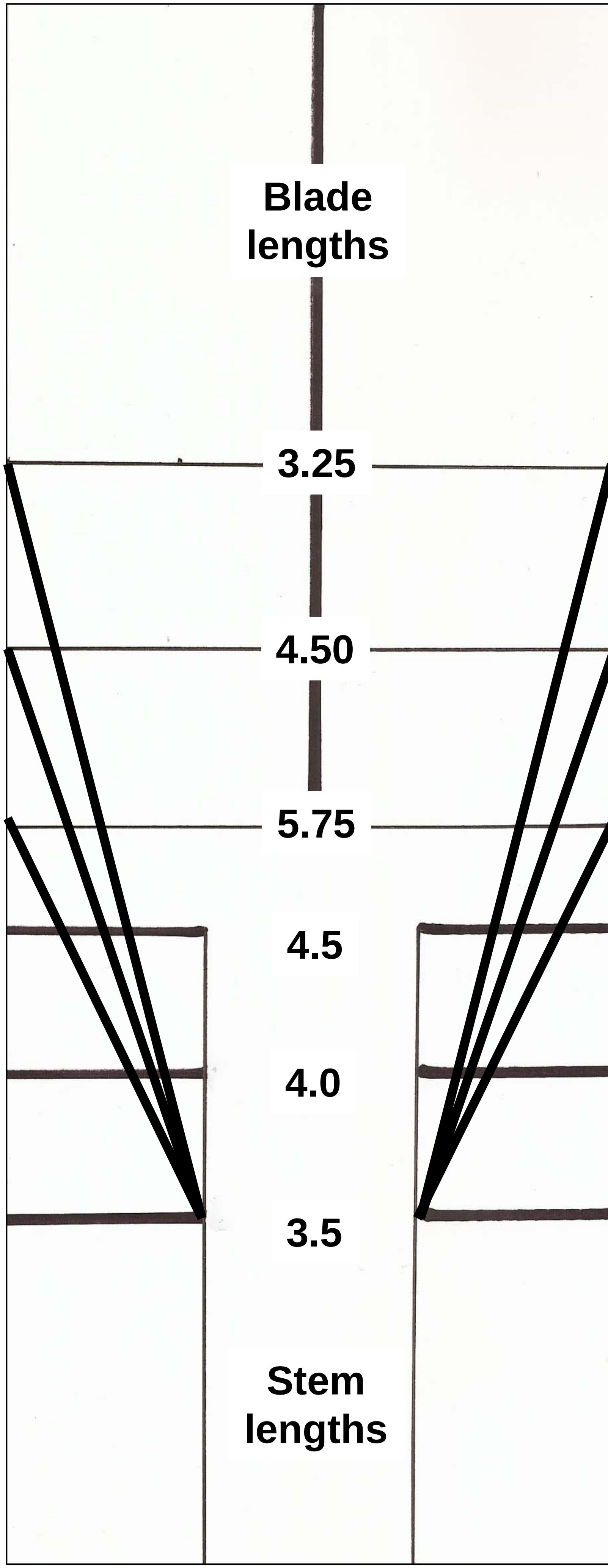
**Tapers with 4.5″ stem**



**Tapers with 4.0″ stem**



**Tapers with 3.5″ stem**



# 24 Multiple Response Optimization



- Experiments may have more than one response variable
- You can optimize each response separately . . .
- . . . but you will get different answers for each response!

## Notes



In real experiments there are always multiple response variables. If you think you have just one, you haven't finished planning your experiment. For example, any change you make to a process could affect multiple Y variables. All of these should be considered as possible response variables for your experiment.

If there are only two factors, we can jointly optimize multiple responses by overlaying their contour plots. The more factors there are, the more difficult this becomes. In this section we introduce and illustrate the most widely used general technique for jointly optimizing multiple responses when there are three or more factors.

# Example 1



**Response BOND**

**Effect Tests**

| Source | Nparm | DF | Sum of Squares | F Ratio | Prob > F |
|---|---|---|---|---|---|
| TEMP(120,180) | 1 | 1 | 1540.835 | 366.0070 | <.0001* |
| PRESS(50,150) | 1 | 1 | 8.439 | 2.0046 | 0.1715 |
| DUR(0.2,2) | 1 | 1 | 1606.813 | 381.6793 | <.0001* |
| TEMP*TEMP | 1 | 1 | 1363.630 | 323.9142 | <.0001* |
| TEMP*PRESS | 1 | 1 | 14.607 | 3.4697 | 0.0766 |
| PRESS*PRESS | 1 | 1 | 1.385 | 0.3290 | 0.5724 |
| TEMP*DUR | 1 | 1 | 20235.249 | 4806.642 | <.0001* |
| PRESS*DUR | 1 | 1 | 0.759 | 0.1804 | 0.6754 |
| DUR*DUR | 1 | 1 | 715.715 | 170.0096 | <.0001* |
| SHIFT | 1 | 1 | 3.578 | 0.8499 | 0.3671 |

**Response PRI NT**

**Effect Tests**

| Source | Nparm | DF | Sum of Squares | F Ratio | Prob > F |
|---|---|---|---|---|---|
| TEMP(120,180) | 1 | 1 | 6.821113 | 85.3929 | <.0001* |
| PRESS(50,150) | 1 | 1 | 25.625986 | 320.8095 | <.0001* |
| DUR(0.2,2) | 1 | 1 | 2.121674 | 26.5611 | <.0001* |
| TEMP*TEMP | 1 | 1 | 2.148242 | 26.8937 | <.0001* |
| TEMP*PRESS | 1 | 1 | 0.300304 | 3.7595 | 0.0661 |
| PRESS*PRESS | 1 | 1 | 0.257674 | 3.2258 | 0.0869 |
| TEMP*DUR | 1 | 1 | 1.613751 | 20.2024 | 0.0002* |
| PRESS*DUR | 1 | 1 | 1.065140 | 13.3344 | 0.0015* |
| DUR*DUR | 1 | 1 | 1.372401 | 17.1810 | 0.0005* |
| SHIFT | 1 | 1 | 0.137813 | 1.7253 | 0.2032 |

## Notes



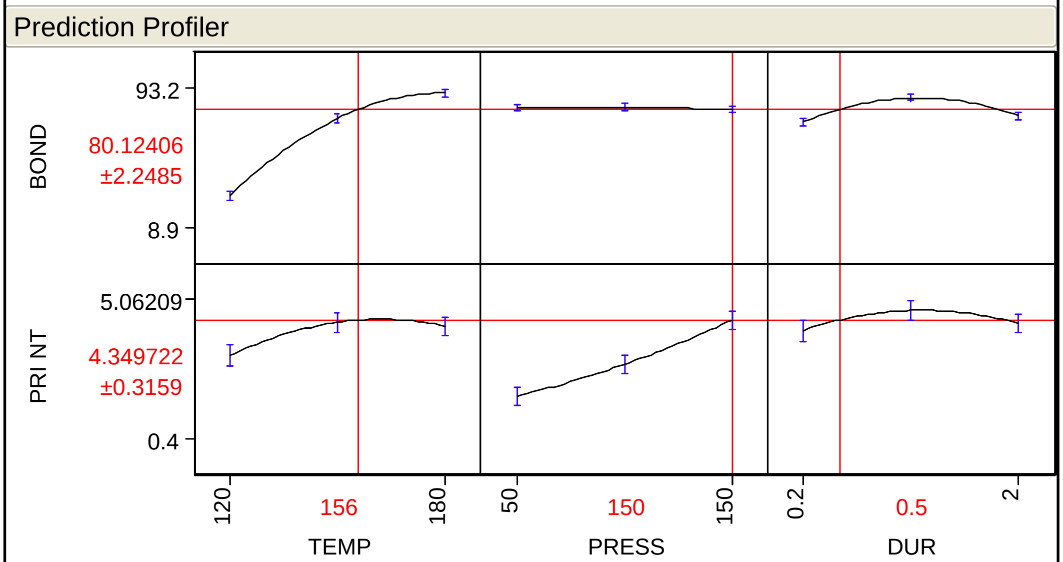
The data table *heat sealing 2.jmp* contains the data from an experiment to optimize the sealing of potato chip bags. In addition to bond strength (BOND) we now have a second Y variable PRINT, a rating of cosmetic quality based on visual inspection. PRINT is defined so that higher is better.

TEMP and DWELL have significant linear and quadratic effects on BOND. PRESS has very little effect on BOND, but a big effect on PRINT. This is good news:  it means we can use PRESS to bring PRINT up as high as possible, then use TEMP and/or DWELL to get the BOND we want.

SHIFT, a blocking factor, came up insignificant for both responses. Is this good or bad?

## Example 1 (cont'd)

We want BOND = 80 and PRINT as large as possible.
Here is a solution:





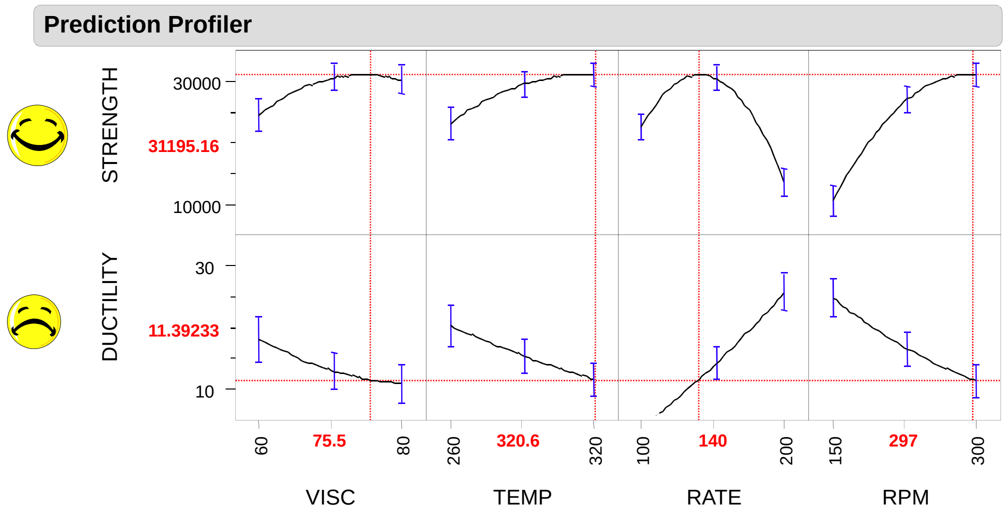

## Notes

In this example is it easy to find solutions using the *Prediction Profiler*. Obviously, PRESS should be 150, because this increases PRINT without significantly affecting BOND. Once we have done that, there are many combinations of TEMP and DWELL that predict 80 psi for BOND. One such combination is shown above.

# Example 2



(VISC, TEMP, RATE, RPM) ≈ (76, 321, 140, 297)
DUCTILITY ≈ 11%




## Notes



The data table *extrusion 2.jmp* contains data from an experiment to optimize the mechanical properties of an extruded plastic material We want STRENGTH as high as possible while maintaining a lower bound of 20 for DUCTILITY.

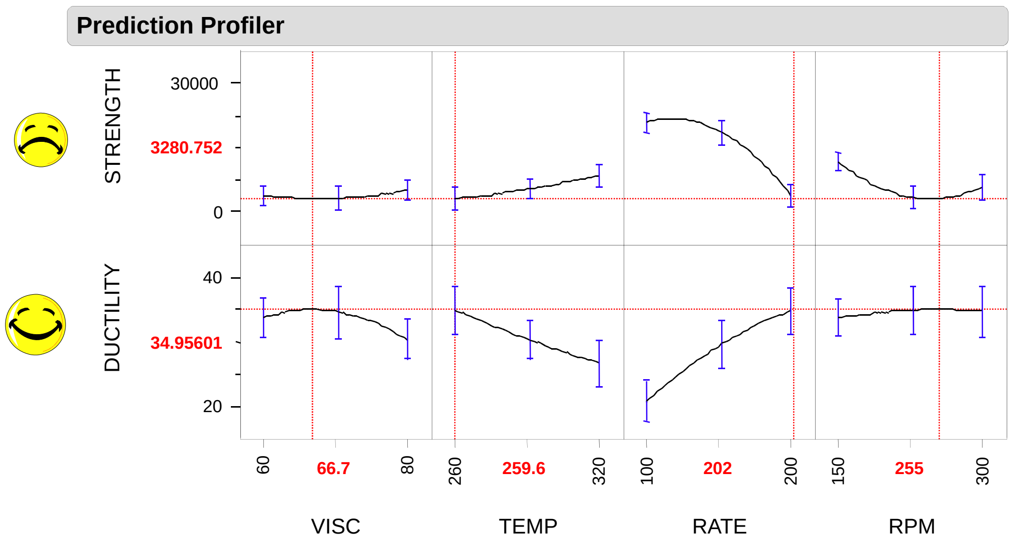
As shown above, it is easy to find a great solution for STRENGTH just by visually exploring the *Prediction Profiler*, but the resulting DUCTILITY is too low.

# Example 2 (cont'd)



(VISC, TEMP, RATE, RPM) ≈ (67, 260, 202, 255)

STRENGTH ≈ 3281




# Notes



As shown above, it is easy to find a great solution for DUCTILITY just by visually exploring the *Prediction Profiler*, but the resulting STRENGTH is very low.

## Joint optimization of responses



- Each response has a goal (minimize, maximize or target)
- Define a "desirability function" for each response
- Combine the individual desirabilities into a single overall desirability function
- Maximize the overall desirability to jointly optimize all responses

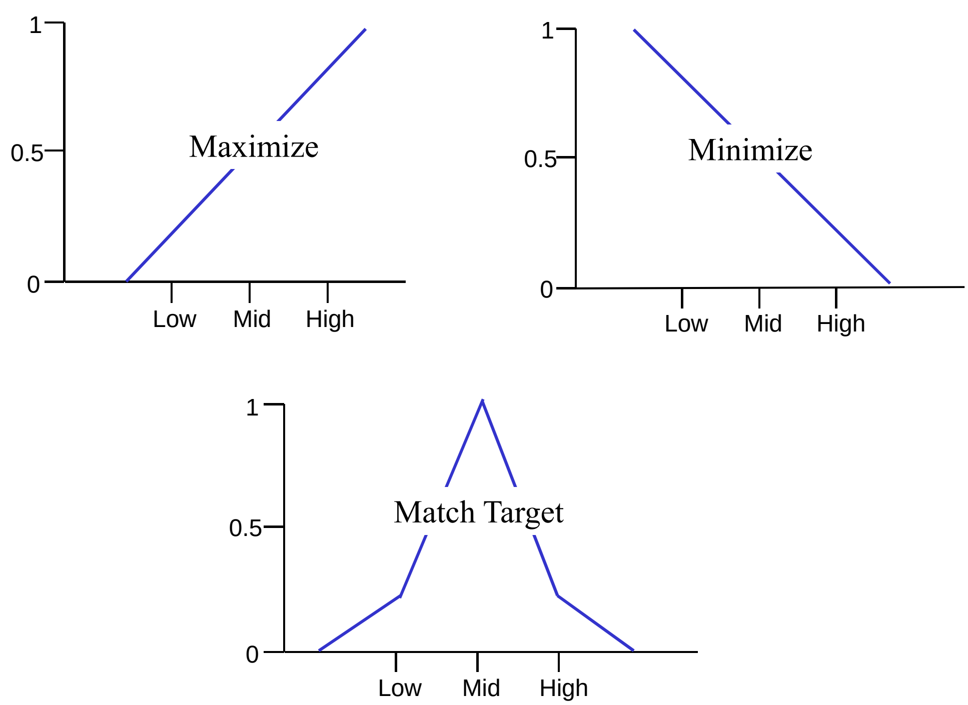
## Notes



*Desirability* is a unitless quantity between 0 and 1, defined so that higher is better. JMP supplies default desirability functions based on the experimental data for your response variables. You must redefine the desirability functions so that they represent your objectives for each response variable.

You start by setting the general goal for each response: *Maximize*, *Minimize* or *Match Target*. Then you specify low, middle, and high data values to fine tune the shape of the desirability functions.

## Default desirability functions






## Notes

The desirability function is increasing for *Maximize* responses and decreasing for *Minimize* responses. It is bell-shaped for *Match Target* responses.

For *Minimize* responses with a lower bound of 0, it is a good idea to make the *Low* value equal to 0. Examples are number of defects, fraction defective, cycle time, standard deviation, cost of waste, etc.

The low and high values for a *Match Target* response are used to define the allowable deviation from the target value.

## Overall desirability



The default overall desirability for $Y_1$ and $Y_2$ is

$$\sqrt{(Y_1 \text{ desirability}) \times (Y_2 \text{ desirability})}$$

This can be customized to

$$(Y_1 \text{ desirability})^{w_1} (Y_2 \text{ desirability})^{w_2}$$

$$w_1 = \text{weight} = \text{relative importance of } Y_1$$
$$w_2 = \text{weight} = \text{relative importance of } Y_2$$
$$w_1 + w_2 = 1$$

## Notes



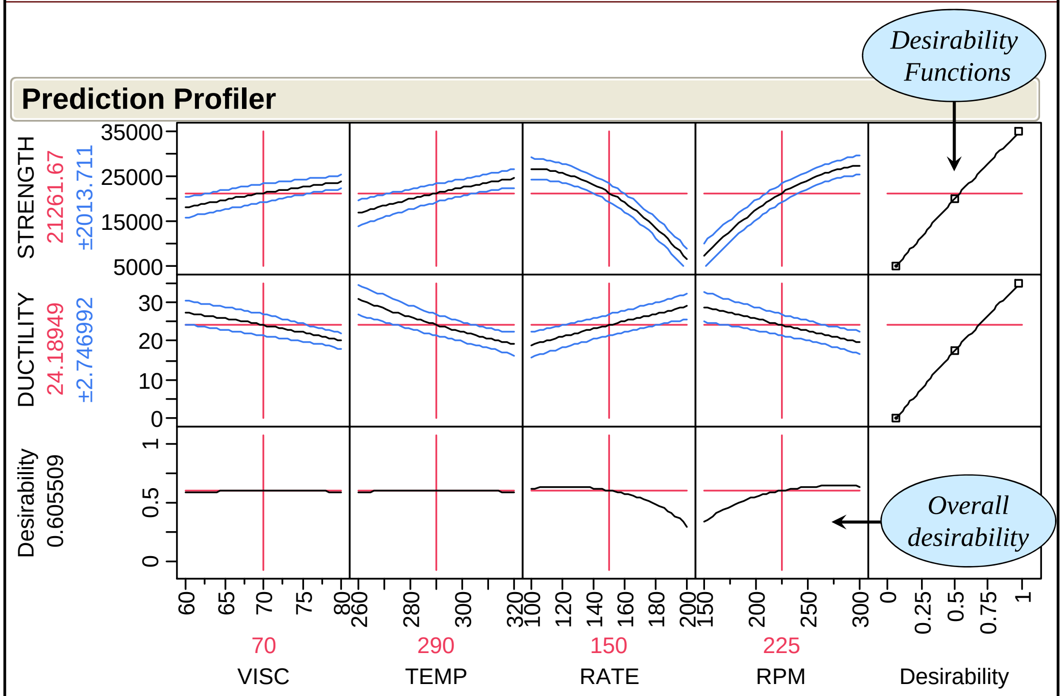
The overall desirability function is calculated as a geometric mean of the desirability functions for individual response variables. It is important to use a geometric mean instead of an arithmetic mean. With the geometric mean, the overall desirability will be zero whenever any individual response desirability is zero. This prevents the optimization algorithm from finding solutions that are excellent for some responses but completely unacceptable for others.

A weighted geometric mean can be used. The weights (called *importance* in JMP) allow users to specify relative priorities for the responses. The higher the importance, the greater the influence the response has in determining the overall solution found by the optimization algorithm.

You can enter any positive numbers for the importance. The program automatically normalizes them to add up to 1.

## Example 2 (cont'd)

405



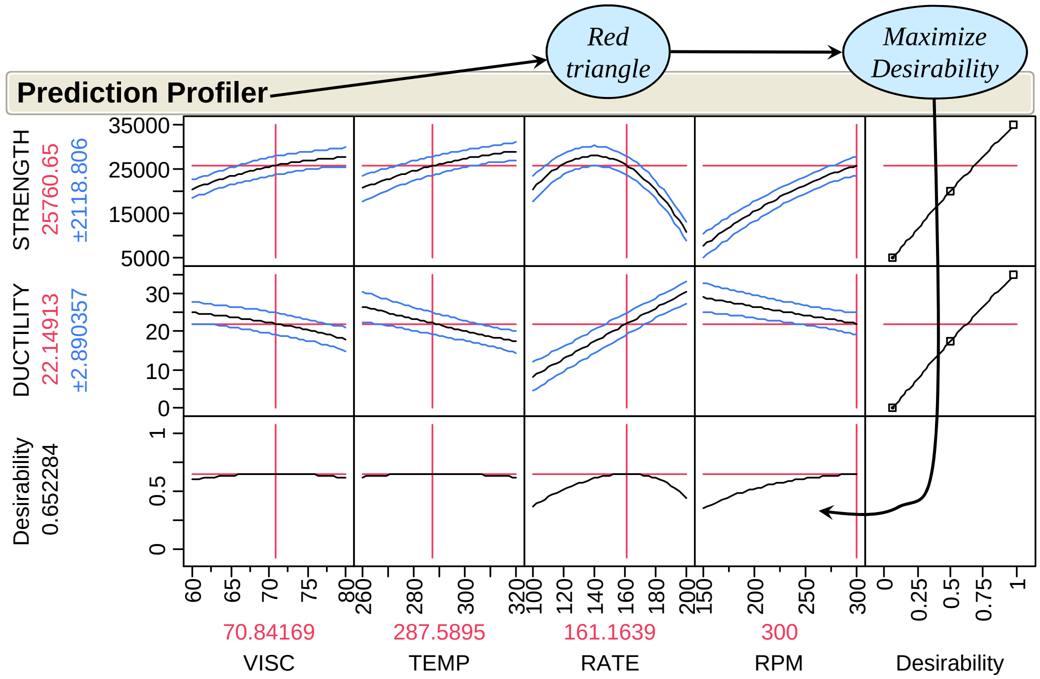

## Notes

406

Here is the default *Prediction Profiler* for the four-factor extrusion experiment. The individual desirability functions are shown in the right-most column. In this case they are both increasing functions because our general objective for both responses is *Maximize*.

The overall desirability is a function of the experimental factors, and is shown in the bottom row. By default, it is the unweighted geometric mean of the individual desirability functions.

# Example 2 (cont'd)





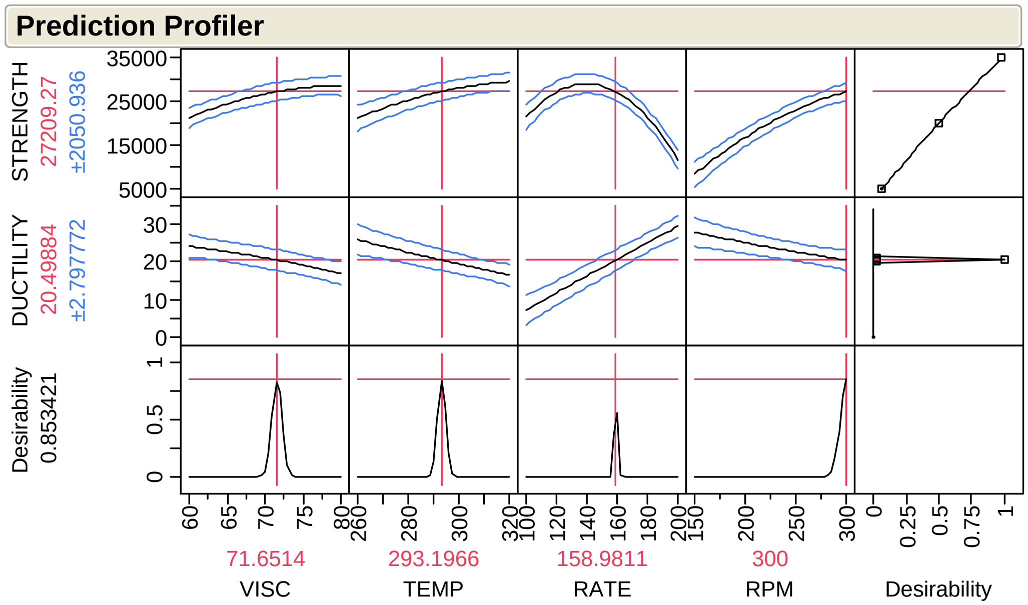

## Notes

Here is the *Prediction Profiler* after selecting *Maximize Desirability* from the red triangle menu. We have increased STRENGTH from 21262 to 25761. DUCTILITY has dropped from 24.2 to 22.1, but it is still greater than 20.

In presenting these results to a group of stakeholders, including some engineers, the project team is challenged to increase the STRENGTH even further. Based on knowledge of the extruded material, it is known that this would require a further drop in DUCTILITY. This is confirmed by examination of the *Prediction Profiler* (please convince yourself this is true). With DUCTILITY currently at 22.1, there is a good possibility we can increase STRENGTH without driving DUCTILITY below 20.

# Example 2 (cont'd)

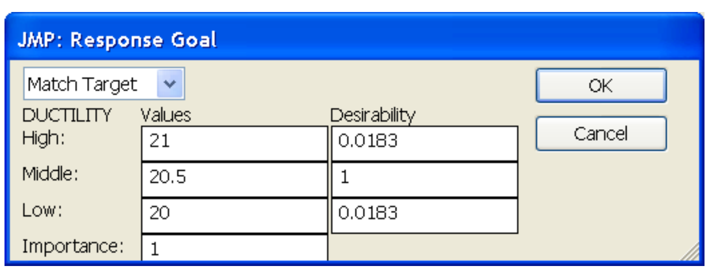





## Notes



To obtain the results shown above, double-click in the individual *Desirability* pane for DUCTILITY (on the right), change the specifications as shown below, then run *Maximize Desirability* again.




We have increased STRENGTH to 27209 by allowing DUCTILITY to fall to 20.5.

From the output shown above, are sure that average DUCTILITY will exceed 20?

## Exercise 24.1



(a) Open or go to *heat sealing 2*. Run the model script. Use the *Effects Tests* elements for *Bond* and *Print* to prune out any model terms with P > 0.15 for both responses. Re-run the model as needed.

(b) Go to the *Prediction Profiler* for your final model. Our target for mean *Bond* is 80, with a tolerance of ±5. The highest (best) possible value for mean *Print* is 5. We require mean *Print* to be greater than 4. Modify the desirability functions for *Bond* and *Print* accordingly. On the red triangle menu for the *Prediction Profiler*, select *Save Desirabilities*.

(c) Use *Maximize Desirability* to find the optimal factor settings. Have we achieved our objectives?

## Exercise 24.1 (cont'd)



d) The Production Manager is unhappy with our solution. It achieves excellent *Bond* and *Print*, but the proposed increase in *Dwell* would reduce throughput from 300 to 50 bags per minute!

To look for a compromise, select *Reset Factor Grid* on the *Prediction Profiler* red triangle. Click OK for *Temp* and *Press*. We want to lock the value of *Dwell* at a low value, say 0.5. Type 0.5 for *Current Value,* check the *Lock Factor Setting* box, then click OK. The vertical line on the *Dwell* profile should now be solid.

e) Use *Maximize Desirability* to find the constrained optimal factor settings. Is everybody happy now?

f) Save your script, close and save the data table.

## Exercise 24.2



a) Assembly of inkjet print cartridges includes an ultrasonic welding operation with control parameters *Force*, *Energy* and *Amplitude*. The Y variables are *Weld Depth* and *Leak Rate*. The noise factor *Cavity* is the tool cavity the plastic cartridge bodies were molded in.

   Open *ultrasonic welding 2*. Run the model script, using a Log transformation for *Leak Rate*. Prune the model as needed, run again.

b) Go to the *Prediction Profiler*. The target for mean *Depth* is 0.20, with a tolerance of ± 0.05. The lowest (best) possible value for mean *Leak Rate* is 0. We require mean *Leak Rate* to be no larger than 0.10. Modify the desirability functions for *Depth* and *Leak Rate* accordingly. Save the desirabilities (red triangle on *Prediction Profiler*).

c) Use *Maximize Desirability* to find the optimal factor settings. Have we achieved our objectives?

## Exercise 24.2 (cont'd)



d) Save your script, close and save the data table.

## Exercise 24.3



a) Open *electron microscope*. Run the *Fit Model* script, but this time go to the *Model Dialog*. Include all 4 response variables, apply Log transformations to all of them, then run the model again. In reality, you should remove all terms that have P > 0.15 for all responses. Consider that a homework assignment. For now, just let it be.

b) Go to the *Prediction Profiler*. We want to minimize all 4 responses. Use the same desirability functions for all 4 responses: High = 2, Middle = 1, Low = 0. Save the desirabilities (red triangle on *Prediction Profiler*).

c) Run *Maximize Desirability* to find the optimal factor settings. Give the predicted mean values for all responses. Have we achieved all of our objectives?

d) Save your script, close and save the data table.

Notes

# 25 Screening Experiments



| Optimization | Screening |
|---|---|
| Smaller number of factors | Larger number of factors |
| Single and interactive effects | Single effects only |
| Quantitative factors have 3 levels | All factors have 2 levels (usually) |
| Identify the best factor levels | Identify the best factors |

## Notes



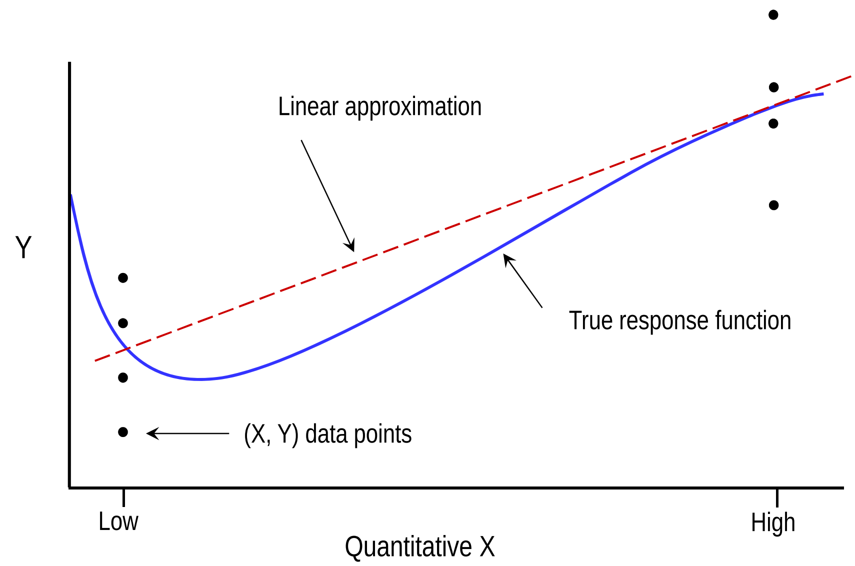
Screening experiments involve a relatively large number of factors, usually all at two levels. The objective of a screening experiment is to identify a smaller set of influential factors for further experimentation. We don't need a screening experiment if we already know what the key factors are from past experience or knowledge of the process.

Screening experiments usually employ linear models with no interaction or quadratic terms. This is done to allow relatively small sample sizes. We don't expect such models to give us good characterizations, but they are usually adequate for identifying the "big hitters."

## Bold strategy

419

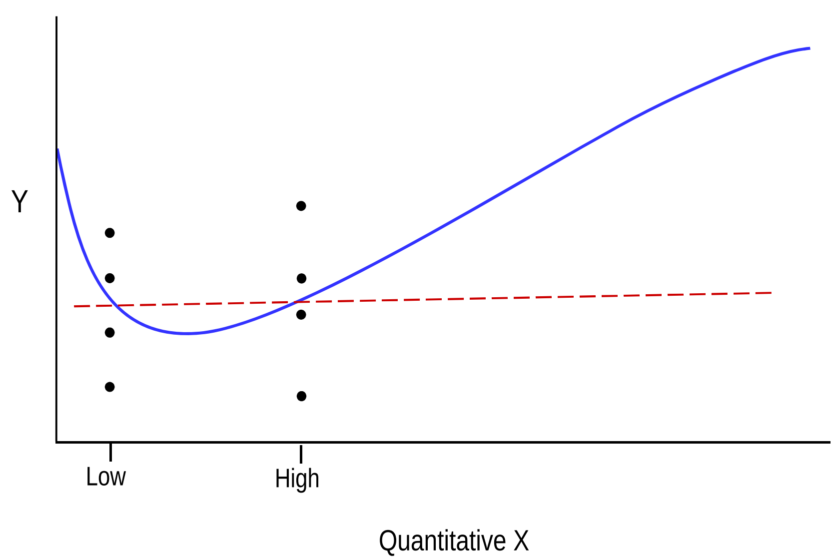
*Use the entire feasible operating range in a first experiment*




## Not bold enough

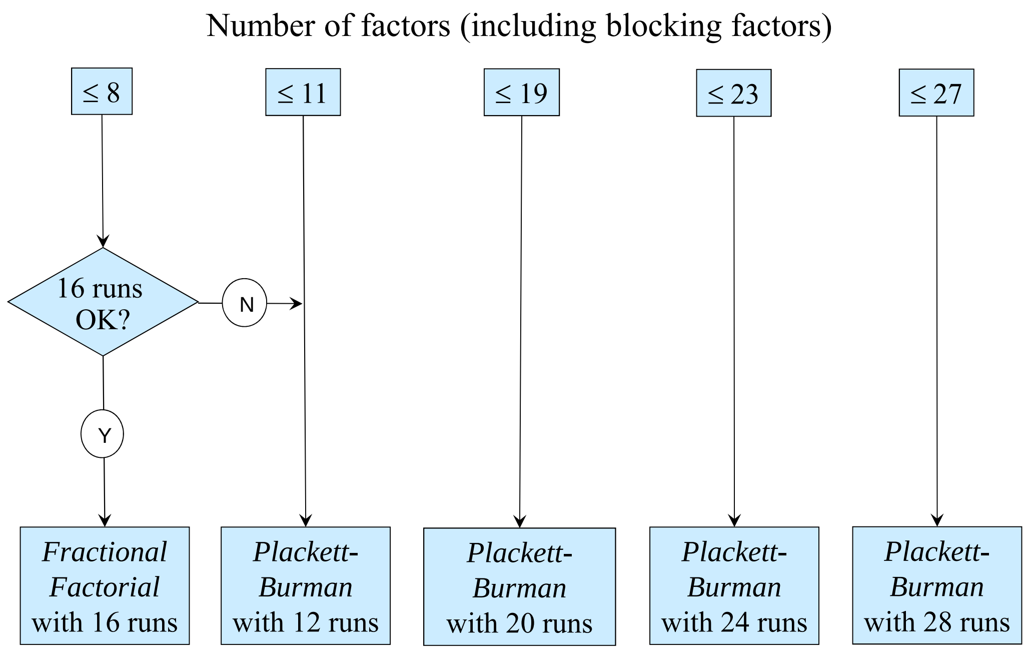
420

- Low and high levels of X are too close together
- We don't get a true picture of the X-Y relationship



## Guidelines for screening experiments

421




## Notes

422

The general strategy for factor screening is to use the smallest design possible, subject to the guidelines given above. To this end, we use main effects models, even though there are significant interaction and quadratic effects in most processes. We use the main effect estimates from a screening experiment to rank the factors in importance. These estimates are biased, because interaction and quadratic terms are (deliberately) left out of the model.

The designs recommended above work well in practice because the biases are distributed evenly among the factors. This makes it relatively safe to rank the factors by comparing the estimates. These remarkable designs were discovered by British statisticians Plackett and Burman during World War II.

The next four slides show how to create screening design matrices in JMP, using the guidelines given above. The path is *DOE* → *Screening Design*.

# Guidelines for screening (cont'd)



## Up to 8 factors in 16 runs

DOE- Screening Design

**Screening Design**

**Responses**

**Factors**

Screening Design
8 Factors

Choose a Design

| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 12 | | Plackett-Burman | 3 - Main Effects Only |
| 16 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 16 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 16 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | | Fractional Factorial | 5 - All 2-factor interactions |
| 64 | 32 | Fractional Factorial | 5 - All 2-factor interactions |
| 64 | 16 | Fractional Factorial | 5 - All 2-factor interactions |
| 64 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | | Fractional Factorial | 5+ - All 2-factor interactions |
| 128 | 64 | Fractional Factorial | 5+ - All 2-factor interactions |
| 128 | 32 | Fractional Factorial | 5+ - All 2-factor interactions |
| 128 | 16 | Fractional Factorial | 5+ - All 2-factor interactions |
| 128 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 2 | Fractional Factorial | 4 - Some 2-factor interactions |

optional item

Continue

Back

## Up to 11 factors in 12 runs

DOE- Screening Design

**Screening Design**

**Responses**

**Factors**

Screening Design
11 Factors

Choose a Design

| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 12 | | Plackett-Burman | 3 - Main Effects Only |
| 16 | | Fractional Factorial | 3 - Main Effects Only |
| 16 | 8 | Fractional Factorial | 3 - Main Effects Only |
| 16 | 4 | Fractional Factorial | 3 - Main Effects Only |
| 20 | | Plackett-Burman | 3 - Main Effects Only |
| 24 | | Plackett-Burman Folded | 4 - Some 2-factor interactions |
| 32 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 32 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | | Fractional Factorial | 5 - All 2-factor interactions |
| 128 | 64 | Fractional Factorial | 5 - All 2-factor interactions |
| 128 | 32 | Fractional Factorial | 5 - All 2-factor interactions |
| 128 | 16 | Fractional Factorial | 5 - All 2-factor interactions |
| 128 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |

optional item

Continue

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# Guidelines for screening (cont'd)



## Up to 19 factors in 20 runs

DOE- Screening Design

**Screening Design**

**Responses**

**Factors**

Screening Design
19 Factors

Choose a Design

| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 20 | | Plackett-Burman | 3 - Main Effects Only |
| 24 | | Plackett-Burman | 3 - Main Effects Only |
| 32 | | Fractional Factorial | 3 - Main Effects Only |
| 32 | 16 | Fractional Factorial | 3 - Main Effects Only |
| 32 | 8 | Fractional Factorial | 3 - Main Effects Only |
| 32 | 4 | Fractional Factorial | 3 - Main Effects Only |
| 64 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 64 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |

optional item

Continue

Back

## Up to 23 factors in 24 runs

DOE- Screening Design

**Screening Design**

**Responses**

**Factors**

Screening Design
23 Factors

Choose a Design

| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 24 | | Plackett-Burman | 3 - Main Effects Only |
| 28 | | Plackett-Burman | 3 - Main Effects Only |
| 32 | | Fractional Factorial | 3 - Main Effects Only |
| 32 | 16 | Fractional Factorial | 3 - Main Effects Only |
| 32 | 8 | Fractional Factorial | 3 - Main Effects Only |
| 32 | 4 | Fractional Factorial | 3 - Main Effects Only |
| 64 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 64 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |

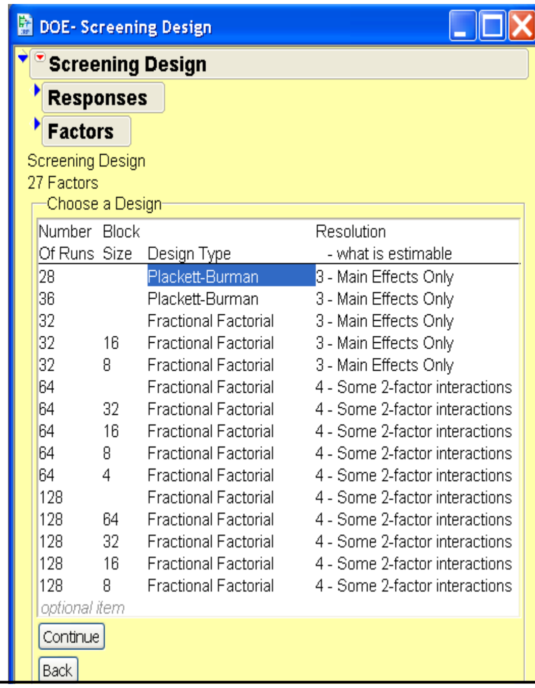
optional item

Continue

# Guidelines for screening (cont'd)



Up to 27 factors in 28 runs



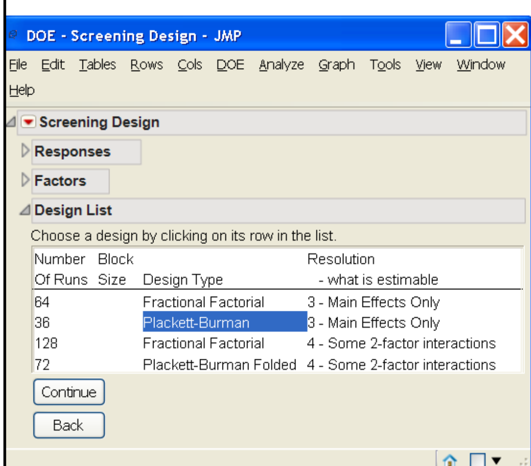


| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 28 | | Plackett-Burman | 3 - Main Effects Only |
| 36 | | Plackett-Burman | 3 - Main Effects Only |
| 32 | | Fractional Factorial | 3 - Main Effects Only |
| 32 | 16 | Fractional Factorial | 3 - Main Effects Only |
| 32 | 8 | Fractional Factorial | 3 - Main Effects Only |
| 64 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |
| 64 | 4 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 64 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 32 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 16 | Fractional Factorial | 4 - Some 2-factor interactions |
| 128 | 8 | Fractional Factorial | 4 - Some 2-factor interactions |

# Guidelines for screening (cont'd)



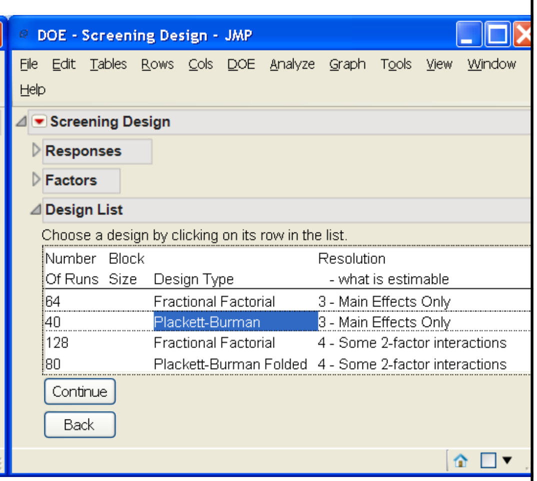
Up to 35 factors in 36 runs





| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 64 | | Fractional Factorial | 3 - Main Effects Only |
| 36 | | Plackett-Burman | 3 - Main Effects Only |
| 128 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 72 | | Plackett-Burman Folded | 4 - Some 2-factor interactions |

Up to 39 factors in 40 runs

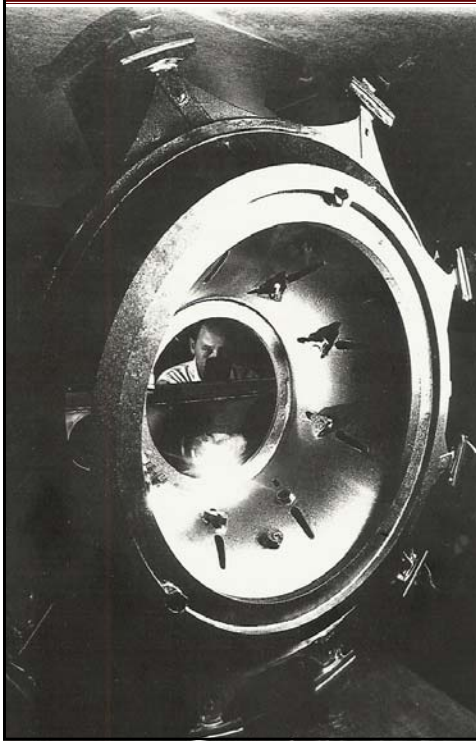




| Number Of Runs | Block Size | Design Type | Resolution - what is estimable |
|---|---|---|---|
| 64 | | Fractional Factorial | 3 - Main Effects Only |
| 40 | | Plackett-Burman | 3 - Main Effects Only |
| 128 | | Fractional Factorial | 4 - Some 2-factor interactions |
| 80 | | Plackett-Burman Folded | 4 - Some 2-factor interactions |

## Example

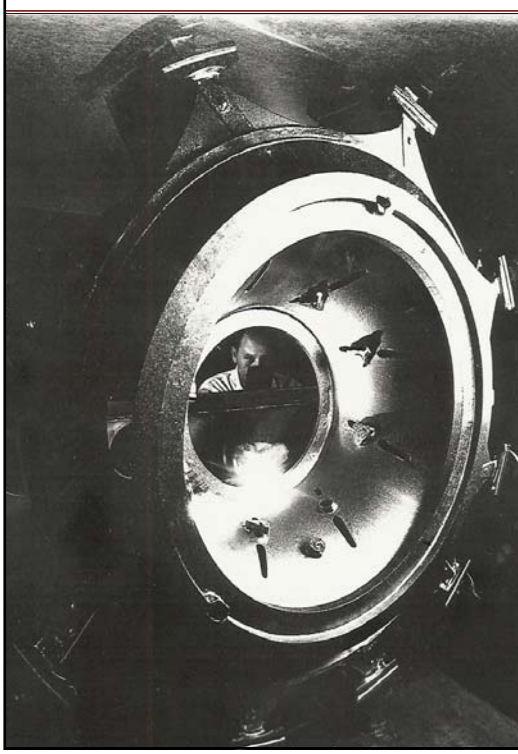
427



- Titanium castings → strong & light
- Ti develops surface oxidation during the cooling phase
- Large Ti castings were failing the customer $O_2$ requirement
- Analysis of file cabinet data yielded no significant correlations

## Example (cont'd)

428



**Black Belt**
"We should brainstorm factors for a DOE."

**Plant manager**
"We can't experiment with such an expensive part!"

**Ti metallurgist**
"The problem doesn't replicate on smaller parts."

**Part engineer**
"What have got to lose? It's been weeks since we shipped any of these!"

## Factors and levels



| Factors | | Levels | |
|---|---|---|---|
| *Shell makeup* | | | |
| | Slurry | 1 | 2 |
| | Dips | 18 | 14 |
| *Shell bake* | | | |
| | Time | 48 | 6 |
| | Temp | 2050 | 1950 |
| *Alloy* | | | |
| | Quality | Low $ | High $ |
| | Status | New | Revert |
| *Cooling* | | | |
| | Shield | SS | Mild |
| | Fan speed | 2400 | 3200 |

## Notes



Here is the list that emerges from the brainstorming session. The first factor *Slurry* is really a noise variable. One batch of shell material is made up each week, and there isn't be enough material from a single batch for the whole experiment.

The other factors are a combination of X variables in the current process and improvement ideas for the future process.

# Design matrix



- 8 factors
- Plant manager agreed to 16 castings
- For screening experiments, it doesn't matter whether factors are entered as quantitative or categorical
- Calling them all categorical allows entry of text for the levels.

→ DOE → Screening Design → Responses → Response Name → O2 → Goal → Minimize

→ Factors → 2-Level Categorical → 8 → Add → enter factor names and values → Continue

→ Design List → Number of Runs → 16 → Design Type → Fractional Factorial → Continue → Make Table

**Screening Design**

**Responses**

Add Response | Remove | N Responses...

| Response Name | Goal | Lower Limit | Upper Limit | Impor |
|---|---|---|---|---|
| O2 | Minimize | . | . | . |

**Factors**

Add [1] Continuous

Add [8] 2-Level Categorical

Add [1] 3-Level Categorical

Remove Selected

| Name | Role | Values | |
|---|---|---|---|
| SLURRY | Categorical | 1 | 2 |
| DIPS | Categorical | 18 | 14 |
| TIME | Categorical | 48 | 6 |
| TEMP | Categorical | 2050 | 1950 |
| QUALITY | Categorical | Low $ | High $ |
| STATUS | Categorical | New | Revert |
| SHIELD | Categorical | SS | Mild |
| FAN SPEED | Categorical | 2400 | 3200 |

Screening Design

Specify Factors

Add a Continuous or Categorical factor by clicking its button. Double click on a factor name or level to edit it.

Continue

# Design matrix



**Fractional Factorial**

Fractional Factorial

Design Fractional Factorial

Model

Columns (10/0)

Pattern
SLURRY ✱
DIPS ✱
TIME ✱
TEMP ✱
QUALITY ✱
STATUS ✱
SHIELD ✱
FAN SPEED ✱
O2 ✱

| | Pattern | SLURRY | DIPS | TIME | TEMP | QUALITY | STATUS | SHIELD | FAN SPEED |
|---|---|---|---|---|---|---|---|---|---|
| 1 | −+−−+−++ | 1 | 14 | 48 | 2050 | High $ | New | Mild | 3200 |
| 2 | ++−−++−− | 2 | 14 | 48 | 2050 | High $ | Revert | SS | 2400 |
| 3 | −++++−−− | 1 | 14 | 6 | 1950 | High $ | New | SS | 2400 |
| 4 | −−+−++−+ | 1 | 18 | 6 | 2050 | High $ | Revert | SS | 3200 |
| 5 | ++−+−−+− | 2 | 14 | 48 | 1950 | Low $ | New | Mild | 2400 |
| 6 | +−+−+−+− | 2 | 18 | 6 | 2050 | High $ | New | Mild | 2400 |
| 7 | −−++−−++ | 1 | 18 | 6 | 1950 | Low $ | New | Mild | 3200 |
| 8 | −++−−++− | 1 | 14 | 6 | 2050 | Low $ | Revert | Mild | 2400 |
| 9 | +++−−−−+ | 2 | 14 | 6 | 2050 | Low $ | New | SS | 3200 |
| 10 | +−++−+−− | 2 | 18 | 6 | 1950 | Low $ | Revert | SS | 2400 |
| 11 | −−−++++− | 1 | 18 | 48 | 1950 | High $ | Revert | Mild | 2400 |
| 12 | −+−+−+−+ | 1 | 14 | 48 | 1950 | Low $ | Revert | SS | 3200 |
| 13 | +−−++−−+ | 2 | 18 | 48 | 1950 | High $ | New | SS | 3200 |
| 14 | −−−−−−−− | 1 | 18 | 48 | 2050 | Low $ | New | SS | 2400 |
| 15 | +−−−−+++ | 2 | 18 | 48 | 2050 | Low $ | Revert | Mild | 3200 |
| 16 | ++++++++ | 2 | 14 | 6 | 1950 | High $ | Revert | Mild | 3200 |

- Factor combinations in coded form (FYI only)
- Perfect balance: every factor is (+) 8 times and (–) 8 times
- For every pair of factors, each +/– combination appears 4 times

# Two months (and many sleepless nights) later…



*LSSV2 data sets \ Ti casting alpha case*

Ti casting alpha case - JMP

File Edit Tables Rows Cols DOE Analyze Graph Tools View Window Help

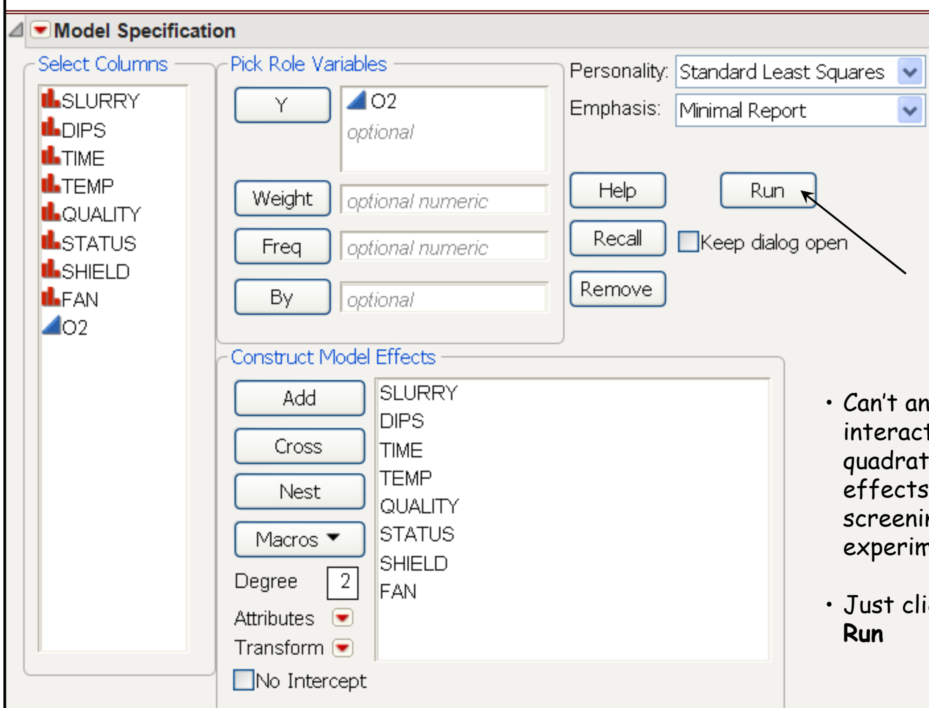
Ti casting alph…
Design Custom Desig
Model

Columns (9/0)
SLURRY
DIPS
TIME
TEMP
QUALITY
STATUS
SHIELD
FAN
O2

| | SLURRY | DIPS | TIME | TEMP | QUALITY | STATUS | SHIELD | FAN | O2 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 14 | 48 | 2050 | Low $ | New | SS | 2400 | 91 |
| 2 | 1 | 14 | 48 | 2050 | High $ | Revert | Mild | 3200 | 191 |
| 3 | 1 | 14 | 6 | 1950 | Low $ | Revert | Mild | 2400 | 90 |
| 4 | 1 | 14 | 6 | 1950 | High $ | New | SS | 3200 | 76 |
| 5 | 1 | 18 | 48 | 1950 | Low $ | New | Mild | 3200 | 132 |
| 6 | 1 | 18 | 48 | 1950 | High $ | Revert | SS | 2400 | 184 |
| 7 | 1 | 18 | 6 | 2050 | High $ | New | Mild | 2400 | 144 |
| 8 | 1 | 18 | 6 | 2050 | Low $ | Revert | SS | 3200 | 197 |
| 9 | 2 | 14 | 48 | 1950 | Low $ | Revert | SS | 3200 | 128 |
| 10 | 2 | 14 | 48 | 1950 | High $ | New | Mild | 2400 | 174 |
| 11 | 2 | 14 | 6 | 2050 | High $ | Revert | SS | 2400 | 166 |
| 12 | 2 | 14 | 6 | 2050 | Low $ | New | Mild | 3200 | 255 |
| 13 | 2 | 18 | 48 | 2050 | Low $ | Revert | Mild | 2400 | 186 |
| 14 | 2 | 18 | 48 | 2050 | High $ | New | SS | 3200 | 318 |
| 15 | 2 | 18 | 6 | 1950 | High $ | Revert | Mild | 3200 | 111 |
| 16 | 2 | 18 | 6 | 1950 | Low $ | New | SS | 2400 | 213 |

# The model dialog






- Can't analyze interactive or quadratic effects in a screening experiment
- Just click on **Run**

# Analysis



Red triangle

Effect Screening
↓
Pareto Plot

**Response O2**

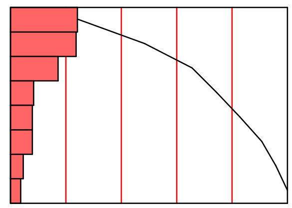
**Effect Screening**

The parameter estimates have equal variances.
The parameter estimates are not correlated.

| | Lenth PSE |
|---|---|
| t-Test Scale | 0.9741692 |
| Coded Scale | 14.625 |

**Pareto Plot of Estimates**

| Term | Estimate |
|---|---|
| SLURRY[1] | -27.87500 |
| TEMP[1950] | -27.50000 |
| DIPS[14] | -19.62500 |
| FAN[2400] | -10.00000 |
| TIME[48] | 9.50000 |
| STATUS[New] | 9.37500 |
| SHIELD[Mild] | -5.62500 |
| QUALITY[High $] | 4.50000 |



**"Big hitters"**
SLURRY
TEMP
DIPS

*Oops…SLURRY is a noise factor!*

## Notes



To interpret screening experiments, we use the *Effects Screening* analysis element as shown above. It gives a Pareto chart showing the relative magnitude of the factor effects. The idea is to use the factors with the largest effects in a subsequent optimization experiment.

The P-values from a screening experiment are not to be trusted. The interactive and quadratic effects left out of the model artificially increase the noise in the analysis. This biases the signal-to-noise ratios downward, so factors appear less significant than they really are.

The three largest effects in the example are SLURRY, TEMP and DIPS. The result for SLURRY was perplexing. There was no doubt that the castings made from shells made from Slurry 1 looked a lot better than those from Slurry 2, but the shell making operators said there were no differences in how the slurries were made up.

## Ideal follow-up plan

437

- Do a screening experiment in the shell-making area
- Include TEMP, DIPS and the important shell-making variables in an optimization experiment

## What actually happened

438

- They changed TEMP to 1950 and DIPS to 14 (easy)
- The problem immediately went away
- 13 of the 16 DOE castings were good to ship as is
- Only 1 eventually scrapped
- Worst-case annual cost avoidance:  $20.8M
- No immediate follow-up

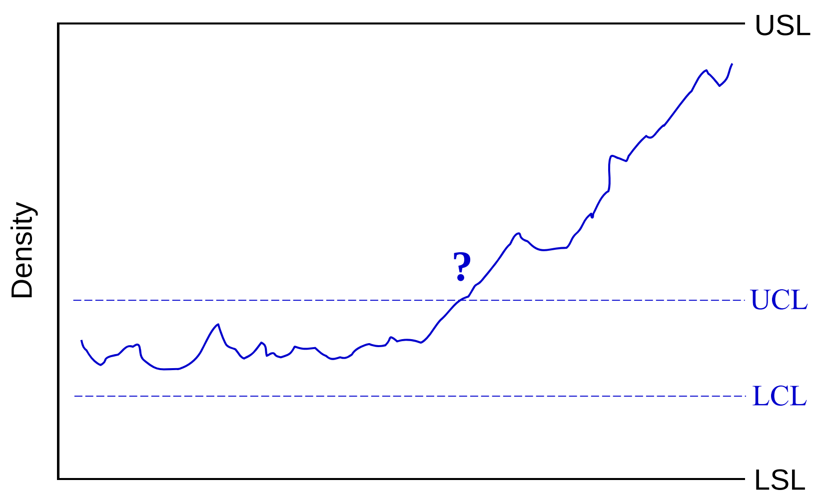
## Root causes



- Investigation of the slurry effect eventually lead to the root cause of the problem
  - → The density of the ceramic powder used to make the shell had increased over time, resulting in heavier shells
  - → The increase had been noted, but no action was taken because the densities were still within spec limits
  - → At the time, shell weights were not monitored
- Why no significant correlations in the "file cabinet" data?
  - → The $O_2$ data in the engineering database was final rather than first pass

## Control limits vs. spec limits






- The data was trying to tell us something
- Disaster could have been averted

## Exercise 25.1



(a) Apply the screening experiment guidelines to create a screening design matrix for the example shown on the slide below. Ask the instructor to review it before you proceed.

(b) Open *extrusion 0.jmp*. Did these experimenters follow the guidelines?

(c) Based on the results for STRENGTH and DUCTILITY, find the best set of 4 factors for a subsequent optimization experiment.

## Exercise 25.1 (cont'd)



| | *Factors* | *Feasible ranges* |
|---|---|---|
| Polymer | | |
| | Smoother | 0.0 to 0.5 |
| | Filler | 2.0 to 4.0 |
| | Viscosity | 60 to 80 |
| | Moisture | 0.1 to 0.25 |
| Process | | |
| | Zone 1 temp | 260 to 320 |
| | Zone 2 temp | 260 to 320 |
| | Zone 3 temp | 260 to 320 |
| | Zone 4 temp | 260 to 320 |
| | Rate | 100 to 200 |
| | RPM | 150 to 300 |

Responses are *strength* and *ductility* of the extrudate

## 26 Simple Regression with Pass/fail Y

a) Raw data — each row represents one part or transaction

b) Tabulated data — each row represents multiple parts or transactions



## Raw data

| 2/0 Cols / 25/0 | Target speed | Result |
|---|---|---|
| 1 | 200 | Hit |
| 2 | 205 | Hit |
| 3 | 210 | Hit |
| 4 | 215 | Hit |
| 5 | 220 | Hit |
| 6 | 225 | Miss |
| 7 | 230 | Hit |
| 8 | 235 | Hit |
| 9 | 240 | Miss |
| 10 | 245 | Hit |
| 11 | 250 | Hit |
| 12 | 255 | Hit |
| 13 | 260 | Hit |
| 14 | 265 | Miss |
| 15 | 270 | Miss |
| 16 | 275 | Hit |
| 17 | 280 | Miss |
| 18 | 285 | Miss |
| 19 | 290 | Miss |
| 20 | 295 | Miss |
| 21 | 300 | Hit |
| 22 | 305 | Miss |
| 23 | 310 | Miss |
| 24 | 315 | Miss |
| 25 | 320 | Miss |

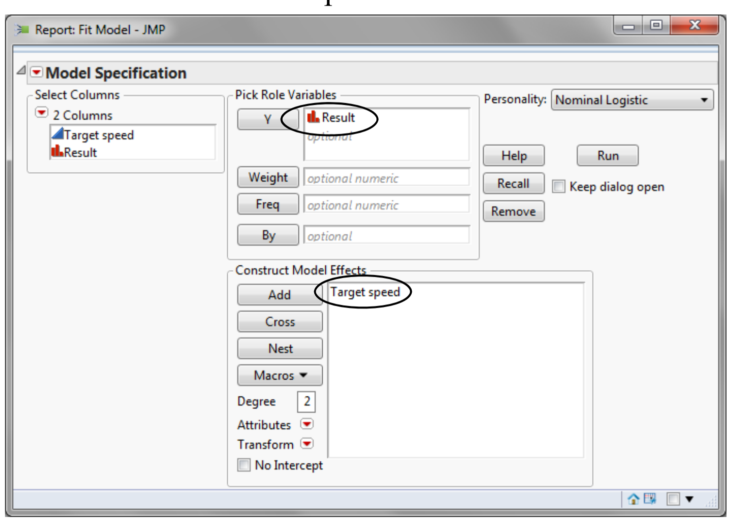
Open *LSSV2 data sets \ target practice* (in JMP)

↓

Fit Model

↓

Set up as shown

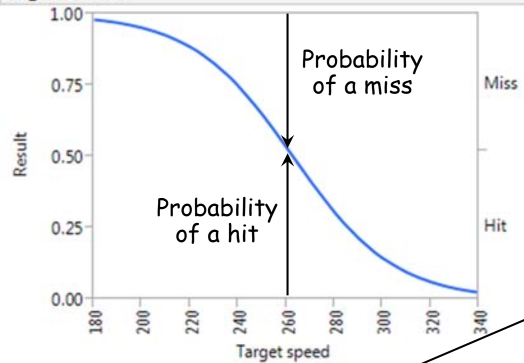


## Analysis output



**Nominal Logistic Fit for Result**

**Logistic Plot**




Converged in Gradient, 5 iterations

**Parameter Estimates**

| Term | Estimate | Std Error | ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Intercept | 12.5297022 | 4.7931154 | 6.83 | 0.0089* |
| Target spee | -0.0476836 | 0.0181939 | 6.87 | 0.0088* |

For log odds of Hit/Miss

**Effect Likelihood Ratio Tests**

| Source | Nparm | DF | L-R ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Target spee | 1 | 1 | 11.1939322 | 0.0008 |

- Model parameters
- P value for correlation (ignore the one labeled *Target speed* — it is not as accurate)
- Very strong evidence of a negative correlation between the speed of a target and the probability of hitting it (big surprise)

## Model equation



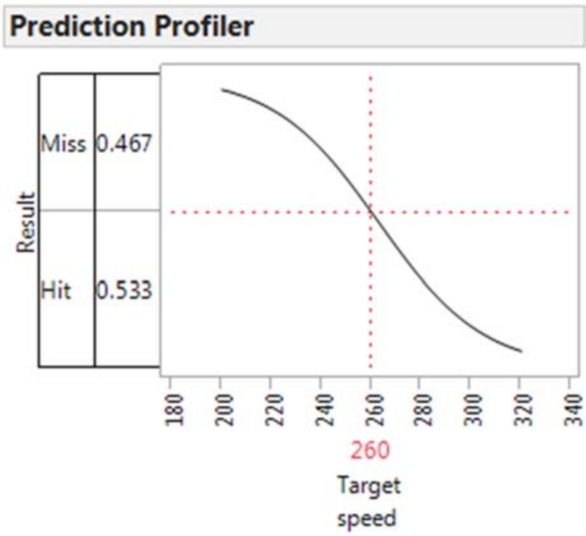
- The Y variable here is “Hit” or “Miss”
- The plotted curve gives the probability of hitting a target as a function of its speed

$$\text{Prob(Hit)} = \frac{1}{1+\exp\left(-12.53+0.048(\text{Target speed})\right)}$$

- One minus the curve gives the probability of missing the target as a function of its speed
- These model equations are nonlinear, and bounded between 0 and 1
- If a straight line model were used, it would produce ridiculous extrapolations (probabilities larger than 1 or less than 0)

## The prediction profiler






Calculates the hit/miss probabilities for any given speed

## Exercise 26.1



Open *LSSV2 data sets \ quotation process.jmp*.

a) Go to *Column Properties* for PO, select *Value Ordering* → *Reverse* → OK.

b) Fit *PO* by *TAT*. Give and interpret the P value for TAT.

c) Use the profiler to find the PO hit rates for 3 day turnarounds and 15 day turnarounds.

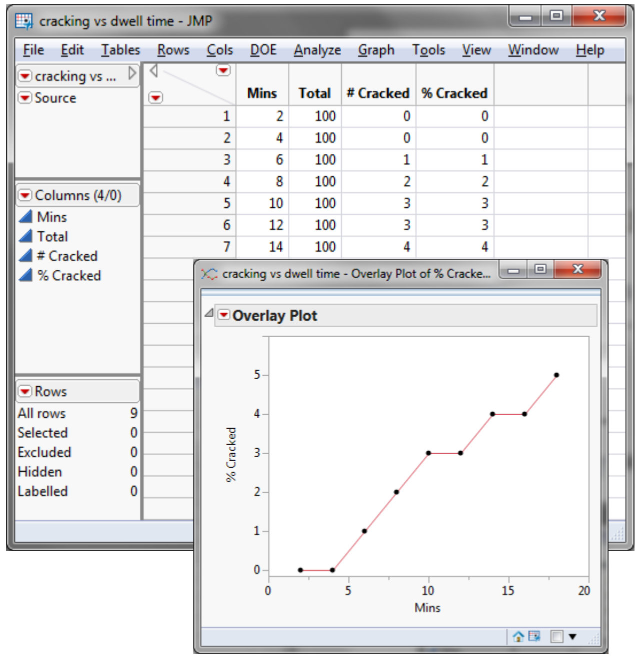
d) Save your script, close and save the data table.

## Tabulated data

449

Open *LSSV2 data sets \ cracking vs dwell time* (in JMP)

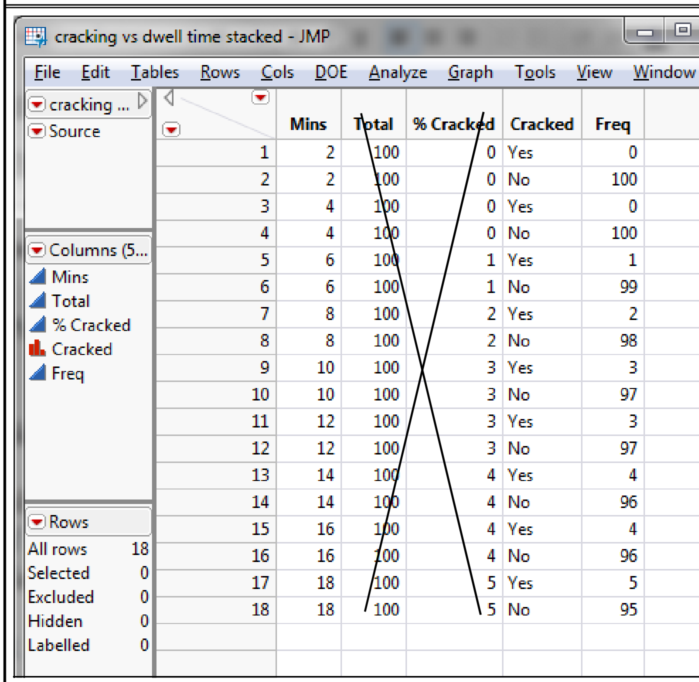
1. Make a plot of *% Cracked* by *Mins*
2. Rename *# Cracked* as *Yes*
3. Create a new column called *No* defined as *Total – Yes* (*Column Properties → Formula*)
4. Tables → Stack
3. Use *Yes* and *No* as the *Stack Columns*
4. Change *Label* to *Cracked*, *Data* to *Freq*
5. Save as *cracking vs dwell time stacked*




| | Mins | Total | # Cracked | % Cracked |
|---|---|---|---|---|
| 1 | 2 | 100 | 0 | 0 |
| 2 | 4 | 100 | 0 | 0 |
| 3 | 6 | 100 | 1 | 1 |
| 4 | 8 | 100 | 2 | 2 |
| 5 | 10 | 100 | 3 | 3 |
| 6 | 12 | 100 | 3 | 3 |
| 7 | 14 | 100 | 4 | 4 |

## Stacked format

450



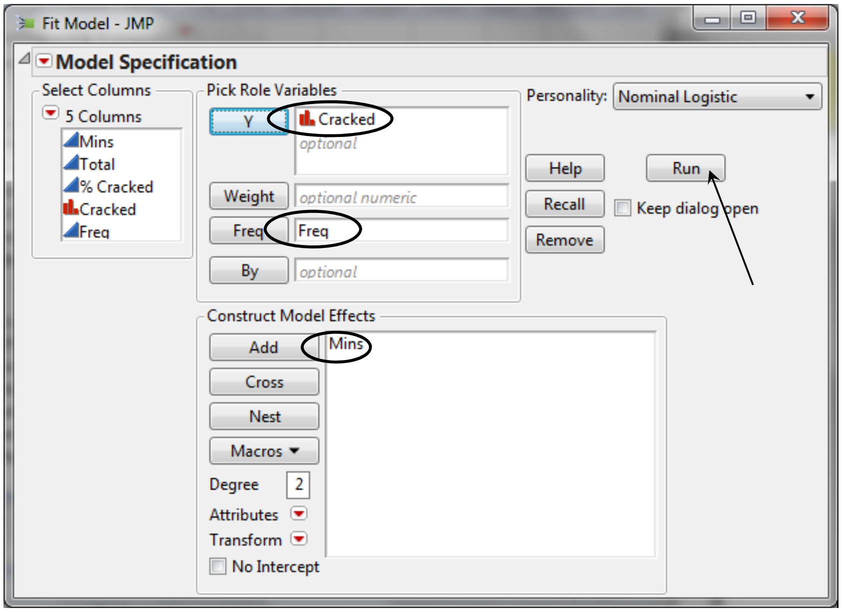

| | Mins | Total | % Cracked | Cracked | Freq |
|---|---|---|---|---|---|
| 1 | 2 | 100 | 0 | Yes | 0 |
| 2 | 2 | 100 | 0 | No | 100 |
| 3 | 4 | 100 | 0 | Yes | 0 |
| 4 | 4 | 100 | 0 | No | 100 |
| 5 | 6 | 100 | 1 | Yes | 1 |
| 6 | 6 | 100 | 1 | No | 99 |
| 7 | 8 | 100 | 2 | Yes | 2 |
| 8 | 8 | 100 | 2 | No | 98 |
| 9 | 10 | 100 | 3 | Yes | 3 |
| 10 | 10 | 100 | 3 | No | 97 |
| 11 | 12 | 100 | 3 | Yes | 3 |
| 12 | 12 | 100 | 3 | No | 97 |
| 13 | 14 | 100 | 4 | Yes | 4 |
| 14 | 14 | 100 | 4 | No | 96 |
| 15 | 16 | 100 | 4 | Yes | 4 |
| 16 | 16 | 100 | 4 | No | 96 |
| 17 | 18 | 100 | 5 | Yes | 5 |
| 18 | 18 | 100 | 5 | No | 95 |

The **Total** and **% Cracked** columns are no longer relevant — you may delete them if you wish

Go to *Column Properties* for *Cracked*
↓
*Value Ordering*
↓
*Reverse*
↓
OK
↓
Analyze
↓
Fit Model
↓
See next slide
↓
Set as shown

# Fit Model



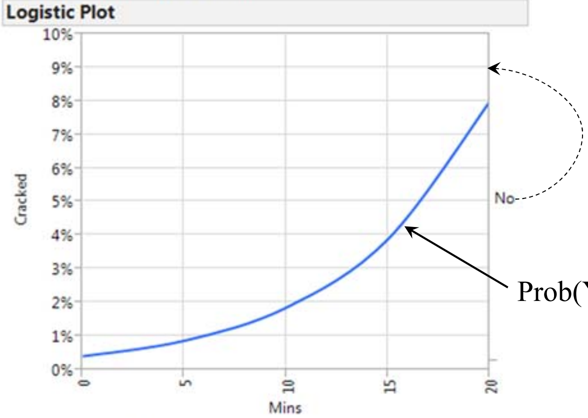



# Analysis output (edited)



**Nominal Logistic Fit for Cracked**

**Logistic Plot**




$$\text{Prob(Yes)} = \frac{1}{1 + \exp[5.49 - 0.152(\text{Mins})]}$$

Converged in Gradient, 7 iterations

Freq: Freq

**Parameter Estimates**

| Term | Estimate | Std Error | ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Intercept | -5.4880044 | 0.7001671 | 61.44 | <.0001* |
| Mins | 0.15230995 | 0.0494273 | 9.50 | 0.0021* |

For log odds of Yes/No

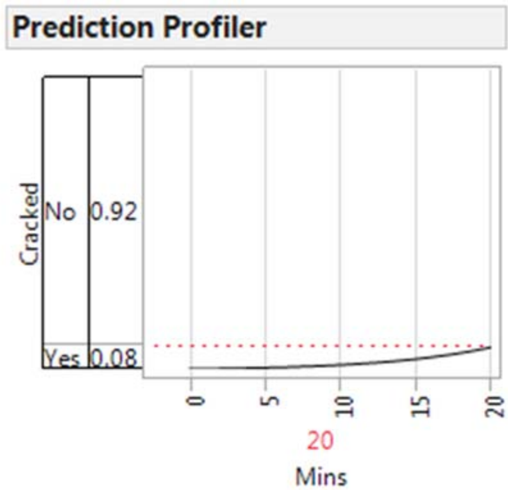
**Effect Likelihood Ratio Tests**

| Source | Nparm | DF | L-R ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Mins | 1 | 1 | 11.2187549 | 0.0008* |

- Model parameters
- P value for correlation
- Very strong evidence of a positive correlation between dwell time and the probability of cracking

## Prediction profiler






| Dwell time (mins) | Probability of cracking |
|---|---|
| 0 | 0.4% |
| 5 | 0.9% |
| 10 | 1.9% |
| 15 | 3.9% |
| 20 | 8.0% |

## Notes

# 27 Multiple Regression with Pass/fail Y



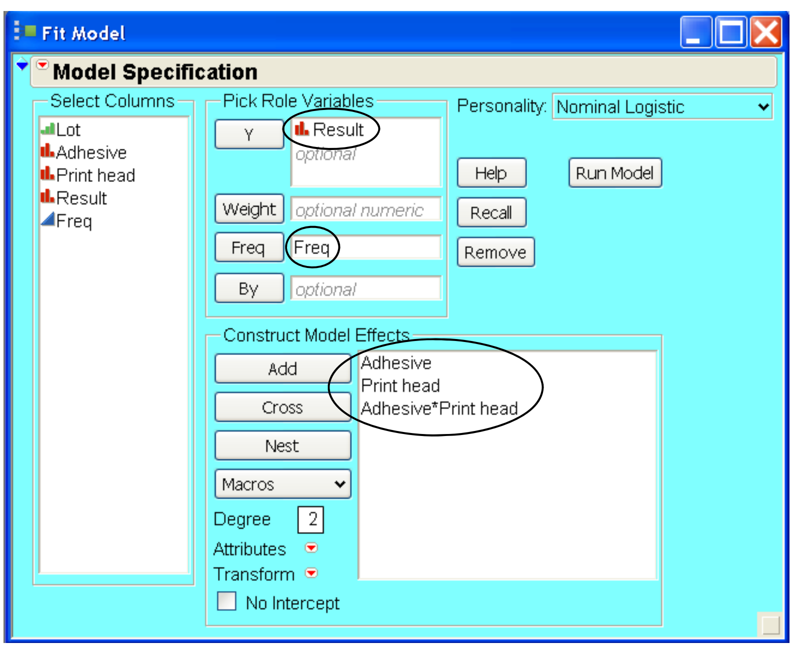
- Project to reduce clogged nozzles in print heads
- Comparison of four types of adhesive and two print head designs
- Each lot = 60 print cartridges
- “Pass” = no customer detectable print defects
- Open *LSSV2 data sets \ clogging pass fail*

| 5/0 Cols 32/0 | Lot | Adhesive | Print head | Result | Freq |
|---|---|---|---|---|---|
| 1 | 1 | A4 | D2 | Fail | 2 |
| 2 | 1 | A4 | D2 | Pass | 58 |
| 3 | 2 | A4 | D1 | Fail | 1 |
| 4 | 2 | A4 | D1 | Pass | 59 |
| 5 | 3 | A2 | D2 | Fail | 13 |
| 6 | 3 | A2 | D2 | Pass | 47 |
| 7 | 4 | A1 | D2 | Fail | 11 |
| 8 | 4 | A1 | D2 | Pass | 49 |
| 9 | 5 | A3 | D2 | Fail | 4 |
| 10 | 5 | A3 | D2 | Pass | 56 |
| 11 | 6 | A4 | D1 | Fail | 5 |
| 12 | 6 | A4 | D1 | Pass | 55 |
| 13 | 7 | A1 | D2 | Fail | 8 |
| 14 | 7 | A1 | D2 | Pass | 52 |
| 15 | 8 | A2 | D1 | Fail | 3 |
| 16 | 8 | A2 | D1 | Pass | 57 |
| 17 | 9 | A3 | D2 | Fail | 1 |
| 18 | 9 | A3 | D2 | Pass | 59 |
| 19 | 10 | A2 | D2 | Fail | 13 |
| 20 | 10 | A2 | D2 | Pass | 47 |
| 21 | 11 | A2 | D1 | Fail | 1 |
| 22 | 11 | A2 | D1 | Pass | 59 |
| 23 | 12 | A1 | D1 | Fail | 1 |
| 24 | 12 | A1 | D1 | Pass | 59 |
| 25 | 13 | A3 | D1 | Fail | 7 |

# Example (cont’d)



*Analyze → Fit Model*



# Example (cont'd)



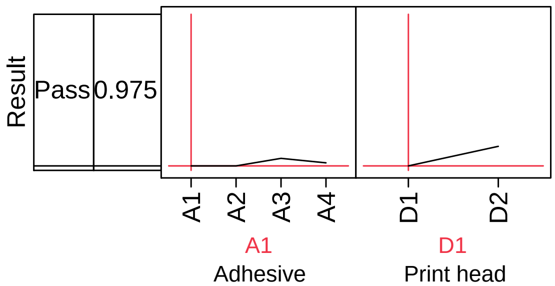
## Nominal Logistic Fit for Result

Freq: Freq

### Effect Likelihood Ratio Tests

| Source | Nparm | DF | L-R ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Adhesive | 3 | 3 | 3.01536018 | 0.3893 |
| Print head | 1 | 1 | 7.68556625 | 0.0056* |
| Adhesive*Print head | 3 | 3 | 19.7623238 | 0.0002* |

### Prediction Profiler




- Reverse *Value Ordering* property for *Result* so that "Pass" probability is plotted
- Remove insignificant *Adhesive* term
- Run model again

# Example (cont'd)



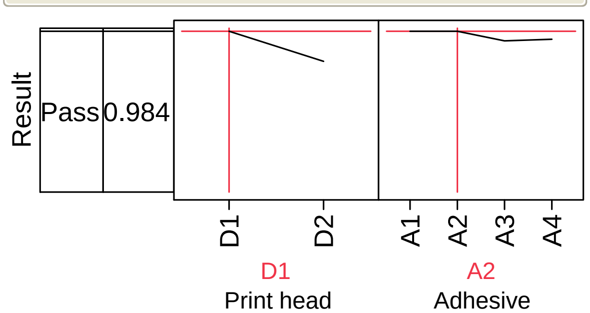
## Nominal Logistic Fit for Result

Freq: Freq

### Effect Likelihood Ratio Tests

| Source | Nparm | DF | L-R ChiSquare | Prob>ChiSq |
|---|---|---|---|---|
| Print head | 1 | 1 | 16.2058108 | <.0001* |
| Adhesive*Print head | 3 | 3 | 29.2140908 | <.0001* |

### Prediction Profiler




- Best combination is D1 with A2
- Baseline failure rate was > 20%
- Predicted failure rate is < 2%

## Exercise 27.1



A Black Belt wants to minimize the occurrence of bubbles and ripples in the urethane coating on truck nameplates. The X variables and ranges are:

- Badge temp: 20 to 40
- Mixing ratio: 92.6 to 94.6
- Curing temp: 30 to 55

- Open *LSSV2 data sets \ urethane coating pass-fail*
- Reverse the *Value Ordering* properties for *Result* so that the “Pass” probability will be plotted on the *Prediction Profiler*.
- Run the *Model* script provided in the left panel, run the model.
- Remove insignificant terms from the model (P > 0.15), then re-run the model.
- Use the *Prediction Profiler* to find a factor combination that maximizes the yield.

## Exercise 27.1 (cont’d)



- The baseline yield was about 95%. What is the predicted yield for the improved process?

# 28 Sample Size for DOE with Pass/fail Y



$$\text{Current state fraction defective}: \quad \phi_{current} \quad (0 < \phi_{current} < 1)$$

$$\text{Future state objective}: \quad \phi_{future} \quad (0 < \phi_{future} < \phi_{current})$$

$$\text{Midpoint}: \quad \phi_{mid} = \frac{\phi_{current} + \phi_{future}}{2}$$

$$\text{Batch size defining the experiment unit}: \quad n_0$$

$$\text{RMSE}: \quad \sqrt{\frac{\phi_{mid}(1-\phi_{mid})}{n_0}}$$

$$\text{DTD}: \quad \phi_{current} - \phi_{future}$$

$$\text{Required AVP}: \quad \frac{1}{8}\left(\frac{\text{DTD}}{\text{RMSE}}\right)^2$$

## Notes



The Greek letter $\phi$ (phi) is used here for the population fraction defective, not to be confused with the fraction defective in a particular sample.

When a pass-fail variable is numerically coded as 0 and 1, it has a standard deviation that depends on $\phi$. This means we need to do a little work on the side to get a correct input value for RMSE. The required calculation for RMSE is shown above. Once RMSE is determined, the rest of the sample size calculation is done the same way as for quantitative Y.

The assumption here is that the experiment unit consists of a batch of $n_0$ items to be processed and tested. All items in a given batch should experience the same factor levels, and move through the process at the same time. The design team must select the batch size $n_0$. If $n_0 = 1$, then the sample size N obtained from JMP is the total number of items tested for the experiment. If $n_0 \geq 2$, then N is the number of rows in the design matrix, but the total number of items tested for the experiment is $N \times n_0$.

## Example



Current state fraction defective : 0.054

Future state objective : 0.01

Midpoint : 0.032

Batch size defining the experiment unit : 20

$$\text{RMSE}: \quad \sqrt{\frac{0.032(0.968)}{20}} = 0.039355$$

$$\text{DTD}: \quad 0.054 - 0.01 = 0.044$$

$$\text{AVP}: \quad \frac{1}{8}\left(\frac{0.044}{0.039355}\right)^2 = 0.156248$$

## Notes



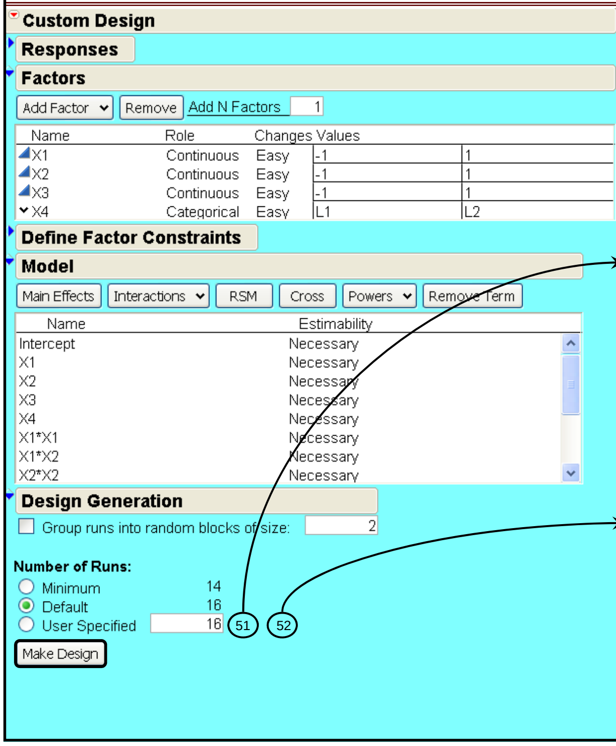
In the example shown above, the project goal is to reduce the current 5.4% failure rate to 1%. The parts in question were routinely processed in batches of 20, so 20 was a good choice for $n_0$.

The top slide on the next page shows the *Custom Design* “mock up” for the same factors as in the previous example, and the AVPs computed for sample sizes 51 and 52. The AVP is greater than our calculated AVP (0.156248) for N = 51 and less than that for N = 52, so we will go with N =52. The design matrix will have 52 rows, the total number of parts produced and tested will be 52 × 20 = 1040.

The sample size requirements for pass-fail Y are much greater than for continuous Y. This is an unpleasant statistical fact of life. People usually experience “sticker shock” the first few times they do sample size calculations for pass-fail Y. The only solution is to develop quantitative measurements for the quantities of interest.

# Example (cont'd)






Design Diagnostics

| I Optimal Design | |
|---|---|
| D Efficiency | 49.76708 |
| G Efficiency | 80.04761 |
| A Efficiency | 38.24845 |
| Average Variance of Prediction | 0.156398 |
| Design Creation Time (seconds) | 0.216667 |

Design Diagnostics

| I Optimal Design | |
|---|---|
| D Efficiency | 50.08028 |
| G Efficiency | 81.49717 |
| A Efficiency | 38.4207 |
| Average Variance of Prediction | 0.153914 |
| Design Creation Time (seconds) | 0.2 |

# Exercise 28.1



We are planning an experiment to optimize an ultrasonic welding process with 3 quantitative factors and a 4-level categorical factor. In addition to *weld depth*, a second response variable of interest is whether or not the welded part passes a leak test. We want to do a sample size calculation based on the following information:

- About 9% of welded assemblies in current production fail the leak test.
- Our goal is to reduce this to 3% leaking.
- The parts in question are routinely processed in totes of 20.

Open *LSSV2 other stuff \ sample size calculator*, go the sheet *AVP for DOE (pass-fail Y)*. Enter the information given above to get the required AVP. Use this to determine the required sample size. Find the total number of parts that need to be welded and tested for this experiment.

# 29 Reformatting Data for Pareto Analysis



- Data on defect types or failure reasons often is available only in tabulated form
- Each row may represent a production lot, work order, time period, machine work center, part number, . . . , or some combination thereof
- Common problem with tabulated data: wrong format for Pareto analysis

# Big example: *molding process - Pareto*



Each row = Date, Machine, P/N, . . . ?

Total parts run = Good + Bad

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Date | Machine | P/N | Primary material | Primary lot # | Concentrate | Concen lot # | Regrind type | Parts palletized | Total defective |
| 2 | 03-Apr-06 | 9 | LSGV0101 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 120 | 7 |
| 3 | 03-Apr-06 | 11 | LSGY0251 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 300 | 17 |
| 4 | 03-Apr-06 | 15 | LSGV0098 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 372 | 18 |
| 5 | 04-Apr-06 | 2 | LSGV0093 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 288 | 6 |
| 6 | 04-Apr-06 | 9 | LSGV0101 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 600 | 2 |
| 7 | 04-Apr-06 | 11 | LSGY0251 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 690 | 33 |
| 8 | 04-Apr-06 | 13 | LSGY0307 | CHEIL HF1690H LtGry | 133232 | NA | NA | NA | 160 | 8 |
| 9 | 04-Apr-06 | 15 | LSGV0098 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 624 | 0 |
| 10 | 05-Apr-06 | 2 | LSGV0102 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 120 | 15 |
| 11 | 05-Apr-06 | 11 | LSGY0251 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 650 | 21 |
| 12 | 05-Apr-06 | 13 | LSGY0252 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 300 | 18 |
| 13 | 05-Apr-06 | 13 | LSGY0307 | CHEIL VE-1877S LtGry | 133232 | NA | NA | NA | 160 | 0 |
| 14 | 05-Apr-06 | 14 | LSGY0308 | CHEIL HF1690H LtGry | 133232 | NA | NA | NA | 240 | 25 |
| 15 | 05-Apr-06 | 15 | LSGV0098 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 336 | 17 |
| 16 | 06-Apr-06 | 2 | LSGV0102 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 780 | 0 |
| 17 | 06-Apr-06 | 11 | LSGY0251 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 600 | 7 |
| 18 | 06-Apr-06 | 13 | LSGY0252 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 500 | 49 |
| 19 | 06-Apr-06 | 14 | LSGV0130 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 108 | 34 |
| 20 | 06-Apr-06 | 15 | LSGV0099 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 276 | 95 |
| 21 | 07-Apr-06 | 2 | LSGV0102 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 300 | 0 |
| 22 | 07-Apr-06 | 2 | LSGV0102 | CHEIL VE-1877S DrkGry | 121642 | NA | NA | 25 | 1020 | 5 |
| 23 | 07-Apr-06 | 11 | LSGY0251 | CHEIL VE-1877S Blk | 101200 | NA | NA | NA | 360 | 6 |
| 24 | 07-Apr-06 | 13 | LSGY0252 | CHEIL VE-1877S Blk | 387487 | NA | NA | NA | 200 | 16 |
| 25 | 07-Apr-06 | 13 | LSGY0252 | CHEIL VE-1877S Blk | 387487 | NA | NA | NA | 700 | 7 |
| 26 | 07-Apr-06 | 14 | LSGV0130 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 72 | 0 |
| 27 | 07-Apr-06 | 14 | LSGV0131 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 120 | 17 |
| 28 | 07-Apr-06 | 15 | LSGV0099 | CHEIL HF1690H DrkGry | 122930 | NA | NA | 8 | 180 | 0 |

# Big example (cont'd)



Total defective × Cost per pc.

| | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z | AA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Cost per pc. | Total cost | Start-up | Sink | Flash | Weld line | Flow mark | Short shot | Warp | Burn marks | Silver | Gas marks | Color/ carbon | Oil | Broken part | Scratches | Bubbles |
| 2 | $2.89 | $20.25 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | $5.08 | $86.43 | 4 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 |
| 4 | $11.10 | $199.76 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | $2.69 | $16.12 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | $2.89 | $5.79 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | $5.08 | $167.77 | 0 | 4 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| 8 | $3.55 | $28.44 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | $11.10 | $0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | $4.13 | $62.00 | 6 | 6 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | $5.08 | $106.76 | 0 | 17 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 12 | $4.96 | $89.28 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 9 |
| 13 | $3.55 | $0.00 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0 |
| 14 | $8.97 | $224.36 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0 |
| 15 | $11.10 | $188.66 | 0 | 0 | 0 | 0 | 0 | 12 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | $4.13 | $0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | $5.08 | $35.59 | 0 | 2 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 18 | $4.96 | $243.04 | 3 | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 27 |
| 19 | $10.33 | $351.07 | 8 | 0 | 0 | 0 | 0 | 14 | 0 | 0 | 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | $14.19 | $1,347.62 | 56 | 30 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | $4.13 | $0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | $4.13 | $20.67 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | $5.08 | $30.50 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 24 | $4.96 | $79.36 | 0 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 25 | $4.96 | $34.72 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 |
| 26 | $10.33 | $0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | $15.15 | $257.56 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 8 | 0 | 0 | 0 |
| 28 | $14.19 | $0.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Counts for each type of defect

## Notes



One of the things we would want from a data set like this is a Pareto breakdown of defect types by frequency of occurrence. For this, we need to calculate the total number of defective parts for each defect type. With the format shown above, we cannot do this by means of a pivot table. As an alternative, we could calculate the totals for the columns representing the defect types. However, compared to a pivot table, this method is extremely tedious for doing anything else, such as comparing Pareto breakdowns for stratifications of the data set.

Another thing we would want from a data set like this is a Pareto breakdown of defect types by total cost. It is not impossible to do this with the format shown above, but, once again, it would be extremely tedious compared to a pivot table.

## Small example

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Open *molding process - small* (in JMP)

| 7/0 Cols / 3/0 Rows | Total defective | Cost per pc. | Total cost | Start-up | Short shot | Silver | Bubbles |
|---|---|---|---|---|---|---|---|
| 1 | 7 | 3 | 21 | 3 | 0 | 4 | 0 |
| 2 | 17 | 5 | 85 | 4 | 4 | 0 | 9 |
| 3 | 18 | 11 | 198 | 0 | 6 | 12 | 0 |

↑
This is what we have

This is what we need →

| 4/0 Cols / 12/0 | Cost per pc. | Defect | Freq | Total cost |
|---|---|---|---|---|
| 1 | 3 | Start-up | 3 | 9 |
| 2 | 3 | Short shot | 0 | 0 |
| 3 | 3 | Silver | 4 | 12 |
| 4 | 3 | Bubbles | 0 | 0 |
| 5 | 5 | Start-up | 4 | 20 |
| 6 | 5 | Short shot | 4 | 20 |
| 7 | 5 | Silver | 0 | 0 |
| 8 | 5 | Bubbles | 9 | 45 |
| 9 | 11 | Start-up | 0 | 0 |
| 10 | 11 | Short shot | 6 | 66 |
| 11 | 11 | Silver | 12 | 132 |
| 12 | 11 | Bubbles | 0 | 0 |

→ How do we get there?

## Stacking a data table

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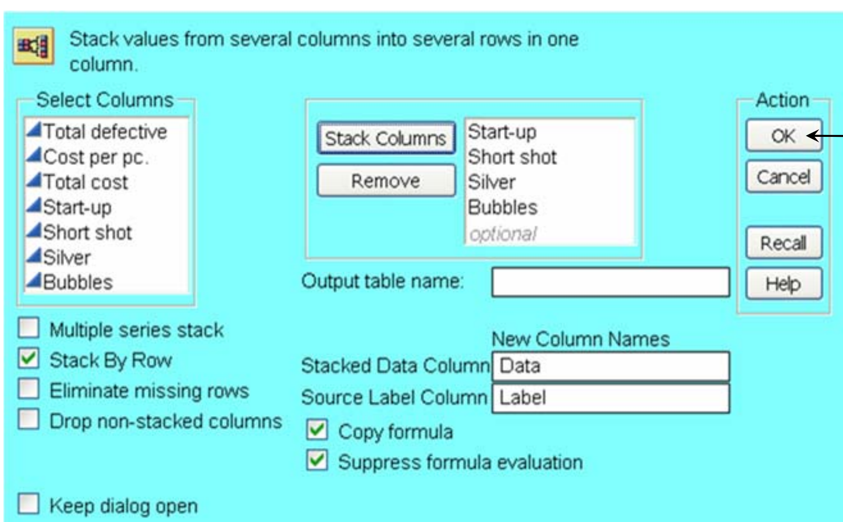
*Tables* → *Stack* → Select the defect columns as the *Stack Columns*



## Editing the columns

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| 12/0 | Total defective | Cost per pc. | Total cost | Label | Data |
|---|---|---|---|---|---|
| 1 | 7 | 3 | 21 | Start-up | 3 |
| 2 | 7 | 3 | 21 | Short shot | 0 |
| 3 | 7 | 3 | 21 | Silver | 4 |
| 4 | 7 | 3 | 21 | Bubbles | 0 |
| 5 | 17 | 5 | 85 | Start-up | 4 |
| 6 | 17 | 5 | 85 | Short shot | 4 |
| 7 | 17 | 5 | 85 | Silver | 0 |
| 8 | 17 | 5 | 85 | Bubbles | 9 |
| 9 | 18 | 11 | 198 | Start-up | 0 |
| 10 | 18 | 11 | 198 | Short shot | 6 |
| 11 | 18 | 11 | 198 | Silver | 12 |
| 12 | 18 | 11 | 198 | Bubbles | 0 |

*Total defective* and *Total cost* are now incorrect row by row

1. Right-click on *Data*
2. Select *Column Info*
3. Rename as *Freq* → OK
4. Rename *Label* as *Defect type*
5. Delete *Total defective*
6. Right-click on *Total cost*
7. Select *Formula* → *Cost per pc.***Freq*
8. Save as *molding data small stacked.xls*

| 12/0 | Cost per pc. | Total cost | Defect type | Freq |
|---|---|---|---|---|
| 1 | 3 | 9 | Start-up | 3 |
| 2 | 3 | 0 | Short shot | 0 |
| 3 | 3 | 12 | Silver | 4 |
| 4 | 3 | 0 | Bubbles | 0 |
| 5 | 5 | 20 | Start-up | 4 |
| 6 | 5 | 20 | Short shot | 4 |
| 7 | 5 | 0 | Silver | 0 |
| 8 | 5 | 45 | Bubbles | 9 |
| 9 | 11 | 0 | Start-up | 0 |
| 10 | 11 | 66 | Short shot | 6 |
| 11 | 11 | 132 | Silver | 12 |
| 12 | 11 | 0 | Bubbles | 0 |

## Pareto plot by frequency

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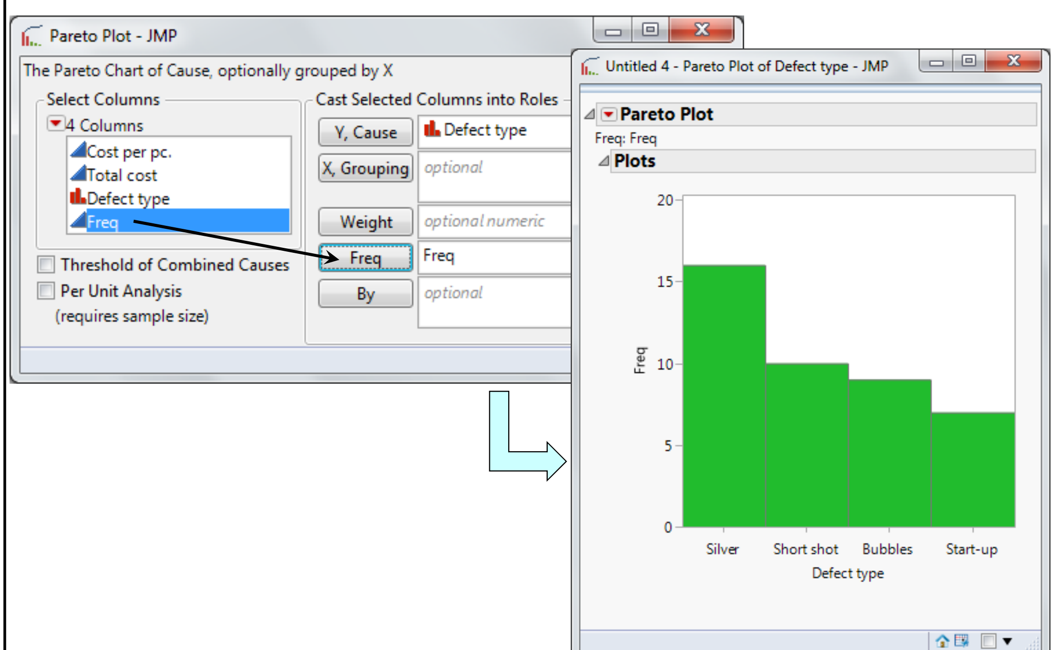
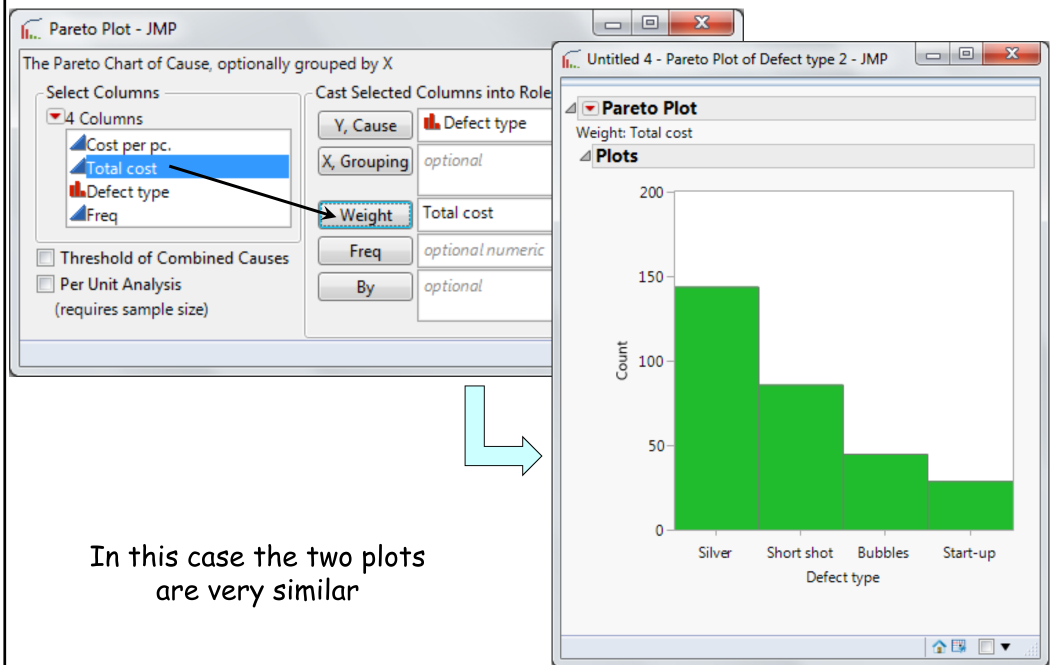
*Analyze* → *Quality and Process* → *Pareto Plot* → *set up as shown* → *OK*



## Pareto plot by total cost

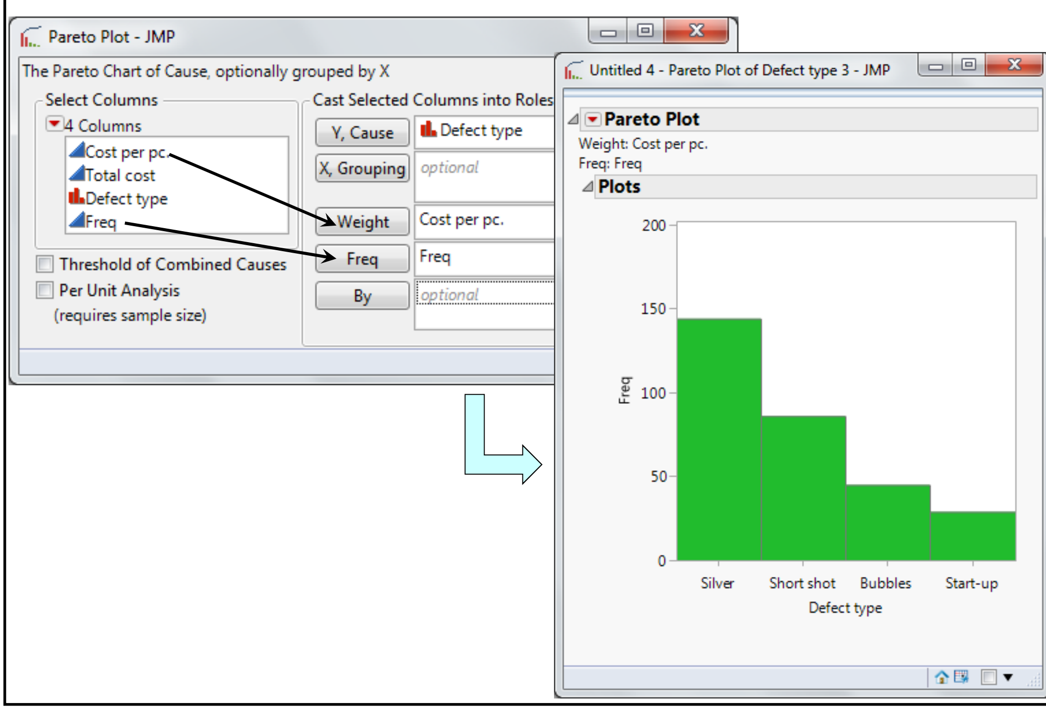
475




In this case the two plots are very similar

## Cost Pareto without calculating the total cost column

476



## Exercise 29.1



Open *molding process* - *Pareto* (in JMP). Use the method described in this section to reformat the file for Pareto analysis. Save the reformatted file as *molding process* - *stacked*. Create Pareto plots of defect types by frequency of occurrence and total cost.

Notes